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A multi-parametric approach to solve flexible fuzzy multi-choice goal programming

Ghorbanali Ramzannia-Keshteli*
Department of Mathematics,
Farhangian University, Tehran, Iran.
Email: ramzannia52@gmail.com, ramzannia52@yahoo.com

Seyed Hadi Nasseri
Department of Mathematics,
University of Mazandaran, Iran.
Email: nasseri@umz.ac.ir

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Abstract. Goal Programming (GP) is one of the most important techniques to solve Multiple Objective Programming (MOP) problem, which for each target an aspirational level is considered by the decision maker. If several aspiration level are considered for each target, then there will be the issue of Multi- Choice Goal Programming (MCGP) problem. Recently a fuzzy version of the MCGP problem is proposed by some researchers. In this paper, we are going to extend the current model to the general form, where a flexible assumption is added to the constraints and the goals which is named as Flexible Fuzzy Multi-Choice Goal Programming (FFMCGP) Problem. In this way, we present a new method to solve FFMCGP model using linear multi-parametric programming while the minimum degree membership for constraints and goals are considered by decision maker. For further understanding, we present a numerical example to illustrate the proposed approach.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Multi-choice goal programming, Multi-parametric linear programming, Flexible fuzzy, Aspiration level.

1. Introduction

In the literature of this subject it is observed that if some activities are dependent on participatory use of limited resources, the study of resource allocation and Then determination of the amount of activities are proposed such problems can be modeled mathematical programming. If in the corresponding mathematical model all mathematical relations are linear functions, the corresponding programming is called linear programming. Linear programming models are mainly based on a framework in which the objectives are summarized in the form of some major objective, such as maximizing profits or minimizing total costs. Multi Objective Decision Making (MODM) aims to resolve optimal design problems in which several (conflicting) objectives must be achieved simultaneously. Resently, application of the classical LP and MODM problems in fuzzy environment, have been used in the generalized models such as [24, 28, 27] and etc. In this way, Ramzannia and Nasseri [13] proposed the flexibility concept for obtaining a new method for solving these problems. The Goal Programming (GP) is One of these effective techniques for decision makers (DMs) to solve (MODM) problems in finding a set of satisfying solutions. This kind of programming was first proposed by Charnce and Cooper [11], and by Lee [16], Ignazio [14], Tamiz et al. [29], Romero [25], and others [17, 10, 21]. In the goal programming model (GP), we minimize the deviation between achievement of goal and their levels of aspreation. In this way, the decision maker takes into account the amount of the expectancy amount of each and decides on how to minimize the positive deviation of the goal or the negative deviation of the target. Often, in real world affairs, the decision makers are not able to consider a crisp amount of aspirational level for goal programming purpose in allocating it (the level of a wish). For these reasons, Charles and Columbus [12] proposed an ideal ballistic programming approach developed by Jones and Tamiz [15], Vittoria et al. [31] and [9]. In some conditions, targets may have ambiguous aspiration levels, in this case fuzzy goal programming is proposed [9, 32]. Also, if we encounter issues where the goals have multiple levels of aspiration, we will have a multi-choice goal programming which was first introduced by Chang.T.C., [7]. In the this presented model, several discrete aspiration levels are considered for targets. A year later, change proposed a revised multi-choice goal programming model for goal programming problems with continuous aspiration level [8]. Valuable studies have been conducted in the theoretical and applied field, such as the transformation of multi-choice goal programming into binary goal programming problems [1], multi-segment goal programming [18], solving multi-choice goal programming by use of interpolation polynomials [2], multi choice goal programming with utilized functions [4],multi-coefficient GP in constraint sets [5], revised multi-segment goal programming [6] and etc., [30, 19, 22, 23, 3] The first research on fuzzy multi-choice goal programming was presented by Bankers Tabrizi etc. al. [26] who considered the multiple aspiration levels of goals programming as triangular fuzzy numbers and solved using the Zimmerman [32] and by Chang.T.C., [7] Another research has been done by Neha Gupta and Abdul Bari, Was in the context of multi-choice goal programming with trapezoidal fuzzy numbers which by use of ranking function, the fuzzy model changed into a crisp model became a definitive model using the fuzzy model ranking function [20]. As we know, there is not any serious work on Multi-Choice Goal Programming with Flexibility in Fuzzy Goal and Constraints (MCGPFFGC). One of our main contribution here is extending the current models to the above mentioned MCGPFFGC problem. The second achievement here is using the various cuts for goals and constraints. This approach leads us to a Multi Parametric Multi-Choice Goal Programming(MPMCGP) model in which eligible the decision maker to analyze of the data in goal and constraint separately. These advantages are so important in the practical situation too. In this paper, a new model of flexible fuzzy multi-choice GP is considered which goals and constraint are of flexible fuzzy type and in order to the solve of this model a new method is presented to solve problem using linear multi-parametric Programming while the minimum degree membership for constraint and goal are considered by decision maker. The rest of this paper is organized as follows: in section 2, the studying some models of multi-choice goal program-ming are presented; In section 3, 4 and 5 we presented a general model of the multi choice GP problem with flexible fuzzy goals and constraints and how to convert them in a crisp form and how to solve them; section 6 and 7 presents a numerical illustration and sensitivity analysis. Finally, section 8 consists of conclusions.

2. Studying some models of multi-choice goal programming

2.1. The general form of MCGP

The general form of MCGP is formulated as follows:

$$\begin{aligned} & G_k(x) \begin{smallmatrix}\geq\\=\\\leq\end{smallmatrix} (g_{k1}\, or\, g_{k2}\, or \ldots or\, g_{kT}),\ k = 1, \ldots, K \\ & s.t. \qquad AX = b,\ X \geq 0. \end{aligned} \tag{2. 1}$$

Which several definite Aspiration Levels(AL) are considered for $k-th$ goal, such as $g_{k1}, \ldots, g_{kt}$. We want to minimize deviations in $k-th$ goals.

$$\begin{aligned} \min \quad & \sum_{k=1}^{K} |G_k(x) - (g_{k1}\, or\, g_{k2}\, or \ldots or\, g_{kT})| \\ s.t. \quad & AX = b,\ A = [a_{ij}]_{m\times n},\ i = 1, \ldots, m, \\ & X = (x_1, \ldots, x_n)^T,\ b = (b_1, \ldots, b_n)^T, \\ & x_j \geq 0,\ j = 1, \ldots, n. \end{aligned} \tag{2. 2}$$

Since there are several AL for each goal the above model will be changed as following:

$$\min \quad \sum_{k=1}^{K} w_k(d_k^- + d_k^+) \tag{2. 3}$$

$$\begin{aligned} s.t. \quad & G_k(X) - d_k^+ + d_k^- = \sum_{t=1}^{T} g_{kt}\, S_{kt}(bi), \\ & AX = b,\ A = [a_{ij}]_{m\times n},\ i = 1, \ldots, m, \\ & X = (x_1, \ldots, x_n)^T,\ b = (b_1, \ldots, b_m)^T, \\ & S_{kt}(bi) \in R_k(x), \\ & d_k^-, d_k^+ \geq 0,\ x_j \geq 0,\ k = 1, \ldots, K,\ j = 1, \ldots, n. \end{aligned}$$

Which $S_{kt}(bi)$ represents multicative terms of binary variable that determine which AL should be selected for $k-th$ goal; $R_k(t)$ is the function of resources limitations.

2.2. Revised multi choice goal programming

The early model of MCGP, there were several choices for goal AL, and multiplicative terms were from binary variable which this resulted in some problems in implementation and also difficulty in understanding for industrial participants, in order to resolve these Difficulties introduced the Revised Multi-Choice Goal Programming (RMCGP) method is introduced.

The advantage of this RMCGP is that it does not involve multiplicative terms of binary variables, for solving problems and RMCGP is easily applicable by softwares.

Remark 2.1. *If we are going to choose the most minimized AL through considered aspiration levels the RMCGP will be as follows:*

$$\begin{aligned}
\min \quad & \sum_{k=1}^{K}[w_k(d_k^- + d_k^+) + \alpha_k(e_k^- + e_k^+)] \\
s.t. \quad & G_k(X) - d_k^+ + d_k^- = y_k,\ k = 1, \ldots, K, && (2.\ 4) \\
& g_{k,\min} \leq y_k \leq g_{k,\max},\ y_k - e_k^+ + e_k^- = g_{k,\min}, && (2.\ 5) \\
& AX = b,\ A = [a_{ij}]_{m\times n},\ X = (x_1, \ldots, x_n)^T, \\
& b = (b_1, \ldots, b_m)^T,\ d_k^-, d_k^+ \geq 0,\ X \geq 0,\ k = 1, \ldots, K.
\end{aligned}$$

Remark 2.2. *If we are going to choose the most maximum AL through considered aspiration levels, the RMCGP will be as follows:*

$$\begin{aligned}
\min \quad & \sum_{k=1}^{K}[w_k(d_k^- + d_k^+) + \alpha_k(e_k^- + e_k^+)] \\
s.t. \quad & G_k(X) - d_k^+ + d_k^- = y_k,\ k = 1, \ldots, K, && (2.\ 6) \\
& g_{k,\min} \leq y_k \leq g_{k,\max},\ y_k - e_k^+ + e_k^- = g_{k,\min}, && (2.\ 7) \\
& AX = b,\ A = [a_{ij}]_{m\times n},\ i = 1, \ldots, m,\ X = (x_1, \ldots, x_n)^T, \\
& b = (b_1, \ldots, b_m)^T,\ e_k^+, e_k^-, d_k^-, d_k^+ \geq 0.
\end{aligned}$$

Notation 2.3. *In all the above recently explained models, usually goals are considered independent of the other goals, but in real decision making situation, the goals are interrelated so in such problems we use the Constrained RMCGP (CRMCGP).*

Remark 2.4.

$$\begin{aligned}
\min \quad & \sum_{k=1}^{K}[w_k(d_k^- + d_k^+) + \alpha_k(e_k^- + e_k^+)] \\
s.t. \quad & (G_k(X))(bi)_k - d_k^+ + d_k^- = (bi)_k y_k, && (2.\ 8) \\
& y_k - e_k^+ + e_k^- = g_{k,\min},\ g_{k,\min} \leq y_k \leq g_{k,\max}, \\
& AX = b, \\
& e_k^+, e_k^-, d_k^-, d_k^+ \geq 0,\ (bi)_k \in \{0, 1\},\ k = 1, \ldots, K.
\end{aligned}$$

Remark 2.5. *The CMCGP model when the most minimum AL is intended, as follow:*

$$\begin{aligned}
\min \quad & \sum_{k=1}^{K}[w_k(d_k^- + d_k^+) + \alpha_k(e_k^- + e_k^+)] \\
s.t. \quad & (G_k(X))(bi)_k - d_k^+ + d_k^- = (bi)_k y_k, && (2.\ 9) \\
& y_k - e_k^+ + e_k^- = g_{k,\max},\ g_{k,\min} \leq y_k \leq g_{k,\max}, \\
& AX = b,\ A = [a_{ij}]_{m\times n},\ i = 1, \ldots, m,\ b = (b_1, \ldots, b_m),
\end{aligned}$$

$$X = (x_1, x_2, \ldots, x_n),\ x_j, e_k^+, e_k^-, d_k^-, d_k^+ \geq 0,\ k = 1, \ldots, K,$$
$$j = 1, \ldots, n,\ (bi)_k \in \{0, 1\}.$$

2.3. Fuzzy multi-choice goal programming

It is possible to encounter several imprecise or fuzzy aspiration levels for goals, or imprecise or fuzzy variable, or imprecise or fuzzy coefficients in the MCGP problems, in these cases the fuzzy MCGP is proposed which can be classified as follows:

Case1. One group of such programming includes models with fuzzy aspiration levels which are proposed by Behzad Bankian-Tabrizi and etc.al.

Generally, they formulated FMCGP problems as follows:

$$\begin{aligned} \min \quad & \sum_{k=1}^{K} w_k |G_k(X) - (\tilde{G}_{k1}\, or\, \tilde{G}_{k2}\, or \ldots or\, \tilde{G}_{kT})| \\ s.t. \quad & AX = b,\ A = [a_{ij}]_{m\times n},\ i = 1, \ldots, m,\ b = (b_1, \ldots, b_n)^T, \\ & X = (x_1, \ldots, x_n)^T,\ x_j \leq 0,\ j = 1, \ldots, n,\ k = 1, \ldots, K, \end{aligned} \tag{2. 10}$$

Using the Zimmerma's methods [32], the above problem becomes the following:

$$Maximize \quad f(\mu) = \sum_{k=1}^{K} w_k \mu_k \tag{2. 11}$$
$$\begin{aligned} s.t. \quad & \mu_k \leq 1 - \sum_{t=1}^{T} \left(\frac{G_k(x) - \tilde{G}_{kt}}{P_{kt}^+}\right) S_{kt}(bi),\ k = 1, \ldots, K, \\ & \mu_k \leq 1 - \sum_{t=1}^{T} \left(\frac{\tilde{G}_{kt} - G_k(x)}{P_{kt}^-}\right) S_{kt}(bi),\ k = 1, \ldots, K, \\ & AX = b,\ A = [a_{ij}]_{m\times n},\ i = 1, \ldots, m,\ X = (x_1, \ldots, x_n)^T, \\ & b = (b_1, \ldots, b_n)^T,\ j = 1, \ldots, n,\ k = 1, \ldots, K,\ \mu_k \geq 0,\ t = 1, \ldots, T. \end{aligned}$$

Which $S_{kt}(bi)$ represents multicative terms of binary variable that determine which AL should be selected for $k-th$ goal, also respectively will be about p_{kt}^+ bigger than form and will be about p_{kt}^- smaller than form $\tilde{G}_{kt}$.

Case2. : In some models of MCGP, variable and their coefficient are fuzzy in goals, which the above model will be as follows:

$$\begin{aligned} & \tilde{G}_k(\tilde{Y}) = (g_{k1}\, or\, g_{k2}\, or \ldots or\, g_{kT}), \\ s.t. \quad & A\tilde{Y} = b,\ A = [a_{ij}]_{m\times n},\ i = 1, \ldots, m, \\ & \tilde{Y} = (\tilde{y}_1, \ldots, \tilde{y}_n)^T,\ b = (b_1, \ldots, b_n)^T, \\ & j = 1, \ldots, n,\ k = 1, \ldots, K. \end{aligned} \tag{2. 12}$$

By using the ranking function and the concept of MCGP, will be converted to the following problem and resolved.

$$\min \quad \sum_{k=1}^{K} w_k (d_k^- + d_k^+) \tag{2. 13}$$

$$s.t. \quad \Re(\tilde{G}_k(\tilde{Y})) - d_k^+ + d_k^- = \sum_{t=1}^{T} g_{kt} S_{kt}(bi),$$
$$A\Re(\tilde{Y}) = b,\ A = [a_{ij}]_{m\times n},\ i = 1,\dots,m,$$
$$\tilde{Y} = (\tilde{y}_1,\dots,\tilde{y}_n)^T,\ b = (b_1,\dots,b_m)^T,$$
$$\Re(\tilde{Y}) = (\Re(\tilde{y}_1),\dots,\Re(\tilde{y}_n))^T,\ \Re(\tilde{y}_j) \geq 0,$$
$$d_k^-, d_k^+ \geq 0,\ j = 1,\dots,n,\ k = 1,\dots,K.$$

3. Multi-Choice Goal Programming with Flexible Fuzzy Goal and Constraints(MCGPFFGC) formulation

A type of fuzzy MCGP problems is a flexible fuzzy MCGP, which goals and constraints are of flexible fuzzy type. The general model of MCGPFFGC problems will be as follows:

$$G_k(X) \prec_{ff}\ g_{k1} or \dots or\, g_{ks_k},\ k = 1,\dots,K, \tag{3. 14}$$
$$G_v(X) \succ_{ff}\ g_{v1}\, or \dots or\, g_{vs_v},\ v = K+1,\dots,V, \tag{3. 15}$$
$$s.t. \quad \sum_{j=1}^{n} a_{l_1 j}\, x_j \prec_{ff}\ b_{l_1},\ l_1 = 1,\dots,r, \tag{3. 16}$$
$$\sum_{j=1}^{n} a_{l_2 j}\, x_j \succ_{ff}\ b_{l_2},\ l_2 = r+1,\dots,m, \tag{3. 17}$$
$$\sum_{j=1}^{n} e_{wj}\, x_j \leq or \geq o_w,\ w = 1,\dots,W, \tag{3. 18}$$
$$X = (x_1,\dots,x_n)^T,\ x_j \geq 0,\ j = 1,\dots,n. \tag{3. 19}$$

Remark 3.1. *The indices and the variable of the Multi-Choice Goal Programming (MCGP) model with flexible fuzzy goals and constraints are defined in Table 1.*

It is notable that the types of goals and constraints will be as follows:

3.1. Goals

The first type: the relation (3. 14) $G_k(X)$ are first type flexible fuzzy goals $(k = 1,\dots,K)$, g_{kh} $(h = 1,\dots,s_k)$, are the multi aspreation level of the $k-th$ goal.
The second type: the relation (3. 15) $G_v(X)$ are second type flexible fuzzy goals $(v = K+1,\dots,V)$, g_{vu} $(u = 1,\dots,s_v)$, are the multi aspreation level of the $v-th$ goal.

3.2. Constraints

The first type contains flexible fuzzy constraints (3. 16) of type $(\prec_{ff})$.
The second type includes flexible fuzzy constraints (3. 17) of type $(\succ_{ff})$.
The third type contains crisp constraints includes the crisp constraints of the problem. The fourth constraints are the corresponding of the non-negative decision variables. In order to transfer the above problem into a crisp problem, it is necessary to form the flexible fuzzy

TABLE 1. The indices and the variable 0f the (MCGP).

$G_k(X)$	The first type flexible fuzzy goals function of type ($\prec_{ff}$), ($k = 1, \ldots, K$).
g_{kh}	The multi aspreation levels of the k-th goal, ($k = 1, \ldots, K$), ($h = 1, \ldots, s_k$).
$G_v(X)$	The second type flexible fuzzy goals function of type ($\succ_{ff}$), ($v = K+1, \ldots, V$).
g_{vu}	The multi aspreation levels of the v-th goal, ($v = K+1, \ldots, V$), ($u = 1, \ldots, s_v$).
$\sum_{j=1}^{n} a_{l_1 j}\, x_j \prec_{ff} b_{l_1}$	The first type contains flexible fuzzy constraints, of type ($\prec_{ff}$), $l_1 = 1, \ldots, r$.
$\sum_{j=1}^{n} a_{l_2 j}\, x_j \succ_{ff} b_{l_2}$	The second type includes flexible fuzzy constraints of type ($\succ_{ff}$), $l_2 = r+1, \ldots, m$.
$\sum_{j=1}^{n} e_{wj}\, x_j \leq or \geq o_w$	The third type contains crisp constraints includes the crisp constraints of the problem, $w = 1, \ldots, W$.
$X = (x_1, \ldots, x_n)^T$	The decision variable vector.

goals and constraints membership function and recognize the minimum membership degree.

4. TRANSFER OF MCGPFFGC INTO A CRISP PROBLEM

4.1. Membership function of flexible fuzzy constrain

If $P_{l_1}(l_1 = 1, \ldots, r)$, $P_{l_2}(l_2 = r+1, \ldots, m)$ are respectively the amount tolerance (maximum allowable positive and negative deviation) of the right hand side of the first and second constraints, then constraints membership function will be in the following form:

$$\mu\Big(\sum_{j=1}^{n} a_{l_1 j}\, x_j \prec_{ff} b_{l_1}\Big) = \frac{b_{l_1} - \sum_{j=1}^{n} a_{l_1 j}\, x_j}{P_{l_1}} + 1, \; l_1 = 1, \ldots, r, \tag{4. 20}$$

$$\mu\Big(\sum_{j=1}^{n} a_{l_2 j}\, x_j \succ_{ff} b_{l_2}\Big) = \frac{\sum_{j=1}^{n} a_{l_2 j}\, x_j - b_{l_2}}{P_{l_2}} + 1, \; l_2 = r+1, \ldots, m. \tag{4. 21}$$

4.2. Transfer of flexible fuzzy constraint to crisp constraint

If $\alpha_{l_1}(l_1 = 1, \ldots, r)$, $\alpha_{l_2}(l_2 = r+1, \ldots, m)$ are respectively the minimum amount of membership degree satisfaction of the first and second goals, then we will have According to the relation of (4. 20), (4. 21) flexible fuzzy constraint (3. 16), (3. 17) will transfer into crisp constraint in the following way.

$$\sum_{j=1}^{n} a_{l_1 j}\, x_j \leq b_{l_1} + P_{l_1}(1-\alpha_{l_1}),\ l_1 = 1,\ldots,r, \tag{4. 22}$$

$$\sum_{j=1}^{n} a_{l_2 j}\, x_j \geq b_{l_2} - P_{l_2}(1-\alpha_{l_2}),\ l_2 = r+1,\ldots,m, \tag{4. 23}$$

$$0 \leq \alpha_{l_1} \leq 1,\ 0 \leq \alpha_{l_2} \leq 1.$$

4.3. Membership function of flexible fuzzy goals

The goals of membership function will be as follows:

$$\mu\big(G_k(X) \prec_{ff}\ g_{k1},\ldots,g_{ks_k}\big) = \frac{g_{k1}-G_k(X)}{P_{k1}} + 1\ or \ldots or\ \frac{g_{ks_k}-G_k(X)}{P_{ks_k}} + 1,$$
$$k = 1,\ldots,K, \tag{4. 24}$$

$$\mu\big(G_v(X) \succ_{ff}\ g_{v1},\ldots,g_{vs_k}\big) = \frac{G_v(X)-g_{v1}}{P_{v1}} + 1\ or \ldots or\ \frac{G_v(X)-g_{vs_v}}{P_{vs_v}} + 1,$$
$$v = K+1,\ldots,V. \tag{4. 25}$$

Where $P_{kh}(k = 1,\ldots,K),(h = 1,\ldots,s_k)$, is the amount tolerance(maximum allowable positive deviation) first type goal from the aspreation level g_{kh}, and $P_{vu}(v = K + 1,\ldots,V),(1,\ldots,s_v)$, is the amount tolerance (maximum allowable negative deviation) second type goal from the aspreation level g_{vu}.

4.4. Transfer of flexible fuzzy goal to crisp goal

If $\lambda_k,\ \lambda v$,are respectively the minimum amount of membership degree satisfaction of the first and second constraints, then we will have:

$$\mu\big(G_k(X) \prec_{ff}\ g_{k1},\ldots,g_{ks_k}\big) = \frac{g_{k1}-G_k(X)}{P_{k1}} + 1 \geq \lambda_k\ or \ldots or\ \frac{g_{ks_k}-G_k(X)}{P_{ks_k}} + 1$$
$$\geq \lambda_k,\ k = 1,\ldots,K, \tag{4. 26}$$

$$\mu\big(G_v(X) \succ_{ff}\ g_{v1},\ldots,g_{vs_k}\big) = \frac{G_v(X)-g_{v1}}{P_{v1}} + 1 \geq \lambda_v\ or \ldots or\ \frac{G_v(X)-g_{vs_v}}{P_{vs_v}} + 1$$
$$\geq \lambda_v,\ v = K+1,\ldots,V. \tag{4. 27}$$

According to the relation of (4. 24), (4. 25) flexible fuzzy goal (3. 14), (3. 15) will transfer into crisp goal in the following way:

$$G_k(X) \leq g_{k1} + P_{k1}(1-\lambda_k)\ or \ldots or\ g_{ks_k} + P_{ks_k}(1-\lambda_k),\ k = 1,\ldots,K, \tag{4. 28}$$

$$G_v(X) \geq g_{v1} - P_{v1}(1-\lambda_v)\ or \ldots or\ g_{vs_v} - P_{vs_v}(1-\lambda_v),\ v = K+1,\ldots,V. \tag{4. 29}$$

4.5. Transfer of MCGPFFGC problem into crisp problem

On the basis of obtained relation from (4. 22), (4. 23), (4. 28), (4. 29), the crisp form of MCGPFGC (3. 14)-(3. 19), will be as follows:

$$G_k(X) \leq g_{k1} + P_{k1}(1-\lambda_k)\, or \ldots or\, g_{ks_k} + P_{ks_k}(1-\lambda_k),\ k = 1, \ldots, K, \tag{4. 30}$$

$$G_v(X) \geq g_{v1} - P_{v1}(1-\lambda_v)\, or \ldots or\, g_{vs_v} - P_{vs_v}(1-\lambda_v),\ v = K+1, \ldots, V, \tag{4. 31}$$

$$\sum_{j=1}^{n} a_{l_1 j}\, x_j \leq b_{l_1} + P_{l_1}(1-\alpha_{l_1}),\ l_1 = 1, \ldots, r, \tag{4. 32}$$

$$\sum_{j=1}^{n} a_{l_2 j}\, x_j \geq b_{l_2} - P_{l_2}(1-\alpha_{l_2}),\ l_2 = r+1, \ldots, m, \tag{4. 33}$$

$$\sum_{j=1}^{n} e_{wj}\, x_j \leq or \geq o_w,\ w = 1, \ldots, W, \tag{4. 34}$$

$$0 \leq \alpha_{l_1}, \alpha_{l_2}, \lambda_k, \lambda_v \leq 1, \tag{4. 35}$$

$$X = (x_1, \ldots, x_n)^T,\ x_j \geq 0,\ j = 1, \ldots, n. \tag{4. 36}$$

We will have named the above problem as Multi Parametric Multi-Choice Goal Programming (MPMCGP) problem.

Remark 4.1. *The indices and the variable of the Multi Parametric Multi-Choice Goal Programming (MPMCGP) are defined in Table 2.*

5. Solution method

In the above problem (MPMCGP) we are faced with the goals, that are multi aspiration levels. Therefore, the obtained MPMCGP problem will be Changed into Multi Parametric Binary Goal Programming (MPBGP) problem by use of MCGP by use of MCGP model which is discussed in subsection 2.1. As we mentioned in section 2-1 the MPMCGP problem will be transfer to a problem in abrected form by MPBGP problem which is defined in the follows:

$$\min\ Z = \sum_{k=1}^{K} d_k^+ + \sum_{v=K+1}^{V} d_v^-$$

$$G_k(X) - d_k^+ + d_k^- = \sum_{h=1}^{s_k} \big(g_{kh} + P_{kh}(1-\lambda_k)\big)\, S_{kh}(bi),\ k = 1, \ldots, K,$$

$$G_v(X) - d_v^+ + d_v^- = \sum_{u=1}^{s_v} \big(g_{vu} + P_{vu}(1-\lambda_v)\big)\, S_{vu}(bi),\ v = K+1, \ldots, V, \tag{5. 37}$$

$$\sum_{j=1}^{n} a_{l_1 j}\, x_j \leq b_{l_1} + P_{l_1}(1-\alpha_{l_1}),\ l_1 = 1, \ldots, r, \tag{5. 38}$$

TABLE 2. The indices and the variable 0f the (MPMCGP).

$G_k(X)$	*The first type crisp goals function, ($k = 1, \dots, K$).*
g_{kh}	*The multi aspreation levels of the k-th goal, ($k = 1, \dots, K$), ($h = 1, \dots, s_k$).*
P_{kh}	*The amount tolerance(maximum allowable positive deviation) first type goal from the aspreation level gkh, ($k = 1, \dots, K$), ($h = 1, \dots, s_k$).*
λ_k	*The minimum amount of membership degree satisfaction of the first type flexible fuzzy goals, ($k = 1, \dots, K$).*
$G_v(X)$	*The second type crisp goals function, ($v = K+1, \dots, V$).*
g_{vu}	*The multi aspreation levels of the v-th goal, ($v = K+1, \dots, V$), ($u = 1, \dots, s_v$).*
P_{vu}	*The amount tolerance (maximum allowable negative deviation) second type goal from the aspreation level g_{vu}, ($v = K+1, \dots, V$), ($u = 1, \dots, s_v$).*
λ_v	*The minimum amount of membership degree satisfaction of the first type flexible fuzzy goals, ($k = 1, \dots, K$).*
$\sum_{j=1}^{n} a_{l_1 j}\, x_j \leq b_{l_1} + P_{l_1}(1 - \alpha_{l_1})$	*The first type flexible fuzzy constraints has become the crisp constraints, $l_1 = 1, \dots, r$.*
$P_{l_1}\, (l_1 = 1, \dots, r)$	*The amount tolerance (maximum allowable positive deviation) of the right hand side of the first type flexible fuzzy constraints.*
$\alpha_{l_1}\, (l_1 = 1, \dots, r)$	*The minimum amount of membership degree satisfaction of the first type flexible fuzzy constraints.*
$\sum_{j=1}^{n} a_{l_2 j}\, x_j \geq b_{l_2} - P_{l_2}(1 - \alpha_{l_2})$	*The second type flexible fuzzy constraints has become the crisp constraints, $l_2 = r+1, \dots, m$.*
$P_{l_2}\, (l_2 = r+1, \dots, m)$	*The amount tolerance (maximum allowable negative deviation) of the right hand side of the second type flexible fuzzy constraints.*
$\alpha_{l_2}\, (l_2 = r+1, \dots, m)$	*The minimum amount of membership degree satisfaction of the second type flexible fuzzy constraints.*
$\sum_{j=1}^{n} e_{wj}\, x_j \leq or \geq o_w$	*The third type contains crisp constraints includes the crisp constraints of the problem, $w = 1, \dots, W$.*
$X = (x_1, \dots, x_n)^T$	*The decision variable vector.*

$$\sum_{j=1}^{n} a_{l_2 j}\, x_j \geq b_{l_2} - P_{l_2}(1 - \alpha_{l_2}),\; l_2 = r+1, \dots, m,$$

$$\sum_{j=1}^{n} e_{wj}\, x_j \leq or \geq o_w,\; w = 1, \ldots, W,$$
$$0 \leq \alpha_{l_1}, \alpha_{l_2}, \lambda_k, \lambda_v \leq 1,$$
$$X = (x_1, \ldots, x_n)^T,\; x_j \geq 0,\; j = 1, \ldots, n.$$

Which $S_{kh}(bi)$ and $S_{vu}(bi)$ respectively represents a multicative terms of binary variable that determine which AL should be selected for $k-th$ and $v-th$ goal; d_k^+, d_k^-, d_v^+ and d_v^- are, respectively ,over and under achievements of the $k-th$ and $v-th$ goal. It should be noted that the variables d_k^+, d_v^- respectively, are the undesirable deviation of first type goals $(k = 1, \ldots, K)$ and second type goals $(v = K+1, \ldots, V)$.
We name the above problem the Multi Parametric Multi Choice Goal Programming (MPM-CGP) problem.

Remark 5.1. *The indices and the variable of the Multi Parametric Binary Goal Programming (MPBGP) are defined in Table 3.*

Now we present the solving process in the form of an algorithm. The following algorithm is prepared based on the above discussion which are given section 4 and 5 for solving MCGPFFGC problem.

Algorithm (5. 37)
Step1: Transfer the given MCGPFFGC problem into an equivalent crisp model which is defined in subsection 4.5.
Step2: Define the associated MPBGP problem based on Equation (5. 37) and (5. 38).
Step3: Obtain the optimal solution of the MCGPFFGC problem by solving the MPBGP problem which is established in step2.

6. An illustrative example

Lets keep in view a MCGPFFGC problem with flexible fuzzy goals and constraints
Goals:

$$\begin{aligned}
&G_1)\; 3x_1 + 2x_2 + x_3 \prec_{ff} 100 \; or \; 120,\\
&G_2)\; 4x_1 + 3x_2 + 2x_3 \prec_{ff} 90 \; or 100 \; or \; 110, \qquad (6.\ 39)\\
&G_3)\; 3.5x_1 + 5x_2 + 3x_3 \succ_{ff} 80 \; or \; 90 \; or \; 130.
\end{aligned}$$

Constraints:

$$3x_2 - x_3 - x_1 \succ_{ff} 15,$$
$$x_1 - x_3 \prec_{ff} 4,$$
$$x_1 + x_2 + x_3 \prec_{ff} 25,$$
$$x_1 \geq 0,\; x_2 \geq 0, x_3 \geq 0.$$

Which the amount tolerance (maximum allowable positive deviation) first goal from the aspreation level are equal to 50 and 40, the amount tolerance (maximum allowable positive deviation) second goal from the aspreation level are equal to 70,30 and 20, the amount tolerance (maximum allowable negative deviation) third goal from the aspreation level are equal to 10 ,40,60 and 80, also 8,2 and 7 are respectively the amount tolerance (maximum allowable negative and positive deviation) of the right hand side of the first, second and third constraints. According to the steps of the Algorithm (5. 37), the problem (6. 39) will change to the following form. According to the relation of the MPMCGP, the problem (6. 39) will change to the following form.

$$\min Z = D_1^+ + D_2^+ + D_3^- \tag{6. 40}$$

$$3x_1 + 2x_2 + x_3 - D_1^+ + D_1^- = (100 + 50(1-\lambda_1))B_1 + (120 + 40(1-\lambda_1))(1-B_1),$$

$$4x_1 + 3x_2 + 2x_3 - D_2^+ + D_2^- = (90 + 70(1-\lambda_2))B_2B_3 + (100 + 30(1-\lambda_2))(1-B_2)B_3$$
$$+ (110 + 20(1-\lambda_2))B_2(1-B_3),$$

$$3.5x_1 + 5x_2 + 3x_3 - D_3^+ + D_3^- = (80 - 10(1-\lambda_3))B_4B_5 + (90 - 40(1-\lambda_3))(1-B_4)B_5$$
$$+ (120 - 60(1-\lambda_3))(1-B_5)B_4 + (130 - 80(1-\alpha_3))(1-B_5)(1-B_4),$$

$$3x_2 - x_3 - x_1 \geq 15 - 8(1-\alpha_1),$$

$$x_1 - x_3 \leq 4 + 2(1-\alpha_2),$$

$$x_1 + x_2 + x_3 \leq 25 + 7(1-\alpha_3),$$

$$B_2 + B_3 \geq 1,\ 0 \leq \alpha_1 \leq 1,\ 0 \leq \alpha_2 \leq 1,\ 0 \leq \alpha_3 \leq 1,\ 0 \leq \lambda_1 \leq 1,\ 0 \leq \lambda_2 \leq 1,\ 0 \leq \lambda_3 \leq 1,$$

$$D_1^+, D_1^-, D_2^+, D_2^-, D_3^+, D_3^- \geq 0,\ x_1, x_2, x_3 \geq 0,\ B_i \in \{0,1\},\ i = 1,2,3,4,5$$

The last problem is solved by lingo software and optimal solution is obtained. Where B_1, B_2, B_3, B_4 and B_5 are binary variables; D_t^+ and D_t^-, $(t = 1,2,3)$ are, respectively, over and under-achievements of the first, second and third goals.

$$Z = 0,\ D_1^+ = 0,\ D_2^+ = 0,\ D_3^- = 0, \tag{6. 41}$$

$$x_1 = 12.46817,\ x_2 = 8.645448,\ x_3 = 6.468173,\ D_1^- = 38.836421,\ \lambda_1 = 1,\ B_1 = 1,$$

$$D_2^- = 1.25463, \lambda_2 = 1,\ B_2 = 1,\ B_3 = 1,\ D_3^+ = 1.250880,\ \alpha_3 = 0,\ \lambda_3 = 0.7503248,$$

$$B_4 = 1,\ B_5 = 0,\ \alpha_1 = 0,\ \alpha_2 = 0.$$

From the result we realize that for the first, second and third goals, the amount aspreation level 100,90,105.019488 were considered and the optimal value 61.163579, 88.74537, 106.273518 were obtained for them respectively.

7. Sensitivity analysis

It is seen that solving MCGPFFGC problem has led to solving MPBGP problems. The most valuable features of These problems is that we can obtain different optimal solution for the given problem by considering different value for $\alpha_1, \alpha_2, \alpha_3, \lambda_1, \lambda_2, \lambda_3$, in different situation. In the real word, the decision maker faces a different situation in the solution of a problem. For this purpose, we can use the sensitivity analysis of the problem. In this section, we give some of the sensitivity analysis of the obtained model.

1. In problem 40, we consider the minimum value of 0.3 for $\alpha_1, \alpha_2, \alpha_3$.

Due to the addition of the constraints ($\alpha_1 \geq 0.3, \alpha_2 \geq 0.3, \alpha_3 \geq 0.3$) to the (6. 40) problem, the optimal solution is changed as It is.

$$Z = 0,\ D_1^+ = 0,\ D_2^+ = 0,\ D_3^- = 0, \tag{7. 42}$$

$$x_1 = 11.94303,\ x_2 = 9.295350,\ x_3 = 6.543025,\ D_1^- = 39.03720,\ \lambda_1 = 1,\ B_1 = 1,$$

$D_2^- = 1.255799, \lambda_2 = 1,\ B_2 = 1,\ B_3 = 1,\ D_3^+ = 1.251840,\ \alpha_3 = 0.3,\ \lambda_3 = 0.7775762,$
$B_4 = 1,\ B_5 = 0,\ \alpha_1 = 0.3,\ \alpha_2 = 0.3.$

2. Due to the addition of the constraint ($\alpha_1 + \alpha_2 + \alpha_3 \geq 1.5$) to the (6. 40) problem, the Optimal solution is changed as it is.

$$Z = 0,\ D_1^+ = 0,\ D_2^+ = 0,\ D_3^- = 0, \tag{7. 43}$$
$$x_1 = 10.71580,\ x_2 = 10.81053,\ x_3 = 6.715800,\ D_1^- = 39.51573,\ \lambda_1 = 1,\ B_1 = 1,$$
$$D_2^- = 1.273597,\ \lambda_2 = 1,\ B_2 = 1,\ B_3 = 1,\ D_3^+ = 0.7713548,\ \lambda_3 = 0.8489002,\ B_4 = 1,$$
$$B_5 = 0,\ \alpha_1 = 1,\ \alpha_2 = 1,\ \alpha_3 = 0.5368380,$$

3. In problem (6. 40),we consider the minimum value of 0.5 for $\alpha_1, \alpha_2, \alpha_3, \lambda_1, \lambda_2, \lambda_3$. Due to the addition of the constraints ($\alpha_1 \geq 0.5, \alpha_2 \geq 0.5, \alpha_3 \geq 0.5, \lambda_1 \geq 0.5, \lambda_2 \geq 0.5, \lambda_3 \geq 0.5$) to the (6. 40) problem, the optimal solution is changed as. It is.

$$Z = 0,\ D_1^+ = 0,\ D_2^+ = 0,\ D_3^- = 0, \tag{7. 44}$$
$$x_1 = 11.59293,\ x_2 = 9.728618,\ x_3 = 6.592927,\ D_1^- = 39.17106,\ \lambda_1 = 1,\ B_1 = 1,$$
$$D_2^- = 1.256586, \lambda_2 = 1,\ B_2 = 1,\ B_3 = 1,\ D_3^+ = 1.252481,\ \lambda_3 = 0.7957439,\ B_4 = 1,$$
$$B_5 = 0,\ \alpha_1 = 0.5, \alpha_2 = 0.5,\ \alpha_3 = 0.5.$$

4. With the addition of

$$\alpha_1 + \alpha_2 + \alpha_3 - D_4^+ + D_4^- = 3,\ \lambda_1 + \lambda_2 + \lambda_3 - D_5^+ + D_5^- = 3,$$

And considering $\min Z = D_1^+ + D_2^+ + D_3^- + D_4^+ + D_4^- + D_5^+ + D_5^-$ as the objective function, the optimal solution to problem 40 varies as follows:

$$Z = 0,\ D_1^+ = 0,\ D_2^+ = 0,\ D_3^- = 0, D_4^+ = 0, D_4^- = 0, D_5^+ = 0, D_5^- = 0, x_1 = 9.5,$$
$$x_2 = 10,\ x_3 = 5.5,\ D_1^- = 46,\ \lambda_1 = 1,\ B_1 = 1, D_2^- = 11, \lambda_2 = 1,\ B_2 = 1,\ B_3 = 1,$$
$$D_3^+ = 19.75,\ \lambda_3 = 1,\ B_4 = 1,\ B_5 = 1, \alpha_1 = 1, \alpha_2 = 1,\ \alpha_3 = 1.$$

8. CONCLUSION

In this study article we presented a new model and also a new method for solving a class of flexible fuzzy MCGP problems which both of their constraint and goals are fuzzy flexible. Considering the different membership degrees for the flexible fuzzy goals and constraints, we obtained the crisp form of the above model, which is an MPMCGP problem. Finally, by defining the multiplicative function of binary variables, we transformed the above problem into a multi parametric binary goal programming problem. And then we presented an illustrative numerical example and sensitivity analysis to show the efficiency of our suggested flexible fuzzy multi choice goal programming. We also emphasized that the mentioned solving approach in this paper with the extended and proposed model will be useful in many disciplines and we are going to concentrate on them in our futures works.

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TABLE 3. The indices and the variable 0f the (MPBGP)

$G_k(X)$	*The first type crisp goals function, ($k = 1, \ldots, K$).*
g_{kh}	*The multi aspiration levels of the k-th goal, ($k = 1, \ldots, K$), ($h = 1, \ldots, s_k$).*
P_{kh}	*The amount tolerance(maximum allowable positive deviation) first type goal from the aspreation level g_{kh}, ($k = 1, \ldots, K$), ($h = 1, \ldots, s_k$).*
λ_k	*The minimum amount of membership degree satisfaction of the first type flexible fuzzy goals, ($k = 1, \ldots, K$).*
d_k^+	*The undesirable deviation variable of first type goals, ($k = 1, \ldots, K$).*
d_k^-	*The desirable deviation variable of first type goals, ($k = 1, \ldots, K$).*
$S_{kh}(bi)$	*A multicative terms of binary variable that determine which aspiration level should be selected for k-th goal, ($k = 1, \ldots, K$), ($h = 1, \ldots, s_k$).*
$G_v(X)$	*The second type crisp goals function, ($v = K + 1, \ldots, V$).*
g_{vu}	*The multi aspiration levels of the v-th goal, ($v = K + 1, \ldots, V$), ($u = 1, \ldots, s_v$).*
P_{vu}	*The amount tolerance (maximum allowable negative deviation) second type goal from the aspiration level g_{vu}, ($v = K + 1, \ldots, V$), ($u = 1, \ldots, s_v$).*
λ_v	*The minimum amount of membership degree satisfaction of the first type flexible fuzzy goals, ($v = K + 1, \ldots, V$).*
d_v^+	*The desirable deviation variable of first type goals.*
d_v^-	*The undesirable deviation variable of first type goals.*
$S_{vu}(bi)$	*A multicative terms of binary variable that determine which aspiration level should be selected for v-th goal, ($v = K + 1, \ldots, V$), ($u = 1, \ldots, s_v$).*
$\sum_{j=1}^{n} a_{l_1 j}\, x_j \leq b_{l_1} + P_{l_1}(1 - \alpha_{l_1})$	*The first type flexible fuzzy constraints has become the crisp constraints, $l_1 = 1, \ldots, r$.*
P_{l_1} ($l_1 = 1, \ldots, r$)	*The amount tolerance (maximum allowable positive deviation) of the right hand side of the first type flexible fuzzy constraints.*
α_{l_1} ($l_1 = 1, \ldots, r$)	*The minimum amount of membership degree satisfaction of the first type flexible fuzzy constraints.*
$\sum_{j=1}^{n} a_{l_2 j}\, x_j \geq b_{l_2} - P_{l_2}(1 - \alpha_{l_2})$	*The second type flexible fuzzy constraints has become the crisp constraints, $l_2 = r + 1, \ldots, m$.*
P_{l_2} ($l_2 = r + 1, \ldots, m$)	*The amount tolerance (maximum allowable negative deviation) of the right hand side of the second type flexible fuzzy constraints.*
α_{l_2} ($l_2 = r + 1, \ldots, m$)	*The minimum amount of membership degree satisfaction of the second type flexible fuzzy constraints.*
$\sum_{j=1}^{n} e_{wj}\, x_j \leq or \geq o_w$	*The third type contains crisp constraints includes the crisp constraints of the problem, $w = 1, \ldots, W$.*
$X = (x_1, \ldots, x_n)^T$	*The decision variable vector.*