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Mean Estimation Using Even Order Ranked Set Sampling

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Abstract. An efficient estimate of the population mean based on ranked set sample is of major concern with the cost and success in ranking. In this research an efficient mean estimator based on even order ranked set sampling (EORSS) is proposed and analyzed. The EORSS scheme presents an unbiased estimator when the distribution is symmetric. The performance of population mean estimator based on EORSS is compared with its counterparts in simple random sampling (SRS), ranked set sampling (RSS) as well as extreme ranked set sampling (ERSS) using theoretical and simulation studies. The simulation results validate the theoretical results and show that EORSS mean estimator is always more efficient than SRS mean estimator, equal or more efficient than RSS mean estimator and more efficient than ERSS mean estimator for symmetric and non-symmetric distributions considered in this study. An explicatory application to real-life data set is also presented to demonstrate the achievement of the suggested EORSS mean estimator.

AMS (MOS) Subject Classification Codes: 62D05
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1. Introduction

The idea of RSS scheme was first proposed by McIntyre [5]. He developed an efficient method based on ranking for data collection that provides more precise estimates than SRS. He used this effective approach to estimate the average yield of pasture. Measuring yield of pasture plots requires cutting and weighing the hay which is time wasting but an experienced person can be able to rank with eye inspection quite accurately the yields of a small number of such plots without real measurement. The statistical properties of RSS scheme are studied by Takahasi and Wakimoto [13] and proved that ranked-set sample mean can be used as a more efficient mean estimator than SRS mean estimator. Dell and Clutter [3] pointed out the role of ranking errors in RSS which cause loss in efficiency for estimating the population mean.

After McIntyre's work, Muttlak [6] proposed modification in RSS known as Pair RSS (PRSS). In this method, two sampling units are selected from each set instead of one specific unit and indicated that PRSS mean is an unbiased estimator with smaller variance. Samawi et al. [12] introduced the modified form of RSS named as ERSS. They studied the properties of the ERSS and presented that the allocation and ranking of the sampling units in matrix is similar to the classical RSS but the selection procedure is changed. They showed that the ERSS estimators are more precise than usual SRS mean estimator as well as unbiased if the probability distribution is symmetrical. Muttlak [7] suggested the new sampling procedure which is known as median ranked set sampling (MRSS). He demonstrated that MRSS has the potential to reduce errors in ranking and gives efficient estimate than the RSS estimate in case of symmetric distribution. Muttlak [8] developed quartile ranked-set sampling (QRSS) design and investigated the performance of proposed estimator with its counterparts in SRS design for some distributions then found QRSS mean as an efficient estimator. Haq et al. [4] proposed a mixed-ranked set sampling (MxRSS) and showed that it is an efficient and cost effective sampling scheme. They illustrated that the MxRSS median and mean estimates are more efficient than those obtained from SRS scheme. They also demonstrated that MxRSS mean estimators of symmetric as well as some non-symmetric populations are also more efficient than those based on Partial RSS. Zamanzade and Al-Omari [15] presented a new modification of RSS named as neoteric ranked-set sampling (NRSS) for estimating the underlying population mean and variance. They proved that NRSS estimators still perform better than RSS and SRS estimators when ranking is not perfect. By using the auxiliary variable, Tayyab et al. [14] introduced an efficient EORSS scheme for bivariate case. They utilized the auxiliary information efficiently in both ranking and estimation stages for estimating the ratio estimator of population mean. Noor-ul-Amin et al. [9], Audu and Adewara [1] and Noor-ul-Amin et al. [10] also estimated the population mean recently.

The rest of this article is presented as: In section 2, a novel mean estimator based on EORSS is proposed and its properties are discussed. In section 3, a comparison is given to study the performance of proposed EORSS mean estimator with its counterparts in SRS, RSS and ERSS schemes. An illustrative theoretical and simulation process is also carried out to estimate the population mean for some symmetrical and non-symmetrical distributions. Section 4 presents the implementation of EORSS mean estimator to real-data set and obtained empirical results. The main findings and recommendations are summarized in section 5.

2. Mean estimator under EORSS

According to Tayyab et al. [14] an efficient sampling scheme namely EORSS for univariate case is suggested for estimation of the population mean. Similar to classical RSS, the EORSS is applicable in such situations where the ranking of the sampling units is much easier than observing their actual measurements. Even order ranked set sample quantifies only one sampling unit at even position from each ranking set. The EORSS procedure can be described in following steps:
Step 1: Randomly pick l^2 units from underlying population, where l denotes set size.
Step 2: Distribute the l^2 units into l sets at random with each size l.
Step 3: Without knowing the real measurements of variable of interest, the l units of each set are ranked in an ascending order visually, on the basis of auxiliary variable or by any inexpensive approach.
Step 4: If l is even, from the first $l/2$ sets, choose second unit from first ranked-set, fourth unit from second ranked-set, sixth unit from third set and so forth last even ranked unit from $l/2th$ set. Similar process uses to choose sample units from remaining $l/2$ sets for completing the sample size. If l is odd, from the first $(l-1)/2$ sets, choose second unit from the first set, fourth unit from the second set, sixth unit from the third set and so forth last even ranked unit from the $(l-1)/2th$ set. Same process adopts to choose sample units from other $(l-1)/2$ sets and select $(l+1)/2th$ ranked unit from remaining lth set for completing the one cycle of the sample size.
Step 5: Repeat the above procedure r cycles, if essential, to acquire the required sample of size $n = lr$ from initial l^2r sample units.

Suppose $Z_i (i = 1, 2, 3, ..., l)$ represent a normal random sample with pdf $f(z)$, cdf $F(z)$, population mean μ_z and variance σ_z^2. The SRS mean is $\bar{Z}_{SRS} = \sum_{i=1}^{n} Z_i/n$ and $E(\bar{Z}_{SRS}) = \mu_z$ along with $Var(\bar{Z}_{SRS}) = \sigma_z^2/n$. Let $Z_{ij}, i, j = 1, 2, 3, ..., l$, indicate the l independent normal random vectors of same size l and $Z_{1(1)c}, Z_{2(2)c}, ..., Z_{l(l)c}$ represent the ranked set sample along with mean $\bar{Z}_{(RSS)} = (1/lr)\sum_{c=1}^{r}\sum_{i=1}^{l} Z_{i(i:l)c}$ and variance $Var(\bar{Z}_{(RSS)}) = \frac{(\sigma_z^2)}{lr} - \frac{1}{(l^2r)}\sum_{i=1}^{l}(\mu_{z(i)} - \mu_z)^2$. Assume $Z_{1(2)c}, Z_{2(4)c}, ...,$ $Z_{k(l)c}, Z_{k+1(2)c}, Z_{k+2(4)c}, ..., Z_{l(l)c}$ be the even order ranked set sample for even sample size and $Z_{1(2)c}, Z_{2(4)c}, ..., Z_{m(l-1)c},$ $Z_{m+1(2)c}, Z_{m+2(4)c}, ..., Z_{l-1(l-1)c},$ $Z_{l((l+1)/2)c}$ be the even order ranked set sample for odd sample size, where $k = l/2$, $m = (l-1)/2$ and $c = 1, 2, 3, ..., r$.

For even and odd sample sizes, proposed mean estimators using EORSS with one cycle are defined as:

$$\bar{Z}_{(EORSS)e} = \frac{1}{l}[\sum_{i=1}^{k} Z_{i(2i:l)} + \sum_{i=1}^{k} Z_{k+i(2i:l)}] \tag{2. 1}$$

$$\bar{Z}_{(EORSS)o} = \frac{1}{l}[\sum_{i=1}^{m} Z_{i(2i:l)} + \sum_{i=1}^{m} Z_{m+i(2i:l)} + Z_{l((l+1)/2:l)}] \tag{2. 2}$$

The variance of $\bar{Z}_{(EORSS)e}$ can be derived as

$$Var(\bar{Z}_{(EORSS)e}) = \frac{1}{l^2}[\sum_{i=1}^{k} Var(Z_{i(2i:l)}) + \sum_{i=1}^{k} Var(Z_{(k+i(2i:l))})] \tag{2. 3}$$

Note that sampling units $Z_{(2)}, Z_{(4)}, ..., Z_{(l)}$ are independently and identically distributed.

$$V(\bar{Z}_{(EORSS)e}) = \frac{2}{l^2}[\sum_{i=1}^{k} \sigma^2_{z(2i:l)}] \tag{2. 4}$$

Similarly, the variance of $\bar{Z}_{(EORSS)o}$ can be derived as

$$Var(\bar{Z}_{(EORSS)o}) = \frac{1}{l^2}[\sum_{i=1}^{m} Var(Z_{i(2i:l)}) + \sum_{i=1}^{m} Var(Z_{k+i(2i:l)}] + \frac{1}{l^2} Var(Z_{l((l+1)/2:l)}) \tag{2. 5}$$

$$V(\bar{Z}_{(EORSS)o}) = \frac{2}{l^2}[\sum_{i=1}^{m} \sigma^2_{z(2i:l)}] + \frac{1}{l^2}[\sigma^2_{z((l+1)/2:l)}] \tag{2. 6}$$

Theorem 1: $\bar{Z}_{(EORSS)}$ is an unbiased estimator of the population mean in case of symmetrical distribution.

Proof: Let l is even then using (2.1) the EORSS mean estimator can be written as:

$$\bar{Z}_{(EORSS)e} = \frac{1}{l}\sum_{i=1}^{k}[Z_{i(2i:l)} + Z_{k+i(2i:l)}]$$

Take expectation on both sides

$$E(\bar{Z}_{(EORSS)e}) = E(\frac{1}{l}\sum_{i=1}^{k}[Z_{i(2i:l)} + Z_{k+i(2i:l)}])$$

$$= \frac{1}{l}\sum_{i=1}^{k}[E(Z_{i(2i:l)}) + E(Z_{k+i(2i:l)})]$$

If the probability distribution is symmetrical regarding μ then using the fact described by David and Nagaraja [2] as $Z_{(i)} - \mu =^d \mu - Z_{(i)}$. Thus $\mu - \mu_{(2i:l)} =^d \mu_{(2i:l)} - \mu$ and then $\mu_{(2i:l)} + \mu_{(2i:l)} = 2\mu$. Therefore

$$E(\bar{Z}_{(EORSS)e}) = \frac{1}{l}\sum_{i=1}^{k}[\mu_{(2i:l)} + \mu_{(2i:l)}]$$

$$= \frac{1}{l}(l/2)[2\mu] = \mu,$$

Hence proved that $\bar{Z}_{(EORSS)}$ is an unbiased estimator of μ. Note that $\sum_{i=1}^{k}[\mu_{(i:t)}] = k\mu$ see[13].

3. THEORETICAL AND SIMULATION STUDY

To investigate the proposal theoretically, Let study variable Z follows uniform $U(0,1)$ distribution with mean and variance of i^{th} perfect ranked unit $Z_{(i)}$ considered by Zamanzade and Al-Omari [15] as $E[Z_{(i)}] = i/(l+1)$ and $Var[Z_{(i)}] = i(l+1-i)/((l+1)^2(l+2))$ respectively.

Let $l = 3$ and $r = 1$ for EORSS, identify 9 units form study population. After ranking the units of each set, it has

$$\begin{bmatrix} Z_{1(1)}, & \underline{Z_{1(2)}}, & Z_{1(3)} \\ Z_{2(1)}, & \underline{Z_{2(2)}}, & Z_{2(3)} \\ Z_{3(1)}, & \underline{Z_{3(2)}}, & Z_{3(3)} \end{bmatrix}$$

Pick the underlined units $Z_{1(2)}, Z_{2(2)}, Z_{3(2)}$ from each set as an EORSS for $l = 3$. Then mean estimator under EORSS is

$$\bar{Z}_{(EORSS)} = (Z_{1(2)} + Z_{2(2)} + Z_{3(2)})/3$$

The expectation of said estimator is

$$E[\bar{Z}_{(EORSS)}] = (E[Z_{(2)}] + E[Z_{(2)}] + E[Z_{(2)}])/3$$

$$E[\bar{Z}_{(EORSS)}] = \frac{1}{3}(\frac{2}{4} + \frac{2}{4} + \frac{2}{4}) = 0.5$$

which is an unbiased-estimator of population mean $\mu = 0.5$. Now

$$Var[\bar{Z}_{(EORSS)}] = (Var[Z_{(2)}] + Var[Z_{(2)}] + Var[Z_{(2)}])/9$$

$$Var[\bar{Z}_{(EORSS)}] = \frac{1}{9}(\frac{4}{80} + \frac{4}{80} + \frac{4}{80}) = 0.01667$$

The variance of mean estimator under SRS of size $l = 3$ is $Var[\bar{Z}_{SRS}] = \frac{\sigma^2}{l} = \frac{(1/12)}{3} = 0.02778$ and efficiency (E_{ff}) $E_{ff}(\bar{Z}_{(EORSS)}, \bar{Z}_{SRS}) = \frac{Var[\bar{Z}_{SRS}]}{Var[\bar{Z}_{(EORSS)}]} = 1.67$ which shows that $\bar{Z}_{(EORSS)}$ is more precise than $\bar{Z}_{SRS}$. Similarly $\bar{Z}_{(EORSS)}$, for $l = 6$ and $r = 1$, is approximately unbiased with variance $Var[\bar{Z}_{(EORSS)}] = \frac{56}{14112}$ and $Var[\bar{Z}_{RSS}] = \frac{56}{14112}$. The results for $l = 6$ are showing that both (EORSS and RSS) schemes are equal efficient for mean estimation.

To compare the performance of proposed estimator of population mean μ using EORSS scheme with SRS, RSS and ERSS schemes, seven non-uniform probability distributions

TABLE 1. E_{ff} of RSS, ERSS and EORSS w.r.to SRS for estimating population mean with $l = 4, 6$.

Distribution	$l = 4$			$l = 6$		
	RSS	**ERSS**	**EORSS**	**RSS**	**ERSS**	**EORSS**
Normal(0,1)	2.339861	2.018671	2.325356	3.196072	2.407745	3.211768
Logistic(0,1)	2.213788	1.700890	2.226868	2.935868	1.824548	2.938200
Laplace (0,1)	2.048759	1.395763	2.056991	2.613326	1.342433	2.598038
Weibull(6,1)	2.337460	2.051694	2.669031	3.201322	2.454750	3.608093
Beta(7,4)	2.406150	2.236662	2.683626	3.242079	2.741540	3.535248
t(3)	1.684720	1.044280	1.680702	2.053852	0.884969	2.095648
Weibull(6)	2.349385	2.043200	2.679813	3.142946	2.407709	3.540128

TABLE 2. E_{ff} of RSS, ERSS and EORSS w.r.to SRS for estimating population mean with $l = 5, 7$.

Distribution	$l = 5$			$l = 7$		
	RSS	**ERSS**	**EORSS**	**RSS**	**ERSS**	**EORSS**
Normal(0,1)	2.778520	2.409582	3.272047	3.616019	2.758821	4.199975
Logistic(0,1)	2.583326	1.994593	3.662545	3.273771	2.016201	4.614688
Laplace (0,1)	2.348454	1.606242	4.232508	2.879161	1.502568	5.078991
Weibull(6,1)	2.751188	2.415031	3.238542	3.582514	2.767917	4.159956
Beta(7,4)	2.823228	2.609561	3.080013	3.779071	3.175007	4.035019
t(3)	1.902628	1.136607	5.511713	2.228815	0.914393	6.627138
Weibull(6)	2.752046	2.417643	3.227892	3.587093	2.770563	4.149690

are considered. 60,000 random samples are generated from population of each distribution and variance or mean squared error (MSE) of the averages of drawn samples are compared. In case of symmetrical distribution, the efficiency (E_{ff1}) of $\bar{Z}_{RSS}$,$\bar{Z}_{(ERSS)}$ and $\bar{Z}_{(EORSS)g}$, where $g = e$, o, with respect to usual $\bar{Z}_{SRS}$ is as follow:

$$E_{ff1}(T, \bar{Z}_{SRS}) = \frac{Var(\bar{Z}_{SRS})}{Var(T)}, T = \bar{Z}_{(RSS)}, \bar{Z}_{(ERSS)}, \bar{Z}_{(EORSS)g}$$

and for non-symmetrical distributions, the efficiency (E_{ff2}) is:

$$E_{ff2}(T, \bar{Z}_{SRS}) = \frac{MSE(\bar{Z}_{SRS})}{MSE(T)}, T = \bar{Z}_{(RSS)}, \bar{Z}_{(ERSS)}, \bar{Z}_{(EORSS)g}$$

In terms of the values of the efficiency, simulation results for symmetric and non-symmetric distributions are given in Table 1 to 2 for sample size 4, 5, 6 and 7.

From Tables 1-2, we can conclude that:

• When the sample size is even or odd, EORSS estimator is more efficient than usual SRS estimator whether distribution is symmetric or non-symmetric.

• If considered distribution is symmetric then EORSS and RSS are approximately equal efficient in estimating the population mean for even sample size and this confirms the theoretical results. For $l = 6$, the E_{ff1} of EORSS population mean estimator of Normal distribution N(0,1) is 3.211768 while E_{ff1} value under RSS is 3.196072.
• If mentioned distribution is non-symmetric then EORSS mean estimator is more efficient than RSS and ERSS mean estimators for both even and odd sample sizes. For $l = 7$, the E_{ff2} value of EORSS mean estimator is 4.159956 while E_{ff2} values under RSS and ERSS mean estimators are 3.582514 and 2.767917 respectively when the underlying distribution is Weibull(6,1).
• If l is odd, an increase in relative efficiency is obtained by using EORSS in estimating population mean when distribution is symmetric. For $l = 7$, the E_{ff1} value of EORSS mean estimator is 5.078991 while E_{ff1} values under RSS and ERSS mean estimators are 2.879161 and 1.502568 respectively when the underlying distribution is Laplace (0,1).

4. APPLICATION

For illustrating the utilization of EORSS scheme in estimating the population mean, a real-life data related to 399 conifer (pinus palustris) trees from Platt et al. [11] is considered. The dataset were composed on seven variables although considered only two in this study: X (diameter) in cm at chest height and Z (height) in feet. For the target variable Z in the population, the mean and variance are μ_z=52.6768 and σ_z^2=3253.44 respectively. To draw $n = 6$ based on a set size $l = 6$ with a single cycle $r = 1$, the EORSS scheme is carried out as follows:

Randomly draw 36 bivariate sampling units from population and distribute these units in 6 sets at random with each set size 6 as:

$$\begin{pmatrix} set1: (6.2,7) & (11.8,32) & (4.1,8) & (13.3,22) & (47.6,154) & (15.9,76) \\ set2: (4.5,8) & (19.3,25) & (40.4,87) & (42.1,70) & (19.9,24) & (20.5,38) \\ set3: (2.6,3) & (8,38) & (3.5,9) & (3.7,6) & (34,42) & (19.8,33) \\ set4: (4.5,8) & (50.8,106) & (3.0,4) & (16.8,28) & (44.6,149) & (34,99) \\ set5: (2.8,3) & (9.4,23) & (18.7,68) & (44.9,87) & (7.2,26) & (40,82) \\ set6: (4.5,8) & (2.6,7) & (4.2,8) & (45.4,140) & (10.9,26) & (17.5,46) \end{pmatrix}$$

Rank the X values and utilize their ordering for Z, then pick the 6 judgment ranked values (in bold) of Z as:

$$\begin{pmatrix} set1: (4.1,8) & (6.2,\mathbf{7}) & (11.8,32) & (13.3,22) & (15.9,76) & (47.6,154) \\ set2: (4.5,8) & (19.3,25) & (19.9,24) & (20.5,\mathbf{38}) & (40.4,87) & (42.1,70) \\ set3: (2.6,3) & (3.5,9) & (3.7,6) & (8,38) & (19.8,33) & (34,\mathbf{42}) \\ set4: (3.0,4) & (4.5,\mathbf{8}) & (16.8,28) & (34,99) & (44.6,149) & (50.8,106) \\ set5: (2.8,3) & (7.2,26) & (9.4,23) & (18.7,\mathbf{68}) & (40,82) & (44.9,87) \\ set6: (2.6,7) & (4.2,8) & (4.5,8) & (10.9,26) & (17.5,46) & (45.4,\mathbf{140}) \end{pmatrix}$$

The values (in bold) of Z using EORSS, RSS, ERSS and SRS schemes are provided in Table 3.

TABLE 3. The values of target variable Z using EORSS, RSS, ERSS and SRS schemes

EORSS	7	38	42	8	68	140
RSS	8	25	6	99	82	140
ERSS	8	8	3	106	87	140
SRS	137	11	244	21	10	22

The estimates of the mean height of conifer trees with their variances using data in Table 3 are:

$$\bar{Z}_{EORSS}=50.50,\ \bar{Z}_{RSS}=60,\ \bar{Z}_{ERSS}=58.67,\ \bar{Z}_{SRS}=74.17$$

and

$$Var(\bar{Z}_{EORSS})=2448.70,\ Var(\bar{Z}_{RSS})=3050,\ Var(\bar{Z}_{ERSS})=3578.27,$$
$$Var(\bar{Z}_{SRS})=9289.37$$

The empirical results demonstrated that EORSS mean estimator is more efficient than its counterparts in SRS, RSS and ERSS.

5. CONCLUDING REMARKS

This manuscript presents an improved EORSS mean estimator and its utility. It is proved that proposed estimator is an unbiased estimator of the population mean for symmetric distribution. The results obtained theoretically and by simulations confirm that the variance of suggested EORSS mean estimator always shows less value than that of the SRS mean estimator. It is also showed that EORSS and RSS schemes are approximately equally efficient in estimating the population mean for even sample size when mentioned distribution is symmetric. For considered non-symmetric distributions, simulation findings show that the EORSS mean estimator is better than RSS and ERSS mean estimators for both even and odd sample sizes. A gain in relative efficiency is found by using EORSS in estimating population mean when distribution is symmetric and size of sample is odd. Therefore, on the basis of above results, the efficient mean estimator under EORSS can be recommended over the RSS, ERSS and SRS mean estimators for estimation of population mean.

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