Punjab University Journal of Mathematics
(ISSN 1016-2526)
Vol. 52(11)(2020) pp. 1-9

The (s,t)-Padovan Quaternions Matrix Sequence

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Received: 17 April, 2020 / Accepted: 06 October, 2020 / Published online: 12 November, 2020

Abstract.: In this paper, we will introduce the (s,t)-Padovan quaternions matrix sequence. Starting the studies based on the generalization of the Padovan quaternion coefficients in relation to their recurrence, their matrix sequence is then defined. Some mathematical theorems are discussed and the Binet formula and the generating function of this matrix sequence are studied.

AMS (MOS) Subject Classification Codes: 11B37; 11B39
Key Words: matrix sequence; Padovan; quaternions.

1. Introduction

Padovan sequence is a linear and recurring sequence of integers, defined by recurrence:

$$P_n = P_{n-2} + P_{n-3}, n \geqslant 3.$$

With initial values equal to $P_0 = P_1 = P_2 = 1$.

Being a third order sequence, it has its characteristic equation given by $x^3 - x - 1 = 0$, with three roots, one real and two complex. Its historical process can be found in some articles found in the literature, explaining the construction of this sequence and its relationship exists with the plastic number [15] [10] [16].

In order to study the matrix form, the matrices studied in the works of Sokhuma [12] and Seenukul [11] are taken into account. We also highlight that there are five more Padovan matrices. Thus we have the matrix form of the Padovan sequence given by:

For $Q = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, we have: $Q^n = \begin{bmatrix} P_{n-2} & P_{n-1} & P_{n-3} \\ P_{n-3} & P_{n-2} & P_{n-4} \\ P_{n-4} & P_{n-3} & P_{n-5} \end{bmatrix}$, for $n \geqslant 5$.

From the definition for Padovan's positive numbers, it is possible to define a new formula to obtain Padovan's non-positive terms, with $n \in \mathbb{N}$.

$$P_{-n} = P_{-n+3} - P_{-n+1}, n \geqslant 1.$$

In this way, it's possible to study the matrix form for the numbers with non-positive integer index, called ϱ. That matrix form, is obtained by calculating the inverse matrix of Q, for $n \in \mathbb{N}$, resulting in:

For $\varrho = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$, we have: $\varrho^n = \begin{bmatrix} P_{-n-2} & P_{-n-1} & P_{-n-3} \\ P_{-n-3} & P_{-n-2} & P_{-n-4} \\ P_{-n-4} & P_{-n-3} & P_{-n-5} \end{bmatrix}$, for $n > 0$.

A quaternion is an extension of complex numbers, studied in Linear Algebra, developed in 1843 by William Rowan Hamilton (1805-1865), with two quaternionic structures, a priori: quaternions in $\mathbb{R}$, having real components, and the complex quaternions (biquaternions) in $\mathbb{C}$, having complex variables. Quaternions are understood as formal sums of scalars with usual vectors of three-dimensional space, with four dimensions [7] [9].

An important application of these numbers is in relation to the modeling of the sphere, since this numerical set is capable of modeling the rotations of the circle. The application of the sequence for real value cases refers to the study in economics, applied mathematics and among other subjects. In relation to complex cases, there is an application in modern physics. For the cases of quaternions, there is the study of modeling and other areas of physics [5] [6] [13].

A quaternion is described by:

$$q = a + bi + cj + dk,$$

where a, b, c are real numbers and i, j, k the orthogonal part at the base $\mathbb{R}^3$.

Definition 1.1. *The quaternion of Padovan is defined, with* $n \geqslant 0, n \in \mathbb{N}, i^2 = j^2 = k^2 = -1$, *by the equation* [14]*:*

$$QP_n = P_n + iP_{n+1} + jP_{n+2} + kP_{n+3}.$$

Definition 1.2. *The recurrence formula for Padovan's quaternion is defined as* [14]*:*

$$QP_n = QP_{n-2} + QP_{n-3}.$$

The current study is kind of quaternion recurrence relation which is a quaternion difference equation in discrete setting [1]. With this, Padovan's matrix sequence will be explored, generalizing his terms of the recurrence relation, calling them s and t.

2. THE (s,t)-PADOVAN QUATERNIONS MATRIX SEQUENCE

The (s,t)-Padovan matrix sequence were defined in [2]. The split (s,t)-Padovan quaternions were studied in [3] [4]. In this research, we will define and investigate the (s,t)-Padovan quaternions matrix sequence, getting the Binet-like formulas and generating functions.

Definition 2.1. *The (s,t)-Padovan sequence, say $P_n(s,t)$, with $s \geqslant 0, t \neq 0$ and $27t^2 - 4s^3 \neq 0$, is defined by* [2]*:*

$$P_{n+3}(s,t) = sP_{n+1}(s,t) + tP_n(s,t), n > 0,$$

where $P_0(s,t) = 0, P_1(s,t) = 1, P_2(s,t) = 0$.

Definition 2.2. *The split (s,t)-Padovan quaternion, say $QP_n(s,t)$, is defined by* [3]*:*

$$QP_n(s,t) = P_n(s,t)\mathrm{e}_0 + P_{n+1}(s,t)\mathrm{e}_1 + P_{n+2}(s,t)\mathrm{e}_2 + P_{n+3}(s,t)\mathrm{e}_3.$$

Definition 2.3. *The (s,t)-Padovan quaternion, say $QP_n(s,t)$, with $s \geqslant 0$, is defined by* [4]*:*

$$QP_{n+3}(s,t) = sQP_{n+1}(s,t) + tQP_n(s,t).$$

Theorem 2.4. *For $n \geqslant 1$ and $n \in \mathbb{N}$, the matrix form of the Padovan quaternions is given by* [14]*:*

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^n \begin{bmatrix} QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \\ QP_0 & QP_{-1} & QP_{-2} \end{bmatrix} = \begin{bmatrix} QP_{n+2} & QP_{n+1} & QP_n \\ QP_{n+1} & QP_n & QP_{n-1} \\ QP_n & QP_{n-1} & QP_{n-2} \end{bmatrix}.$$

Proof. Using the principle of finite induction, we have that:

For $n = 1$:

$$\begin{aligned} Q^1 \begin{bmatrix} QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \\ QP_0 & QP_{-1} & QP_{-2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \\ QP_0 & QP_{-1} & QP_{-2} \end{bmatrix} \\ &= \begin{bmatrix} QP_1 + QP_0 & QP_0 + QP_{-1} & QP_{-1} + QP_{-2} \\ QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \end{bmatrix} \\ &= \begin{bmatrix} QP_3 & QP_2 & QP_1 \\ QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \end{bmatrix}. \end{aligned}$$

Assuming it is valid for any $n=k, k\in\mathbb{N}$:

$$Q^k\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}=\begin{bmatrix}0&1&1\\1&0&0\\0&1&0\end{bmatrix}^k\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$$
$$=\begin{bmatrix}P_{k-2}&P_{k-1}&P_{k-3}\\P_{k-3}&P_{k-2}&P_{k-4}\\P_{k-4}&P_{k-3}&P_{k-5}\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$$
$$=\begin{bmatrix}QP_{k+2}&QP_{k+1}&QP_k\\QP_{k+1}&QP_k&QP_{k-1}\\QP_k&QP_{k-1}&QP_{k-2}\end{bmatrix}.$$

Verifying that it is valid for $n=k+1$:

$$Q^{k+1}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}=\begin{bmatrix}0&1&1\\1&0&0\\0&1&0\end{bmatrix}^k\begin{bmatrix}0&1&1\\1&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$$
$$=\begin{bmatrix}P_{k-2}&P_{k-1}&P_{k-3}\\P_{k-3}&P_{k-2}&P_{k-4}\\P_{k-4}&P_{k-3}&P_{k-5}\end{bmatrix}\cdot\begin{bmatrix}0&1&1\\1&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$$
$$=\begin{bmatrix}P_{k-1}&P_k&P_{k-2}\\P_{k-2}&P_{k-1}&P_{k-3}\\P_{k-3}&P_{k-2}&P_{k-4}\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$$
$$=\begin{bmatrix}QP_{k+3}&QP_{k+2}&QP_{k+1}\\QP_{k+2}&QP_{k+1}&QP_k\\QP_{k+1}&QP_k&QP_{k-1}\end{bmatrix}.$$

□

Definition 2.5. *For $n\geqslant 0$, the (s,t)-Padovan quaternions matrix sequence, $\Theta P_n(s,t)$ To simplify notation, we take $\Theta P_n(s,t)=\Theta P_n$, with $s>0, t\neq 0$ and $27t^2-4s^3\neq 0$, is defined by:*

$$\Theta P_{n+3}(s,t)=s\Theta P_{n+1}(s,t)+t\Theta P_n(s,t), n\geqslant 0,$$

where $\Theta P_0(s,t)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$, $\Theta P_1(s,t)=\begin{bmatrix}0&s&t\\1&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$
and $\Theta P_2(s,t)=\begin{bmatrix}s&t&0\\0&s&t\\1&0&0\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$.

Theorem 2.6. *For* $n\geqslant 0$, $\Theta P_n(s,t)=\begin{bmatrix}0&s&t\\1&0&0\\0&1&0\end{bmatrix}^n\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}$

Proof. Using the second principle of finite induction on n to prove this theorem.

For $n=0$:

$$
\begin{aligned}
\Theta P_3(s,t) &= s\Theta P_1(s,t) + t\Theta P_0(s,t)\\
&= s\left(\begin{bmatrix}0 & s & t\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}\right) + t\left(\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}\right)\\
&= \left(s\begin{bmatrix}0 & s & t\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix} + t\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}\\
&= \begin{bmatrix}t & s^2 & st\\ s & t & 0\\ 0 & s & t\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}\\
&= \begin{bmatrix}o & s & t\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}^3\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}
\end{aligned}
$$

For $n=k$:

$$
\begin{aligned}
\Theta P_{k+3}(s,t) &= s\Theta P_{k+1}(s,t) + t\Theta P_k(s,t)\\
&= s\left(\begin{bmatrix}0 & s & t\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}^{k+1}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}\right) + t\left(\begin{bmatrix}0 & s & t\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}^{k}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}\right)\\
&= \left(s\begin{bmatrix}P_{k+2} & sP_{k+1}+tP_k & tP_{k+1}\\ P_{k+1} & sP_k+tP_{k-1} & tP_k\\ P_k & sP_{k-1}+tP_{k-2} & tP_{k-1}\end{bmatrix} + t\begin{bmatrix}P_{k+1} & P_{k+2} & tP_k\\ P_k & P_{k+1} & tP_{k-1}\\ P_{k-1} & P_k & tP_{k-2}\end{bmatrix}\right)\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}\\
&= \begin{bmatrix}P_{k+4}(s,t) & P_{k+5}(s,t) & tP_{k+3}(s,t)\\ P_{k+3}(s,t) & P_{k+4}(s,t) & tP_{k+2}(s,t)\\ P_{k+2}(s,t) & P_{k+3}(s,t) & tP_{k+1}(s,t)\end{bmatrix}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}\\
&= \begin{bmatrix}0 & s & t\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}^{k}\begin{bmatrix}QP_2 & QP_1 & QP_0\\ QP_1 & QP_0 & QP_{-1}\\ QP_0 & QP_{-1} & QP_{-2}\end{bmatrix}
\end{aligned}
$$

For $n = k+1$:

$$\begin{aligned}
\Theta P_{k+4}(s,t) &= s\Theta P_{k+2}(s,t) + t\Theta P_{k+1}(s,t) \\
&= s\left(\begin{bmatrix} 0 & s & t \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{k+2} \begin{bmatrix} QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \\ QP_0 & QP_{-1} & QP_{-2} \end{bmatrix}\right) + t\left(\begin{bmatrix} 0 & s & t \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{k+1} \begin{bmatrix} QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \\ QP_0 & QP_{-1} & QP_{-2} \end{bmatrix}\right) \\
&= \left(s \begin{bmatrix} sP_{k+1}+tP_k & sP_{k+2}+tP_{k+1} & tP_{k+2} \\ sP_k+tP_{k-1} & sP_{k+1}+tP_k & tP_{k+1} \\ sP_{k-1}+tP_{k-2} & stP_k+tP_{k-1} & tP_k \end{bmatrix} + t\begin{bmatrix} P_{k+2} & sP_{k+1}+tP_k & tP_{k+1} \\ P_{k+1} & sP_k+tP_{k-1} & tP_k \\ P_k & sP_{k-1}+tP_{k-2} & tP_{k-1} \end{bmatrix}\right) \\
&\cdot \begin{bmatrix} QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \\ QP_0 & QP_{-1} & QP_{-2} \end{bmatrix} \\
&= \begin{bmatrix} P_{k+5}(s,t) & P_{k+6}(s,t) & tP_{k+4}(s,t) \\ P_{k+4}(s,t) & P_{k+5}(s,t) & tP_{k+3}(s,t) \\ P_{k+3}(s,t) & P_{k+4}(s,t) & tP_{k+2}(s,t) \end{bmatrix} \begin{bmatrix} QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \\ QP_0 & QP_{-1} & QP_{-2} \end{bmatrix} \\
&= \begin{bmatrix} 0 & s & t \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{k+1} \begin{bmatrix} QP_2 & QP_1 & QP_0 \\ QP_1 & QP_0 & QP_{-1} \\ QP_0 & QP_{-1} & QP_{-2} \end{bmatrix}
\end{aligned}$$

□

Definition 2.7. *The characteristic equation for the (s,t)-Padovan quaternion matrix sequence is* [2]*:*

$$x^3 - sx - t = 0$$

Theorem 2.8. *The Binet-like formulas for the nth (s,t)-Padovan, for $27t^2 - 4s^3 \neq 0$ and $t \neq 0$, is* [2] [3]*:*

$$P_n(s,t) = a\alpha^n + b\beta^n + c\gamma^n, n \geqslant 0,$$

where α, β, γ are the roots of the characteristic equation, and $a = \frac{(\beta-1)(\gamma-1)}{(\alpha-\beta)(\alpha-\gamma)}$, $b = \frac{(\alpha-1)(\gamma-1)}{(\beta-\alpha)(\beta-\gamma)}$ and $c = \frac{(\alpha-1)(\beta-1)}{(\alpha-\gamma)(\beta-\gamma)}$.

Proof. The discriminant $\Delta = \frac{-t^2}{4} + \frac{s^3}{27}$ concerning the 3rd degree equation, determines the how the roots of the equation will be. Where $\Delta \neq 0$ all roots will be distinct, easily coming to the conclusion that $27t^2 - 4s^3 \neq 0$. Note also that $\alpha\beta\gamma = t$, this condition implies that when $t \neq 0$.The coefficient values are obtained by solving the linear system with three variables.

□

Theorem 2.9. *The Binet-like formulas for the nth (s,t)-Padovan quaternion matrix sequence is:*

$$\Theta P_n(s,t) = \overline{a}\alpha^n + \overline{b}\beta^n + \overline{c}\gamma^n, n \geqslant 0$$

where α,β,γ *36/5000 are roots of the characteristic equation,* $\overline{a}=\frac{\Theta P_2+\Theta P_1\alpha+\Theta P_0\beta\gamma}{(\alpha-\beta)(\alpha-\gamma)}$, $\overline{b}=\frac{\Theta P_2+\Theta P_1\beta+\Theta P_0\alpha\gamma}{(\beta-\alpha)(\beta-\gamma)}$ *and* $\overline{c}=\frac{\Theta P_2+\Theta P_1\gamma+\Theta P_0\alpha\beta}{(\gamma-\alpha)(\gamma-\beta)}$.

Proof. From the definition of nth (s,t)-Padovan quaternion matrix sequence and Binet formula for the nth (s,t)-Padovan, we write:

$$\begin{cases} a+b+c & =\Theta P_0 \\ a\alpha+b\beta+c\gamma & =\Theta P_1 \\ a\alpha^2+b\beta^2+c\gamma^2 & =\Theta P_2 \end{cases}$$

Solving the system, we have:

$$\begin{aligned}
\overline{a} &= \frac{\Theta P_2-\Theta P_1\beta-\Theta P_1\gamma+\Theta P_0\beta\gamma}{\alpha^2-\alpha\beta-\alpha\gamma+\beta\gamma} \\
&= \frac{\Theta P_2-\Theta P_1\beta-\Theta P_1\gamma+\Theta P_0\beta\gamma}{(\alpha-\beta)(\alpha-\gamma)} \\
\overline{b} &= \frac{\Theta P_2-\Theta P_1\alpha-\Theta P_1\gamma+\Theta P_0\alpha\gamma}{\beta^2-\alpha\beta-\beta\gamma+\alpha\gamma} \\
&= \frac{\Theta P_2-\Theta P_1\alpha-\Theta P_1\gamma+\Theta P_0\alpha\gamma}{(\beta-\alpha)(\beta-\gamma)} \\
\overline{c} &= \frac{\Theta P_2-\Theta P_1\alpha-\Theta P_1\beta+\Theta P_0\alpha\beta}{\gamma^2+\alpha\beta-\alpha\gamma-\beta\gamma} \\
&= \frac{\Theta P_2-\Theta P_1\alpha-\Theta P_1\beta+\Theta P_0\alpha\beta}{(\gamma-\alpha)(\gamma-\beta)}
\end{aligned}$$

Using Girard Relations $\alpha\beta\gamma=1$, we have:

$$\begin{aligned}
\overline{a} &= \frac{\Theta P_2+\Theta P_1\alpha+\Theta P_0\beta\gamma}{(\alpha-\beta)(\alpha-\gamma)} \\
\overline{b} &= \frac{\Theta P_2+\Theta P_1\beta+\Theta P_0\alpha\gamma}{(\beta-\alpha)(\beta-\gamma)} \\
\overline{c} &= \frac{\Theta P_2+\Theta P_1\gamma+\Theta P_0\alpha\beta}{(\gamma-\alpha)(\gamma-\beta)}
\end{aligned}$$

□

Theorem 2.10. *The generating function for the* n*th* (s,t)*-Padovan quaternion matrix sequence is:*

$$G\Theta P(x)=\frac{\Theta P_0+\Theta P_1x+(\Theta P_2-s\Theta P_0)x^2}{(1-sx^2-tx^3)}$$

Proof. Assume that the function:

$$G\Theta P(x)=\sum_{n=0}^{\infty}\Theta P_nx^n=\Theta P_0+\Theta P_1x+\Theta P_2x^2+\ldots+\Theta P_nx^n$$

be generating function of the (s,t)-Padovan quaternion matrix sequence. Multiply both of side of the equality by the terms $-sx^2,-tx^3$ such as:

$$-sx^2 G\Theta P(x) = \Theta P_n x^n = -s\Theta P_0 x^2 - s\Theta P_1 x^3 - s\Theta P_2 x^4 - \ldots - s\Theta P_n x^{n+2}$$
$$-tx^3 G\Theta P(x) = \Theta P_n x^n = -t\Theta P_0 x^3 - t\Theta P_1 x^4 - t\Theta P_2 x^5 - \ldots - t\Theta P_n x^{n+3}$$

Then,we write $(1 - sx^2 - tx^3)G\Theta P(x)$:

$$\begin{aligned}(1 - sx^2 - tx^3)G\Theta P(x) &= \Theta P_0 + \Theta P_1 x + \Theta P_2 x^2 + \ldots + \Theta P_n x^n \\ &- s\Theta P_0 x^2 - s\Theta P_1 x^3 - s\Theta P_2 x^4 - \ldots - s\Theta P_n x^{n+2} \\ &- t\Theta P_0 x^3 - t\Theta P_1 x^4 - t\Theta P_2 x^5 - \ldots - t\Theta P_n x^{n+3} \\ &= \Theta P_0 + \Theta P_1 x + (\Theta P_2 - s\Theta P_0)x^2 \\ G\Theta P(x) &= \frac{\Theta P_0 + \Theta P_1 x + (\Theta P_2 - s\Theta P_0)x^2}{(1 - sx^2 - tx^3)}\end{aligned}$$

□

3. CONCLUSION

In this work it was possible to study the (s, t)-Padovan quaternion matrix sequence, identifying some mathematical theorems of this sequence. This sequence allows to generalize the quaternions of the Padovan sequence, since the recurrence coefficients have been generalized.

With this we can establish the process of generalizing the Padovan quaternions, continuing the work of Cerda [2] and Diskaya and Menken [3]. For future work, it is investigated the continuation of this process for the octonions [8] and the application in other areas of these numbers.

The difficulty found in relation to the study of Padovan's quaternions, was in relation to the introduction to the process of complexification of this sequence, studying Padovan's real quaternions. With that, the imaginary units were inserted, remaining with the coefficients of the definition of the quaternions in a real way.

For future work, it is suggested an application of this sequence in the study of modeling, present in the area of modern physics. Performing its visualization, as well as its relation to the generalization of that sequence [6]. In addition, the complexity of the generalization of this sequence can be analyzed.

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