Punjab University
Journal of Mathematics (ISSN 1016-2526)
Vol. 48(2)(2016) pp. 113-124

Modified Jungck Mann and Modified Jungck Ishikawa Iteration Schemes For Zamfirescue Opertor

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Received: 23 December, 2015 / Accepted: 30 May, 2016 / Published online: 02 September, 2016

Abstract. In this paper, we have modified Jungck Mann and Jungck Ishikawa iteration schemes, and their convergence has been proved in the arbitrary Banach space. Comparison of these modified iteration schemes with Jungck Mann and Jungck Ishikawa, iteration schemes respectively have been made by solving some scalar nonlinear equations.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Modified, Jungck Mann, Jungck Ishikawa, Nonlinear equations.

1. Introduction

Several iterative methods have been devolped to construct the fixed point of certian operators (see$[1, 2, 3, 4]$). The stability results of these iterative methods have been discussed under different contractive conditions. Also many modifications have been made in these iteration schemes to make their convergence fast. These iteration schemes give the fixed point of self operator. In 1986, Jungck $[5]$ proved some results regarding the common fixed points of nonself commuting mappings. In $2005,$ Singh et al. $[6]$ introduced Jungck Mann and Jungck Ishikawa iteration schemes. In this paper, we have modified Jungck Mann and Jungck Ishikawa iteration schemes and proved their convergence in any arbitrary Banach space. We have also applied these modified iteration schemes for solving some nonlinear scalar equations and have compared them with Jungck Mann and Jungck Ishikawa iteration schemes respectively.

2. PRELIMINARIES

We consider the following iteration schemes to establish our result.

[7] Let $(X, \|.\|)$ be a normed linear space and $S, T : Y \rightarrow X$ be two non-self mappings such that $T(Y) \subseteq S(Y)$, Then for $x_0 \in Y$, the sequence $\{Sx_n\}_{n=0}^{\infty}$ defined by

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n Ty_n, \\ Sy_n &= (1-\beta_n)Sx_n + \beta_n Tx_n, \quad n = 0, 1, 2... \end{aligned} \tag{2. 1}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in $[0, 1]$, is called Jungck Ishikawa iteration scheme.

We note that if S is injective and $\beta_n = 0$, then Jungck Ishikawa reduces to Jungck Mann iteration scheme.

(1) If S is identity function and $Y = X$ then Jungck Ishikawa is simply Ishikawa iteration scheme and further if $\beta_n = 0$ then it reduces to Mann iteration scheme.

(2) If S is injective, $\beta_n = 0, \alpha_n = 1$, then Jungck Ishikawa iteration reduces to Junck iteration scheme and furthermore if S is identity function and $\beta_n = 0$, $\alpha_n = 1$, then it reduces to Picard iteration.

Definition 1.1 [7] Let X and Y be two non empty sets and $S, T : Y \rightarrow X$ be two functions then any $x^* \in X$ is called the coincident point of functions S and T iff $Tx^* = Sx^*$ .The set of coincident point of S and T is denoted by $C(S, T)$.

Definition 1.2 [7] For two non-self mappings $S, T : Y \rightarrow X$ with $T(Y) \subseteq S(Y)$, there exist real numbers α, β, γ with $0 \leq \alpha < 1$, $0 \leq \beta < \frac{1}{2}$, $0 \leq \gamma < \frac{1}{2}$, respectively such that for every $x, y \in Y$ at least one of the following must holds:

$$d(Tx, Ty) \leq \alpha d(Sx, Sy), \tag{2. 2}$$

$$d(Tx, Ty) \leq \beta[d(Tx, Sx) + d(Ty, Sy)], \tag{2. 3}$$

$$d(Tx, Ty) \leq \gamma[d(Tx, Sy) + d(Ty, Sx)]. \tag{2. 4}$$

The above conditions are called generalized Zamfirescue contractive conditions for the pair (S, T).

3. MAIN RESULT

Consider the nonlinear equation

$$f(x) = 0; \ x \in \mathbb{R}. \tag{3.1}$$

Let $S, T : Y \rightarrow X$ be two non-self mappings with $T(Y) \subseteq S(Y)$, S is onto and T is differentiable. Suppose α is simple root of $f(x) = 0$ and x_0 is initial guess close to α then (3.1) can be written as

$$Sx = Tx. \tag{3.2}$$

Following the approach of Shin et al. [8], we can modify (3.2) by multiplying $\theta \neq -1$ to both sides as follows;

$$\theta Sx = \theta Tx,$$

yields

$$Tx = \frac{\theta Sx + Tx}{1+\theta}, \tag{3.3}$$

where θ is any real number. In order to make (3.3) efficient, we choose $\theta = -cT^{'}x$ and $c \in \mathbb{R}$.

We shall modify Jungck Mann and Jungck Ishikawa iteration schemes by using (3.3) as follows;

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n T_\theta x_n, \tag{3.4}$$

and

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n T_\theta y_n \\ Sy_n &= (1-\beta_n)\,Sx_n + \beta_n T_\theta x_n, 3.5 \end{aligned} \tag{3. 5}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$, $T_\theta x_n = \frac{-c\,T^{'}x_n Sx_n + Tx_n}{1-c\,T^{'}x_n}$ and $T_\theta y_n = \frac{-c\,T^{'}y_n Sy_n + Ty_n}{1-c\,T^{'}y_n}$; $c \in \mathbb{R}$.

Equations (3.4) and (3.5) are called Modified Jungck Mann and Modified Jungck Ishikawa iteration schemes. To show that T_θ coincides with S and T at their coincident point, let $z \in C(S,T)$, that is $Sz = Tz$, then

$$\begin{aligned} T_\theta z &= \frac{-cT^{'}zSz + Tz}{1-cT^{'}z} \\ &= \frac{Tz(1-cT^{'}z)}{1-cT^{'}z} \\ &= Tz. \end{aligned}$$

Similarly one can show that $T_\theta z = Sz$. Hence T_θ coincides with S and T.

4. Convergence Analysis

We discuss the convergence analysis of our modified iteration schemes.

Theorem 4.1 Let $(X, \|.\|)$ be a complete normed space and Y be any arbitrary set. Let us suppose that $S,T : Y \to X$ be two maps with $T(Y) \subseteq S(Y)$, where $S(Y)$ is complete subspace of X, S is injective and T is differentiable. Let z be coincident point of T and S (i. e. $Tz = Sz = p$). Suppose that S and T satisfy the contractive conditions $(2.2-2.4)$. If $\sum_{n=1}^{\infty}\alpha_n = \infty$ then for $x_0 \in Y$, the sequence $\{Sx_n\}_{n=0}^{\infty}$ obtained from (3.5) converges strongly to p.

Proof First we will establish that the conditions $(2.2-2.4)$ imply

$$\begin{aligned} \|T_\theta x - T_\theta y\| &\leq \beta[\|T_\theta x - Sx\| + \|T_\theta y - Sy\|] \\ &\leq \beta[\|T_\theta x - Sx\| + \|Sx - Sy\| + \|T_\theta x - T_\theta y\| + \|T_\theta x - Sx\|], \end{aligned}$$

implies

$$\|T_\theta x - T_\theta y\| \leq \frac{2\beta}{1-\beta}\|T_\theta x - Sx\| + \frac{\beta}{1-\beta}\|Sx - Sy\|. \tag{4.1}$$

Similarly

$$\|T_\theta x - T_\theta y\| \leq \frac{2\gamma}{1-\gamma}\|T_\theta x - Sx\| + \frac{\gamma}{1-\gamma}\|Sx - Sy\| \tag{4.2}$$

From (4.1) and (4.2) we have

$$d(T_\theta x, T_\theta y) \leq 2\delta d(T_\theta x, Sx) + \delta d(Sx, Sy), \tag{4.3}$$

$0 \leq \delta = max(\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}) < 1.$

To prove uniqueness, suppose if $z_1, z_2 \in C(T, S, T_\theta)$ such that $Sz_1 = Tz_1 = T_\theta z_1 = p_1$ and $Sz_2 = Tz_2 = T_\theta z_2 = p_2$. If $p_1 = p_2$ then $Sz_1 = Sz_2$, since S is injective, it follows that $z_1 = z_2$. If $p_{1 \neq} p_2$ then $0 < \|p_1 - p_2\| = \|T_\theta z_1 - T_\theta z_2\| = 2\delta \|p_1 - p_1\| + \delta \|p_1 - p_2\| = \delta \|p_1 - p_2\|$ implies $(1-\delta)\|p_1 - p_2\| \leq 0$, since $\delta \in [0, 1)$, so we have $\|p_1 - p_2\| \leq 0$, since norm can not be negative. Therefore, we have $||p_1 - p_2|| = 0 \Longrightarrow p_1 = p_2$ that is $Sz_1 = Sz_2$, injectivity of S implies that $z_1 = z_2$. Hence $z = z_1 = z_2$ is the unique coincidence point.

Now we will prove that the sequence $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to p. Substituting $x = z$ and $y = x_n$ in (4.3), we have

$$\begin{aligned} \|T_\theta x_n - p\| &\leq 2\delta \|p - p\| + \delta \|Sx_n - p\| \\ &= \delta \|Sx_n - p\| . \end{aligned}$$

Consider,

$$\begin{aligned} \|Sx_{n+1} - p\| &= \|(1-\alpha_n)Sx_n + \alpha_n T_\theta y_n - p\| \\ &\leq (1-\alpha_n)\|Sx_n - p\| + \alpha_n \|T_\theta y_n - p\| \\ &\leq (1-\alpha_n)\|Sx_n - p\| + \alpha_n \delta \|Sy_n - p\| . \end{aligned} \tag{4.4}$$

Again using (4.3) with replacing $x = z$ and $y = y_n$, we have

$$\|T_\theta y_n - p\| \leq \delta \|Sy_n - p\| .$$

$$\begin{aligned} \|Sy_n - p\| &= \|(1-\beta_n)Sx_n + \beta_n T_\theta x_n - p\| \\ &\leq (1-\beta_n)\|Sx_n - p\| + \beta_n \|T_\theta x_n - p\| \\ &\leq (1-\beta_n)\|Sx_n - p\| + \beta_n \delta \|Sx_n - p\| \\ &= (1-(1-\delta)\beta_n)\|Sx_n - p\| , \end{aligned}$$

and by substituting in (4.4), we obtain

$$\begin{aligned} \|Sx_{n+1} - p\| &\leq (1-(1-\delta)\alpha_n(1+\delta\beta_n))\|Sx_n - p\| \\ & \quad (1-(1-\delta)\alpha_n)\|Sx_n - p\| \\ &\leq {}_{k=0}^{n}[1-(1-\delta)\alpha_k]\|Sx_0 - p\| \\ &\leq e^{-(1-\delta){}_{k=0}^{n}\alpha_k}\|Sx_0 - p\| \to 0 \text{ as } n \to \infty, \end{aligned}$$

because $1 + \delta\beta_n \geq 1$. Hence $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to p.

Similarly one can prove the following result.

Theorem 4.2 Let $(X, \|.\|)$ be a complete normed space and Y be any arbitrary set. Let us suppose that $S, T : Y \to X$ be two maps with $T(Y) \subseteq S(Y)$, where $S(Y)$ is complete subspace of X, S is injective and T is differentiable. Let z be coincident point of T and S (i. e. $Tz = Sz = p$). Suppose that S and T satisfy the contractive conditions $(2.2 - 2.4)$. If ${}_{n=1}^{\infty}\alpha_n = \infty$ then for $x_0 \in Y$, the sequence $\{Sx_n\}_{n=0}^{\infty}$ obtained from (3.4) converges strongly to p.

5. Applications

In this section, we present application of our newly modified results namely, Modified Jungck Mann and Modified Jungck Ishikawa, iteration schemes. We solve some nonlinear equations by these methods and make comparison with Jungck Mann and Jungck Ishikawa, iteration schemes respectively.

Example 5.1 Consider the nonlinear equation $\ln x + \tan x = 0$. Let us take $Tx = -\ln x$ and $Sx = \tan x$. If we choose the initial guess $x_0 = 0.5$, then the comparison tables correct upto 10 decimal of Jungck Mann iteration scheme, Modified Jungck Mann iteration scheme for $\alpha = 0.45$, $c = 0.048356087$ and Jungck Ishikawa iteration scheme, Modified Jungck Ishikawa iteration scheme for $\alpha = 0.4$, $\beta = 0.0001$, $c = -0.013633$ are shown below:

Jungck Mann iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.6931471806	0.6123826007	0.5494746374	0.0494746374
1	0.5987926619	0.6062671282	0.5450149444	0.0044596931
2	0.6069420638	0.6065708493	0.5452370042	0.0002220598
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
7	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Modified Jugck Mann iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.6931471806	0.6065554098	0.5452257174	0.0452257174
1	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Jungck Ishikawa iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.6931471806	0.6050313189	0.5441107891	0.0441107891
1	0.6086023964	0.6064595577	0.5451556425	0.0010448534
2	0.6066839426	0.6065492996	0.5452212506	0.0000656081
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
7	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Modified Jungck Ishikawa iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	0.6931471806	0.6066770347	0.5453146256	0.0453146256
1	0.6063923562	0.6065551691	0.5452255414	0.0000890841
2	0.6065557324	0.6065554098	0.5452257174	0.0000001759
3	0.6065554098	0.6065554098	0.5452257174	0.0000000000

Example 5.2 Consider the nonlinear equation $f(x) = x^2 + 10\cos x = 0$. Let us take $Tx = x^2$, $Sx = -10\cos x$. If we choose the initial guess $x_0 = 1$, then the comparison tables correct upto 10 decimal of Jungck Mann iteration scheme, Modified Jungck Mann iteration scheme for $\alpha = 0.99999999$, $c = 0.10848589$ and Jungck Ishikawa iteration scheme,

Modified Jungck Ishikawa iteration scheme for $\alpha = 0.9999999999$, $\beta = 0.99999$, $c = 0.10855$ are shown below:

Jungck Mann iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	1.0000000000	0.9999999360	1.6709637415	0.6709637415
1	2.7921198255	2.7921198076	1.8537696808	0.1828059392
2	3.4364620293	3.4364620229	1.9215931391	0.0678234584
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
26	3.8764606452	3.8764606452	1.9688729378	0.0000000000

Modified Jungck Mann iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	1.0000000000	2.7742339432	1.8519075182	0.8519075182
1	3.4295614562	3.8697543240	1.9681455310	0.1162380128
2	3.8735968312	3.8764606452	1.9688729378	0.0007274055
3	3.8764606452	3.8764606452	1.9688729378	0.0000000000

Jungck Ishikawa iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	1.0000000000	2.7920983326	1.8537674443	0.8537674443
1	3.4364537377	3.6925141638	1.9489997091	0.0952322648
2	3.7985998662	3.8433294839	1.9652814934	0.0162817843
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
13	3.8764606452	3.8764606452	1.9688729378	0.0000000000

Modified Jungck Ishikawa iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	1.0000000000	3.8702066607	1.9681945871	0.9681945871
1	3.8737899327	3.8702079442	1.9681947263	0.0000001392
2	3.8737904806	3.8764606452	1.9688729378	0.0006782114
3	3.8764606452	3.8764606452	1.9688729378	0.0000000000

Example 5.3 Consider the nonlinear equation $f(x) = e^x - \tan x = 0$. Let us take $Tx = \tan x$, $Sx = e^x$. If we choose the initial guess $x_0 = 1$, the comparison tables correct upto 10 decimal of Jungck Mann iteration scheme, Modified Jungck Mann iteration scheme for $\alpha = 0.45$, $c = 0.448449111$ and Jungck Ishikawa iteration scheme, Modified Jungck Ishikawa iteration scheme for $\alpha = 0.35$, $\beta = 0.35$, $c = 0.38043426726$ are shown below:

Modified Jungck Mann iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\lvert e_n \rvert$
0	1.5574077246	3.6925856854	1.3063269404	0.3063269404
1	3.6925856854	3.6925856854	1.3063269404	0.0000000000

Modified Jungck Ishikawa iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	1.5574077246	3.2273108338	1.1716492313	0.1716492313
1	2.3708580788	3.6852254478	1.3043317036	0.1326824723
2	3.6635985444	3.6927611965	1.3063744700	0.0020427663
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
7	3.6925856854	3.6925856854	1.3063269404	0.0000000000

Note: In this example Jungck Mann and Jungck Ishikawa iteration schemes are diverged.

Example 5.4 Consider the nonlinear equation $f(x) = \cos x - xe^x = 0$. Let us take $Tx = \cos x$, $Sx = xe^x$. If we choose the initial guess $x_0 = 0$, then the comparison tables correct upto 10 decimal of Jungck Mann iteration scheme, Modified Jungck Mann iteration scheme for $\alpha = 0.84$, $c = 0.00641955$and Jungck Ishikawa iteration scheme, Modified Jungck Ishikawa iteration scheme for $\alpha = 0.99999$, $\beta = 0.6$, $c = 0.392454$ are shown below:

Jungck Mann iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	1.0000000000	0.8400000000	0.5062907610	0.5062907610
1	0.8745492658	0.8690213832	0.5177927213	0.0115019603
2	0.8689138177	0.8689310282	0.5177572499	0.0000354714
3	0.8689313742	0.8689313188	0.5177573640	0.0000001141
4	0.8689313177	0.8689313179	0.5177573637	0.0000000004
5	0.8689313179	0.8689313179	0.5177573637	0.0000000000

Modified Jungck Mann iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	1.0000000000	0.8400000000	0.5062907610	0.5062907610
1	0.8745492658	0.8689313179	0.5177573637	0.0114666026
2	0.8689313179	0.8689313179	0.5177573637	0.0000000000

Jungck Ishikawa iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	1.0000000000	0.9204417549	0.5376485777	0.5376485777
1	0.8589152599	0.8660953370	0.5166429583	0.0210056194
2	0.8694823338	0.8690874648	0.5178186622	0.0011757039
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
8	0.8689313179	0.8689313179	0.5177573637	0.0000000000

Modified Jungck Ishikawa iteration scheme

n	Tx_n	Sx_{n+1}	x_{n+1}	$\|e_n\|$
0	1.0000000000	0.8778243050	0.5212385873	0.5212385873
1	0.8672030847	0.8696390589	0.5180351512	0.0032034361
2	0.8687937982	0.8689313179	0.5177573637	0.0002777876
3	0.8689313179	0.8689313179	0.5177573637	0.0000000000

5.1. **Graphical Comparison.** In this subsection, we present graphical comparison of approximate solution obtained from our modified results, Jungck Mann and Jungck Ishikawa iteration schemes. Number of iterations are taken on X-axis and approximate solutions are taken on Y-axis. Graphs of approximate solutions of examples 5.1, 5.2, 5.3, and 5.4 obtained from Modified Jungck Mann, Modified Jungck Ishikawa, Jungck Mann and Jungck Ishikawa iterations schemes are given below;

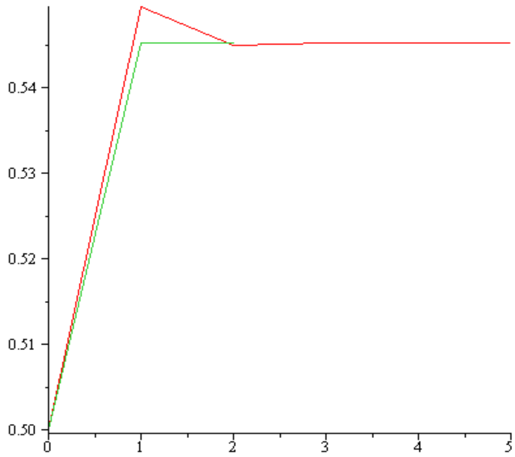

Fig. 1 Evaluation of approximate solution x_k compared to k^{th} iteration for example 5.1 with Modified Jungck Mann (green line) and Jungck Mann (red line) iteration schemes for $\alpha = 0.45$, $c = 0.048356087$.

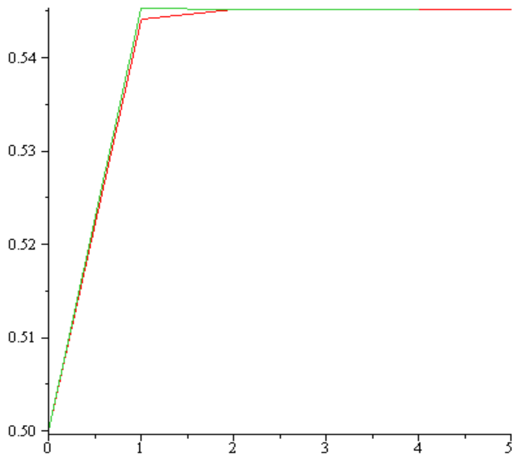

Fig. 2 Evaluation of approximate solution x_k compared to k^{th} iteration for example 5.1 with Modified Jungck Ishikawa (green line) and Jungck Ishikawa (red line)iteration schemes for $\alpha = 0.4$, $\beta = 0.0001$, $c = -i0013633$

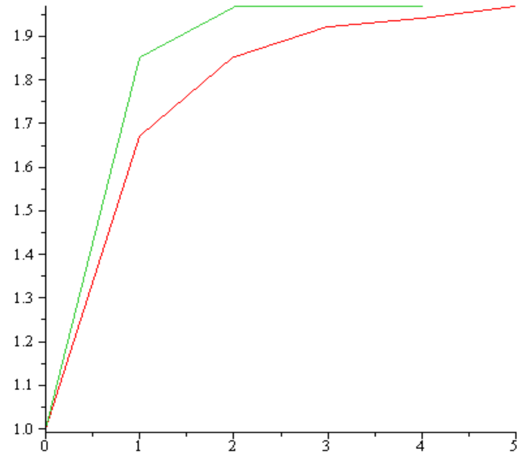

Fig. 3 Evaluation of approximate solution x_k compared to k^{th} iteration for example 5.2 with Modified Jungck Mann (green line)and Jungck Mann (red line) iteration schemes for $\alpha = 0.99999999$, $c = 0.10848589$.

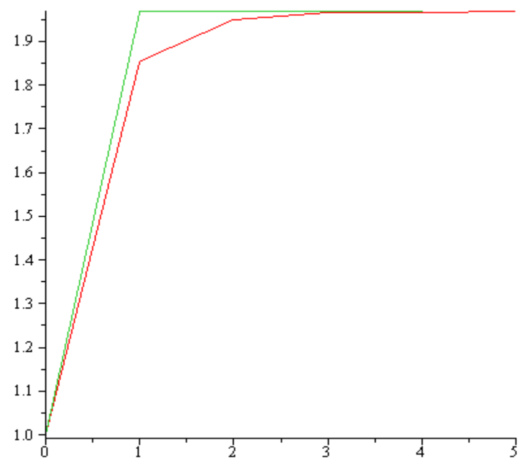

Fig. 4 Evaluation of approximate solution x_k compared to k^{th} iteration for example 5.2 with Modified Jungck Ishikawa (green line) and Jungck Ishikawa (red line)iteration schemes for $\alpha = 0.9999999999$, $\beta = 0.99999$, $c = 0.10855$.

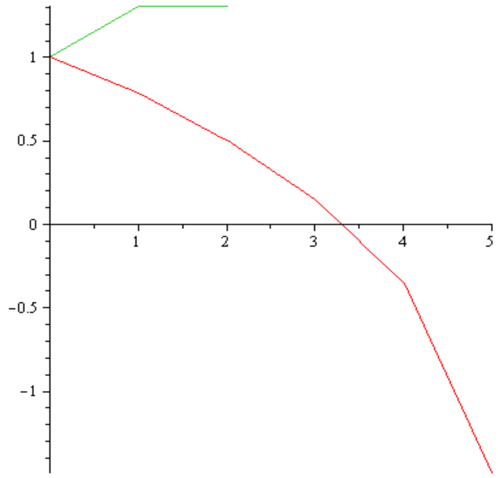

Fig. 5 Evaluation of approximate solution x_k compared to k^{th} iteration for example 5.3 with Modified Jungck Mann (green line) and Jungck Mann (red line) iteration schemes for $\alpha = 0.45$, $c = 0.448449111$.

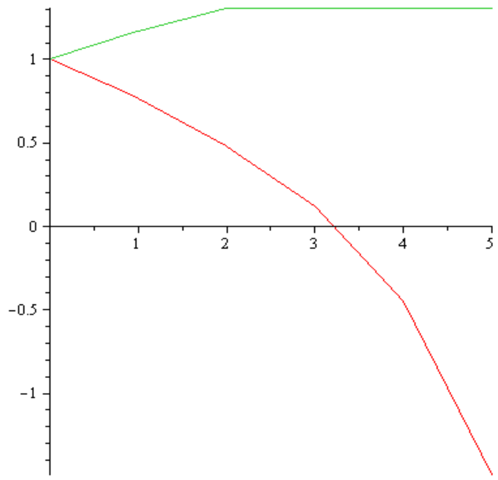

Fig. 6 Evaluation of approximate solution x_k compared to k^{th} iteration for example 5.3 with Modified Jungck Ishiakwa (green line) and Jungck Ishikawa (red line)iteration schemes for $\alpha = 0.35$, $\beta = 0.35$, $c = 0.38043426726$.

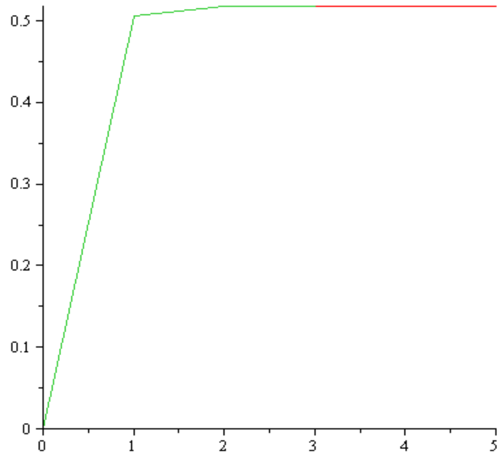

Fig. 7 Evaluation of approximate solution x_k compared to k^{th} iteration for example 5.4 with Modified Jungck Mann (green line)and Jungck Mann (red line) iteration schemes for $\alpha = 0.84$, $c = 0.00641955$.

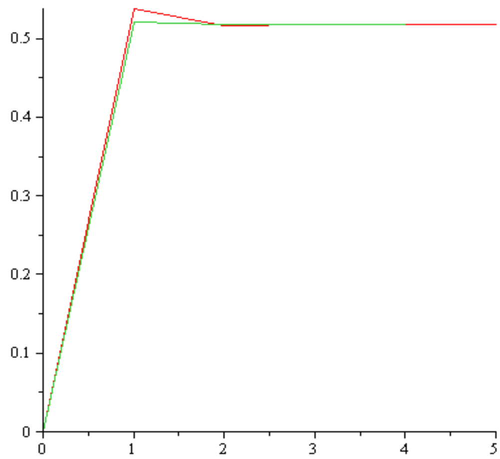

Fig. 8 Evaluation of approximate solution x_k compared to k^{th} iteration for example 5.4 with Modified Jungck Ishikawa (green line) and Jungck Ishikawa (red line) iteration schemes for $\alpha = 0.99999$, $\beta = 0.6$, $c = 0.392454$.

5.2. **Comments.** The graphs of approximations of zeros of nonlinear scalar equations obtained from our modified results, Jungck Mann and Jungck Ishikawa iteration schemes upto 5 iterations are displayed. Green line shows Modified Jungck Mann and Modified Jungck Ishikawa iteration schemes and red line shows Jungck Mann and Jungck Ishikawa iteration schemes. From graphs, we see that our modified results show very higher predictive abilities for approximating zeros of nonlinear scalar equations.

6. Conclusions

From numerical and graphical results, we conclude that our modified, Jungck Mann and Jungck Ishikawa iteration schemes are more efficient than Jungck Mann and Jungck Ishikawa iteration schemes. Modified Jungck Mann and Modified Jungck Ishikawa iteration schemes are faster than Jungck Mann and Jungck Ishikawa iteration schemes for every possible choice of α, β. From example 5.3, we see that our modified Jungck Mann and Jungck Ishikawa iteration schemes also converge for those α, and β for which Jungck Mann and Jungck ishikawa iteration schemes diverge.

7. Acknowledgments

I would like special thanks to referees for their suggestions to improve the quality of paper.

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