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Probabilistic Flood Analysis of Indus River Flow

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Abstract. The assessment for long-term and extreme river flow events is necessary for water resource management and early preparation for any expected hazardous event in the country. This paper considers the probabilistic estimation of the annual maximum flow (AMF) series (1967 to 2015) of the Indus River, Pakistan at Kotri station. Four extreme value distributions Gumbel, Frechet, Weibull and Generalized Extreme Value (GEV) models were tested. Based on the goodness of fit tests of Kolmogorov Smirnov (KS) and Anderson Darling (AD), it was found that GEV is the most suitable distribution for modelling AMF series. The Bayesian and Maximum Likelihood Estimation (MLE) techniques have been used to estimate the parameters of GEV distribution and also compute their return levels. The relative absolute squared error (RASE), relative root mean squared error (RRMSE), maximum absolute error (MAE) and probability plot correlation coefficient (PPCC) were employed to evaluate the forecasting efficiency. Moreover, the forecast efficiency indicators show that the Bayesian technique is relatively better than MLE for observing the AMF in Indus River Pakistan. The improved results of this paper demonstrate that can be helpful for government in establishment of water resources management policies of the country. This is in order to utilise their efforts to reduce or control the risk of significant losses in future upcoming events.

Key Words: Annual maximum flows, extreme value distributions, return level

1. INTRODUCTION

The hydrological extremes are frequently described as processes rooted by hydro meteorological situations which fluctuate noticeably from typical weather conditions. Hydrological variations result in floods and droughts[14], such situations are temporal in character and are connected to occasional or periodic changes in geographic conditions [8]. Climate change and other global drivers put water resources under increasingly severe pressure. It alters soil moisture, glacier mass balance, humidity, and river flow and rainfall patterns. Simultaneously, hydrological extremes like droughts and floods are getting higher in frequency, severity and intensity, causing serious threats to welfare and human life because of their disastrous nature [22], [45] and [47]. Particularly flash floods provoked by heavy rainstorms which have produced extensive property damage as well as loss of human life [5], [28]. Humans are combating with extreme hydrological events and have not been quite victorious because these are natural phenomenon that will continue to occur. It is essential to learn to live with these extremes event and trying to improve in preparedness systems. The societal and economic consequences of flood events became more important recently [23]. Pakistan has five major rivers Indus, Jhelum, Chenab, Ravi and Sutlej [30]. Indus River originates from Tibetan Himalaya, having diverse geography with glaciers covering northern alpines flowing in a south westerly direction through southern plains adjoining the Arabian Sea [4]; [17]. The seasonal changes vastly affect its water volume; it gradually increases with the approaching summer season when the snow starts to melt in the mountainous areas of the river basin and gradually decreases in the winter season. The Indus River basin shows its highest water discharge during the well-marked monsoon season (July-September) [16]. There is regular annual riverine flooding in the low lying region, semi hilly and hilly areas during extreme monsoon rainfall [17]. These floods sometimes become one of the most devastating natural hazards [30].

Pakistan has faced severe historical floods during the last 66 years, affected 599,459 square kilometres of area, life losses of around 11,239, caused financial losses worth over 39 billion rupees and inundated 180,234 villages [27]. To reduce these damages, it is urged to assess historical events in detail and make optimal plans for the future flood disasters. Statistical analysis of their regional variation through extreme value distributions and the GEV distribution has extensive application for unfolding the appearance of the flood peaks (volume), wind speeds, rainfall, snow depths, wave heights, and other maxima. Applications of the extreme value distributions and chosen methods (Bayesian, MLE) have been investigated widely by [11], [15], [26], [29], [31], [34], [40], [46] and [3]. Consequently, this paper analyzes AMF of the Indus river at Kotri station from 1967 to 2015 (Fig. 1) to explore the best fit probability distribution.

Moreover, to explore the best fitted model for flood, employed family of extreme value distribution models, namely Gumbel, Frechet, Weibull and generalized extreme value (GEV). The parameters of each model estimated by Bayesian and maximum likelihood estimator (MLE). The best-fitted distribution are analysed by the Kolmogorov Smirnov (KS) and Anderson Darling (AD) [32] tests. Furthermore, the selected distribution may utilise them to estimate the AMF return levels for 5, 10, 25, 50, 75 and 100 years. The improved results of this paper demonstrate that can be helpful for government in establishment of water resources management policies of the country. This is in order to utilise their efforts to reduce or control the risk of significant losses in future upcoming events.

2. Material and Methods

2.1. Maximum Likelihood Estimation (MLE). The MLE method is a more consistent approach and generally it is less biased in parameter estimations. The simple idea behind this technique is to determine the distribution parameters that maximize the likelihood function of the given sample [42]. Let $X = (x_1, x_2, ..., x_n)$ denote independent annual maximum flow series having GEV probability density function[10], [12], [33].

$$g(X;\mu,\rho,\xi) = \begin{cases} \{\frac{1}{\rho}[1+\xi(\frac{x_i-\mu}{\rho})]^{-1-\frac{1}{\xi}} exp\{-(1+\xi\frac{x_i-\mu}{\rho})^{-\frac{1}{\xi}}\}\}, & for 1+\xi\frac{x_i-\mu}{\rho} > 0. \\ 0, & elsewhere. \end{cases} \tag{2. 1}$$

where μ is the location, ρ is the scale and ξ is the shape parameter. The likelihood function is the product of the probability density functions for sample of n observations $(x_1, x_2, ..., x_n)$ [10], [12], [31], [33] and [21]. It can be written as:

$$L(\mu,\rho,\xi,x_1,...,x_n) = \Pi_{i=1}^{n} g(x_i;\mu,\rho,\xi)$$
$$= \Pi_{i=1}^{n}\frac{1}{\rho}[1+\xi(\frac{x_i-\mu}{\rho}]^{-1-\frac{1}{\mu}} exp\{-(1+\mu\frac{x_i-\mu}{\rho})^{-\frac{1}{\mu}}\}$$
$$= \frac{1}{\rho^n}\Pi_{i=1}^{n}[1+\xi(\frac{x_i-\mu}{\rho}]^{-1-\frac{1}{\xi}} \times \exp\{-\sum_{i=1}^{n}(1+\xi\frac{x_i-\mu}{\rho})^{-\frac{1}{\xi}}\} \tag{2. 2}$$

Now this can written as corresponding log-likelihood function [33] and [9] as:

$$ln[L(\mu,\rho,\xi;x_1,...,x_n)] = -nln\rho - (1+\xi)\sum_{i=1}^{n} z_i - \sum_{i=1}^{n} e^{-z_i} \tag{2. 3}$$

where $z_i = \xi^{-1} ln(1+\xi\frac{x_i-\mu}{\rho})$

Actually, the logarithmic likelihood function is easier to maximize. The MLE estimators $\hat{\theta} = (\hat{\mu},\hat{\rho},\hat{\xi})$ [21] of parameters $\theta = (\mu,\rho,\xi)$ [10] are the solution of a system of equation produced by the first partial derivatives (with respect to each parameter) of the $ln[L(\mu,\rho,\xi;x_1,...,x_n)]$ function setting to zero [1], [21], [26].

2.2. Bayesian Analysis. The parameters of posterior distribution are obtained by Bayesian based Markov chain Monte Carlo (MCMC) technique from an arbitrary distribution turn into an increasingly popular [3], [6], [7], [10], [12], [13], [18], [25], [35], [36], [37], [43], [44], [48] of extremes values [6] and for forecasting of river Rhine [38].

Suppose that the data $X = (x_1, ..., x_n)$ are the realisations of the independent random variable whose density falls inside a parametric family $[g(X;\theta) : \theta(\mu,\rho,\xi) \in \Theta$ (parameter space)]. Then Bayesian model defined by a likelihood function $g(x|\theta)$ and a prior distribution $g(\theta)$, leading to the posterior distribution

$$g(\theta|x) = \frac{g(\theta)g(x|\theta)}{\int g(\theta)g(x|\theta)d\theta}$$

Here g denotes pdf (probability density function)and the likelihood $g(x|\theta)$ can be express for θ as:

$$L(\mu,\rho,\xi;x_1,...,x_n) = L(\theta;X) = g(x|\theta) = \Pi_{i=1}^{n} g(x_i;\mu,\rho,\xi)$$

So simply as;

$$g(\theta|x) = \frac{g(\theta)L(\theta;X)}{\int g(\theta)L(\theta;X)d\theta} \alpha g(\theta)L(\theta;X) \tag{2. 4}$$

The density of the posterior distribution is directly proportional to the product of the likelihood function and prior distribution. Here, when θ is a high dimensional vector then it can be problematic to compute the normalising constant $(\int g(\theta)L(\theta;X)d\theta)$. To avoid this problem the posterior distribution is determined by an MCMC simulation technique [10]. For simulation R-statistical software is used with extreme value modelling packages.

2.3. **Kolmogorov-Smirnov (KS) test.** The KS test is used, whether a sample comes from hypothesized continuous distribution based on the empirical cumulative distribution function (CDF) [2] & [24]. Suppose that a random sample, $x_1, ..., x_n$ from some distribution with the CDF is, $G(x_i)$. The empirical CDF is $G_n(x) = 0, i/N_0, 1$ for $x < x_1, x_i \leq x$ and $x \leq x_i + 1$, $x \geq x_n$ respectively, where N_0 is the number of observations. The KS test statistic D is defined as:

$$D = max_{1\leq i\leq N_0}[G(x_i) - \frac{i-1}{N_0}, \frac{i}{N_0} - G(x_i)] \tag{2. 5}$$

If $D > D_n^{\alpha}$ (where α is the significance level of critical value), in that case the null hypothesis H_0: the data follow a particular distribution is rejected [2], [24], [41].

2.4. **Anderson-Darling (AD) test.** The AD test is considered to be a refinement of the KS test and also to be comparatively more powerful[2]. It is used to find out if a known sample belongs to a particular probability distribution[39]. The statistic A^2 in the AD test is defined as:

$$A^2 = -N_0 - \sum_{i=1}^{n} \frac{(2i-1)\{lnG(x_i) + ln[1 - G(x_{x_{n-i+1}})]\}}{N_0} \tag{2. 6}$$

where x_i are an increasing order observation. If $A^2 > A_{\alpha}^2$ the hypothesis concerning the distributional form (H_0) is rejected [2], [24].

2.5. **Estimation of probabilistic model.** The best fit probability distribution was evaluated by using the following systematic sub sections.

2.5.1. *Fitting of probability distribution.* This paper is an effort to explore the best fitting distribution that explains the maximum river flow data series (1967 to 2015) of the Indus River at Kotri station. In a univariate process, the distribution of the extreme behaviour can be widely defined by the three types of families: Gumbel (type I), Frechet (type II) and Weibull (type III) [6], [42]. The combination of these three types of distributions into one family is referred to as the GEV distribution [6]. In this paper, the mentioned four extreme value distribution models are candidate distributions, in which a more efficient model that suitably capture the uncertainties pattern of data. The CDF of the above distributions is as follows [10], [20], [42]:

Gumbel Distribution (or Type I):

$$G(X, \mu, \rho, \xi) = exp\{-exp[-(\frac{X-\mu}{\rho})]\}, -\infty < X > \infty \tag{2. 7}$$

Frechet Distribution (or Type II):

$$G(X, \mu, \rho, \xi) = \begin{cases} 0, & x \leq \mu. \\ exp\{-(\frac{X-\mu}{\rho})^{-\xi}\}, & X > \mu. \end{cases} \tag{2. 8}$$

Weibull Distribution (or Type III):

$$G(X, \mu, \rho, \xi) = \begin{cases} exp\{-[-(\frac{X-\mu}{\rho})^{\xi}]\}, & X < \mu. \\ 1, & X \geq \mu. \end{cases} \tag{2. 9}$$

Generalized Extreme Value (GEV) Distribution:

$$G(X,\mu,\rho,\xi) = exp\{-(1+\xi\frac{X-\mu}{\rho})^{\frac{-1}{\xi}}\}, 1+\xi\frac{X-\mu}{\rho} > 0; \quad (2.\ 10)$$

Where $\mu \in \Re$, $\rho > 0$ and $\xi \in \Re$

2.5.2. *Goodness of fit Test.* The two tests KS [2], [24] and AD [42] are executed for evaluating the best fitted probability distribution[19] model among the candidate distribution. Furthermore, Q-Q (Quantile-Quantile) plot and probability difference graph are also used as a visual analysis to select the best fitted model.

2.5.3. *Selection of the best distribution.* Table 1 presented descriptive statistics of maximum flow data which revealed the range ($935200.0m^3/sec$), right-skewed and negative value of excess kurtosis indicate a platykurtic distribution. The test statistic D and A^2 of each distribution (Table 2) shows GEV has lowest statistic values. This means that it is best fitted distribution among the other candidate distributions. Moreover, the PDF curve (Fig. 2) mostly cover the observed distribution, the Q-Q plots (Fig. 3) mainly occupied the reference line, and probability difference chart (Fig.4) shows the GEV distribution belongs to innermost Quintiles very closedwith the horizontal reference line. These analyses are supported GEV to most appropriate to observed distribution.

2.5.4. *Parameter estimation of selected distribution.* The Bayesian and MLE techniques have been implemented for parameters estimation of GEV distribution and also compute return levels. The test statistics like Relative Absolute Squared Error (RASE), relative root mean squared error (RRMSE), maximum absolute error (MAE) and probability plot correlation coefficient (PPCC) are employed to evaluate the forecasting performance and accuracy [11], [46]. These test statistics are written as follows :

$$RASE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\frac{x_i - q_i}{x_i})^2} \quad (2.\ 11)$$

$$RRMSE = \frac{1}{n}\sum_{i=1}^{n}|\frac{x_i - q_i}{x_i}| \quad (2.\ 12)$$

$$MAE = max|x_i - q_i| \quad (2.\ 13)$$

$$PPCC = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(q_i-\bar{q})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(q_i-\bar{q})^2}} \quad (2.\ 14)$$

where x_i and q_i is the ith order observed value, estimated value and the $\bar{x}$ and $\bar{q}$ are their mean values. Now the annual maximum flow data from1967-2005 is used for parameter estimation, and remaining (2006-2015) is used for the comparison of forecasts.

2.5.5. *Return level.* When the best significant model for the data series has been established, then estimate the R year return levels (Y_R) of maximum flow. The return level estimated by inversing the cumulative distribution function $G(X,\mu,\rho,\xi)$ $(Eq.2.10)$ of GEV distribution [31], [33], [42] and [24] and written as:

$$Y_R = \mu - \frac{\rho}{\xi}[1 - log(1-\frac{1}{R})^{-\xi}] \quad (2.\ 15)$$

3. RESULT AND DISCUSSION

The Table 2 shows that the GEV is the best fitted distributional model,criteria based on the least values of KS and AD statistic. The MCMC technique is utilized for simulation of parameter value of GEV distribution. It is found that all value have been found together in an alike zone (Fig. 5). It is also observed (Fig. 5) that the posterior density and trace plots for location, scale and shape parameters are symmetrical. The parametric values determined by MLE and Bayesian methods (Table 3) and are used for forecasting and return level evaluation with the help of computer software R-package. For this purpose first forecast the return levels from 2006 to 2015 and compare with observed values of annual maximum flow (Fig. 6). The comparative analysis of forecasting efficiency indicator (Table 3) shows that both methods have no big differences in estimation and are appropriate to forecast. However, Bayesian has smaller values of RASE, RRMSE and MAE than MLE shows slightly improved results. Accordingly the forecast result for different return levels for 5, 10, 25, 50, 75 and 100 years (Table 4 and Fig. 7) depict that annual maximum flow is consistently increasing over the 100 years using Bayesian method. The Fig. 7 shows that after 2015 the return level of 5 years, approaches 1983, 1986, 1975, 1967; 10 years approaches 2015, 1988, 1992; 25 years approaches 1978, 1976, 1995, 1973: 50 years approaches 1994, and 75 & 100 years approach 2010. Moreover, the return level for 100 years $(956710m^3/sec)$ gets exceeded maximum $(939442m^3/sec; 2010)$ of observed maximum flow. The Fig. 7 also reveals about return levels by the MLE method after 2015, for 5 years approaches 1969, 1968; 10 years approach 2015, 1988, 1992; 25 years approach 1994; for 50, 75 & 100 years the forecast exceeded the maximum $(939442m^3/sec; 2010)$ of observed maximum flow.

4. CONCLUSION

Annual maximum flow series from 1967 to 2015 of the Indus River at Kotri station were utilized to analyse for the best fitted probability distribution. Based on the goodness of fit tests (KS and AD) it is found that the GEV distribution being the most suitable distribution for modelling the maximum flow series at Kotri station. Moreover, the estimation of the flood return levels against return periods through maximum likelihood method and Bayesian technique shows that Bayesian MCMC is better than the MLE method. So it is concluded that Bayesian technique can explain more accurately the annual maximum flow behaviour of selected station. These predicted return levels provide valuable information to the concerned departments in planning and management of the water resources, especially for proper river structures, drainage systems and reservoirs. The results of this study can be used to facilitate the related parties and government in prioritizing water resources in their efforts to reduce or control the risk of large losses.

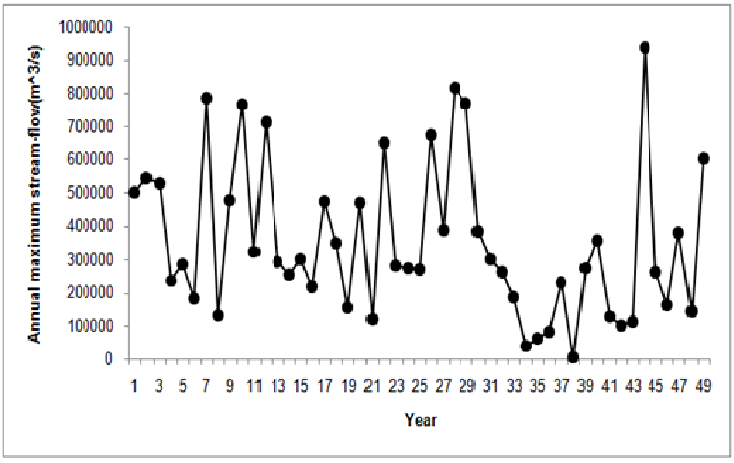

FIGURE 1. Fluctuations in observed Annual maximum flows (1967-2015).

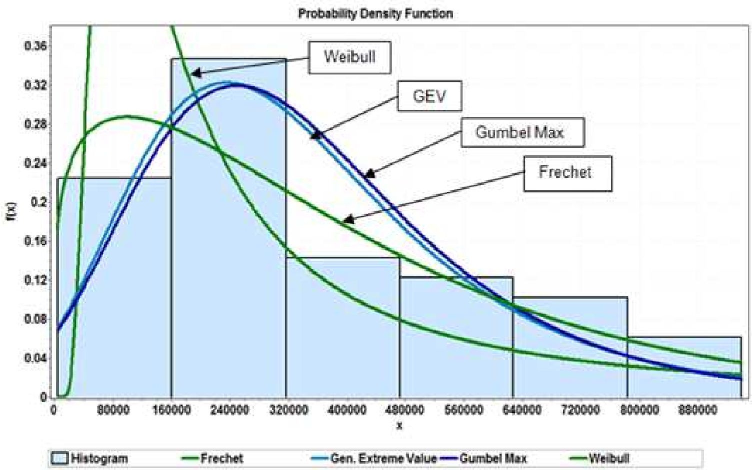

FIGURE 2. Probability Density Function (PDF) Plot of candidate distributions and observed histogram.

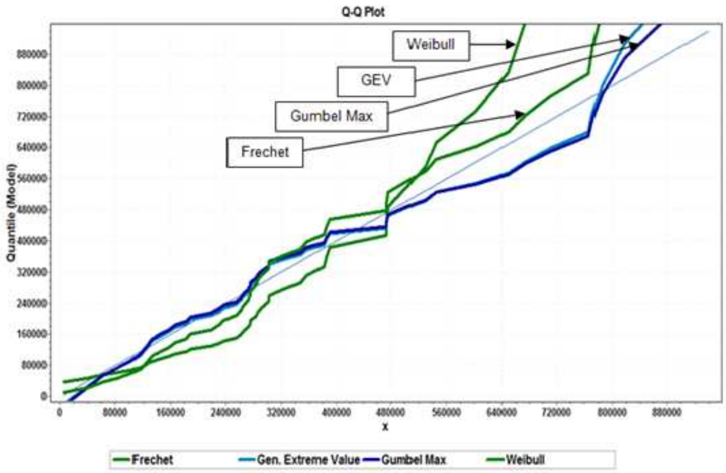

FIGURE 3. Q-Q Plot of candidate distributions.

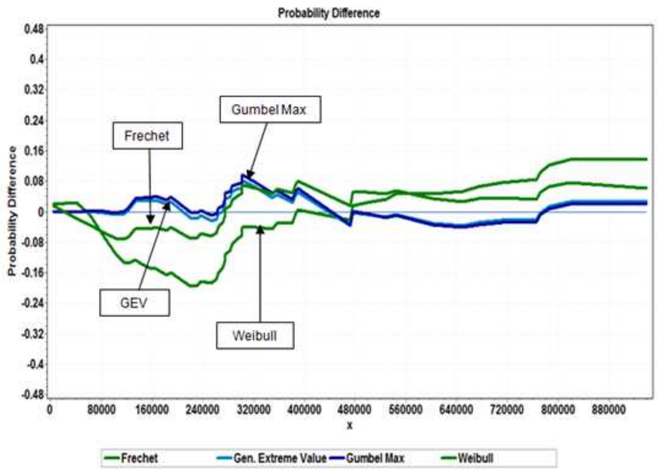

FIGURE 4. Probability difference chart for the fitted candidate distributions.

TABLE 1. Descriptive statistics of Annual maximum flows (m^3/sec).

Statistic	**Value**
Sample Size	49
Range	9.3520E+5
Mean	3.5276E+5
Variance	5.2923E+10
Std. Deviation	2.3005E+5
Coef. of Variation	0.65213
Std. Error	32864.0
Skewness	0.79726
Excess Kurtosis	-0.15486

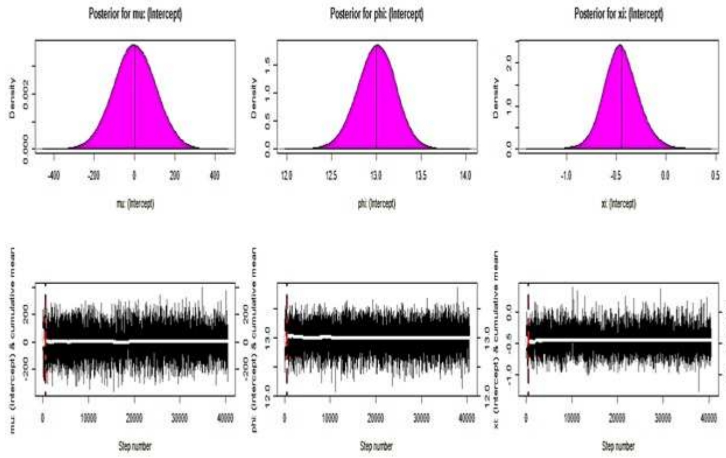

FIGURE 5. Posterior density plots and Trace plots for location, scale and shape parameters of GEV distribution.

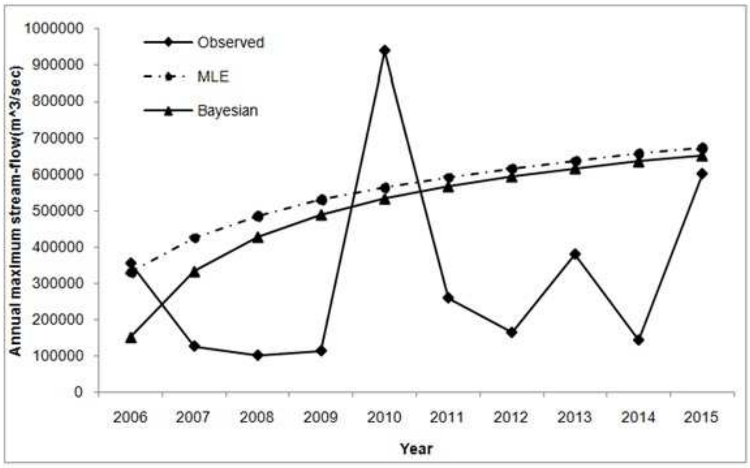

FIGURE 6. Comparison of forecasted values with observed values of Annual maximum flow (2006-2015).

TABLE 2. Goodness of fit test(AD & KS) results for the fitted models, critical Value at 5% level of significant.

Distribution	Kolmogorov Smirnov(KS)		Anderson Darling(AD)	
	Critical Value	Statistic	Critical Value	Statistic
Gumbel Max	0.19028	0.09642	2.5018	0.35163
Frechet	0.19028	0.21622	2.5018	4.14
Weibull	0.19028	0.09235	2.5018	0.85 609
GEV	0.19028	0.08251	2.5018	0.27801

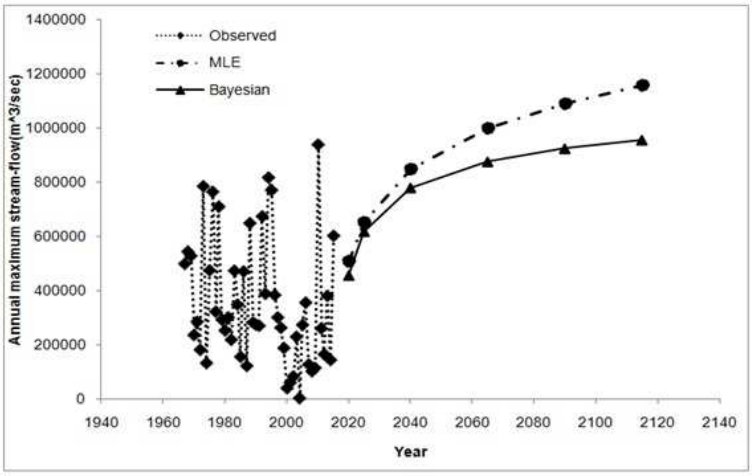

FIGURE 7. Analysis of observed Annual maximum flow (1967-2015) with Return levels (5, 10, 25, 50, 75 and 100 years) of MLE and Bayesian methods.

TABLE 3. Parameters of fitted Extreme Value Distribution(GEV)and Forecasting efficiency indicators.

Method	**GEV Parameters**			**Forecast Errors (2006-2015)**			
	Location (μ)	Shape (ξ)	Scale (ρ)	RASE	RRMSE	MAE	PPCC
MLE	264000	-0.0428	12.1	1.8370	2.3230	511075	0.193
Bayesian	-2.4161	-0.4502	13.0026	1.6759	2.0743	488681	0.172

TABLE 4. Return level of Annual maximum flows (m^3/sec) from 2015 onwards.

Year	**P=5**	**P=10**	**P=25**	**P=50**	**P=75**	**P=100**
MLE	510537	655309	849090	1000000	1092878	1159376
Bayesian	454454	616901	779326	876116	924967	956710

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