Punjab University Journal of Mathematics
(ISSN 1016-2526)
Vol. 52(4)(2020) pp. 41-51

$nI\alpha g$-closed sets and Normality via $nI\alpha g$-closed sets in Nano Ideal Topological Spaces

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Received: 10 June, 2019 / Accepted: 05 March, 2020 / Published online: 01 April, 2020

Abstract.: We have defined a new generalised closed set called $nI\alpha g$ closed sets in nano ideal topological spaces. Also, association of $nI\alpha g$ closed sets with various existing closed sets are studied. Characterisations and equivalent conditions of $nI\alpha g$ closed sets are proved. Normality via $nI\alpha g$ closed sets are also been studied in this paper.

AMS (MOS) Subject Classification Codes: 54A05, 54A10, 54B05.
Key Words: nIg closed sets, $nI\alpha g$ closed sets, $nI\alpha g$-normal spaces.

1. Introduction

Introduction of ideals in topology was initiated by Kuratowski [10]. Ideals we mean a subset $\jmath$ satisfying
i. $\mathcal{P} \in \jmath$ and if $\mathcal{Q} \subset \mathcal{P}$ for any subset $\mathcal{Q}$,then $\mathcal{Q} \in \jmath$
ii. $\mathcal{P} \in \jmath$ and $\mathcal{Q} \in \jmath$, then $\mathcal{P} \cup \mathcal{Q}$ should also be in $\jmath$.
Jankovic et al. [7] defined the local function $\mathcal{P}^*$, in ideal topology. $\mathcal{P}^*$=$\{x \in \chi : \upsilon \cap \mathcal{P} \notin \jmath, \forall \upsilon \in \tau\}$. Here $\tau(x) = \{\upsilon \in \tau : x \in \upsilon\}$. Kuratowski's closure operator is defined as $cl^*(\mathcal{P}) = \mathcal{P} \cup \mathcal{P}^*$ in the $*$-topology $\tau^*(I, \tau)$. I-open sets were introduced by Jankovic et al. [8] and the work was extended by Hamlett et al. [7]. Various decompositions in

ideal spaces under various closed sets are endeavoured by Ekici [4]. A significant approach on Ig^*-openness as Ig^*-openness can be placed between topology and Levine's openness in ideal spaces was made by Erdal [2]. Recently, generalised closed sets in ideal spaces have been studied in some papers [[2],[3],[4],[5],[6], [9], [13], [14]]. The initiation of nano topology was done by Lellis Thivagar [12]. He has defined nano topology by considering a subset χ of the the universal set υ and its equivalence relation on υ, combined with approximations and the boundary regions. Bhuvaneshwari et al. [1] initiated ng closed sets in nano topology. Parimala et al. [15] had worked on ideals in nano topological spaces. Characteristics of nano local function were studied by Parimala et al. [15]. nIg closed sets were best studied by Parimala et al. [16]. This work aims the introduction of $nI\alpha g$ closed sets in nano ideal topological spaces (NITS). The $nI\alpha g$ closed sets are compared with some existing sets. Many equivalent condition on these sets are proved. $nI\alpha g$-normal spaces are also discussed.

2. Notations and Preliminaries

In the following sequel the following notations are used.
i. nano topological spaces - NTS.
ii. nano ideal topological spaces - NITS .
iii. nano open sets - NOS.
iv. nano closed set - NCS.
v. open set - OS.
vi. closed set - CS.

Definition 2.1. [11] When υ be the universal set and $\Re$, the equivalence relation defined on υ, then lower approximation $Low_{(\Re)}$, upper approximation $Upp_{(\Re)}$ and the boundary region $Bou_{(\Re)}$ are defined as follows.

i) $Low_{(\Re)} = \cup\{\Re(\chi) : \Re(\chi) \subseteq \chi, x \in \upsilon\}$.
ii) $Upp_{(\Re)} = \cup\{\Re(\chi) : \Re(\chi) \cap \chi \neq \phi, x \in \upsilon\}$.
iii) $Bou_{(\Re)} = Low_{(\Re)} - Upp_{(\Re)}$.

Definition 2.2. [12] Properties of $Low_{(\Re)}$, $Upp_{(\Re)}$ and $Bou_{(\Re)}$ are stated below.

i) $Low_{\Re(L)} \subseteq L \subseteq Upp_{\Re(L)}$.
ii) $Low_{(\Re)}(\phi) = Upp_{(\Re)}(\phi) = \phi$.
iii) $Low_{(\Re)}(\upsilon) = Upp_{(\Re)}(\upsilon) = \upsilon$.
iv) $Upp_{(\Re)}(L \cup M) = Upp_{(\Re)}(L) \cup Upp_{(\Re)}(M)$.
v) $Upp_{(\Re)}(L \cap M) \subseteq Upp_{(\Re)}(L) \cap Upp_{(\Re)}(M)$.
vi) $Low_{(\Re)}(L \cup M) \supseteq Low_{(\Re)}(L) \cup Low_{(\Re)}(M)$.
vii) $Low_{(\Re)}(L \cap M) = Low_{(\Re)}(L) \cap Low_{(\Re)}(M)$.
viii) $Low_{(\Re)}(L) \subset Low_{(\Re)}(M)$ and $Upp_{(\Re)}(L) \subseteq Upp_{(\Re)}(M)$ while $L \subseteq M$.
ix) $Upp_{(\Re)}(L^c) = [Low_{(\Re)}(L)]^c$ and $Low_{(\Re)}(L^c) = [Upp_{(\Re)}(L)]^c$.
x) $Upp_{(\Re)}[Upp_{(\Re)}(L)] = Low_{(\Re)}[Upp_{(\Re)}(L)] = Upp_{(\Re)}(L)$.
xi) $Low_{(\Re)}[Low_{(\Re)}(L)] = Upp_{(\Re)}[Low_{(\Re)}(L)] = Low_{(\Re)}(L)$.

Definition 2.3. [12] Let $(\upsilon, \Re)$ be the approximation space and the set $\tau_{\Re}(X) = \{\upsilon, \phi, Low_{(\Re)}, Upp_{(\Re)}, Bou_{(\Re)}\}$, $X \subseteq \upsilon$ is called nano topology based on υ according to X. Here $\tau_{\Re}(X)$ satisfies the axioms of topology. $(\upsilon, \tau_{\Re}(X)$ is called NTS. The members of $\tau_{\Re}(X)$ are NOS and the complements are NCS.

Remark 2.4. [12] In $(\upsilon, \tau_{\Re}(X)$, the set $\mathcal{B} = \{\upsilon, , Low_{(\Re)}, Bou_{(\Re)}\}$ is called the basis.

Definition 2.5.[15] The nano local function of $(\upsilon, \mathcal{N}, I)$ can be defined as $\mathcal{P}^*_n = \{x \in \upsilon, G_n \cap \mathcal{P} \notin I$, for every $G_n \in G_n(x)\}$.

Definition 2.6. [15] In $(\upsilon, \mathcal{N})$ let the ideals be $\jmath$, $\jmath'$. $\mathcal{P}$, $\mathcal{Q}$ be subsets of υ. Then

i) $\mathcal{P}$ subset $\mathcal{Q} \Rightarrow \mathcal{P}^*_n$ subset $\mathcal{Q}^*_n$.
ii) $\jmath \subseteq \jmath' \Rightarrow \mathcal{P}^*_n(\jmath') \subseteq \mathcal{P}^*_n(\jmath)$.
iii) $\mathcal{P}^*_n = n\text{-}cl(\mathcal{P}^*_n)$ subset $n\text{-}cl(\mathcal{P})$ where $\mathcal{P}^*_n \subset n\text{-}cl(\mathcal{P})$.
iv) $(\mathcal{P}^*_n)^*_n \subseteq \mathcal{P}^*_n$.
v) $\mathcal{P}^*_n \cup \mathcal{Q}^*_n = (\mathcal{P} \cup \mathcal{Q})^*_n$.
vi) $\mathcal{P}^*_n - \mathcal{Q}^*_n = (\mathcal{P} - \mathcal{Q})^*_n - \mathcal{Q}^*_n \subseteq (\mathcal{P} - \mathcal{Q})^*_n$.
vii) $\mathcal{V} \in \mathcal{N} \Rightarrow \mathcal{V} \cap \mathcal{P}^*_n = \mathcal{V} \cap (\mathcal{V} \cap \mathcal{P})^*_n$ subset $(\mathcal{V} \cap \mathcal{P})^*_n$.
viii) $J \in I \Rightarrow (\mathcal{P} \cup J)^*_n = \mathcal{P}^*_n = (\mathcal{P} - J)^*_n$.

Lemma 2.7. [15] In $(\upsilon, \mathcal{N}, I)$ if $\mathcal{P} \subseteq \mathcal{P}^*_n$, for any subset $\mathcal{P}$, then $\mathcal{P}^*_n = n\text{-}cl(\mathcal{P}^*_n) = n\text{-}cl(\mathcal{P})$.

Lemma 2.8. [15] In $(\upsilon, \mathcal{N}, I)$, the set operator $n\text{-}cl^*(\mathcal{P}) = \mathcal{P} \cup \mathcal{P}^*_n$ for $\mathcal{P} \subseteq \chi$.

Definition 2.9.[15] The characteristics of $n\text{-}cl^*$ are as follows

i) $\mathcal{P} \subseteq n\text{-}cl^*(\mathcal{P})$.
ii) $n\text{-}cl^*(\phi) = \phi$ and $n\text{-}cl^*(\upsilon) = \upsilon$.
iii) $\mathcal{P} \subset \mathcal{Q}$, implies $n\text{-}cl^*(\mathcal{P}) \subseteq n\text{-}cl^*(\mathcal{Q})$.
iv) $n\text{-}cl^*(\mathcal{P}) \cup n\text{-}cl^*(\mathcal{Q}) = n\text{-}cl^*(\mathcal{P} \cup \mathcal{Q})$.
v) $n\text{-}cl^*(n\text{-}cl^*(\mathcal{P})) = n\text{-}cl^*(\mathcal{P})$.

Definition 2.10.[15] An ideal $\jmath$ is called $\mathcal{N}$-codense ideal if $\mathcal{N} \cap \jmath = \{\phi\}$.

Definition 2.11.[15] A subset $\mathcal{P}$ of $(\upsilon, \mathcal{N}, I)$ is $n*$-dense in itself (resp. $n*$-perfect) if $\mathcal{P} \subseteq \mathcal{P}^*_n$ (resp. $\mathcal{P} = \mathcal{P}^*_n$).

Remark 2.12.[15] If $\mathcal{P}$ is $n*$-dense in itself, then $\mathcal{P}^*_n = n\text{-}cl(\mathcal{P}^*_n) = n\text{-}cl(\mathcal{P}) = n\text{-}cl^*(\mathcal{P})$.

Definition 2.13. A subset $\mathcal{P}$ of $(\upsilon, \mathcal{N}, I)$ is

i) nIg-closed, if $\mathcal{P}^*_n \subseteq \mathcal{V}$, $\mathcal{P} \subseteq \mathcal{V}$ and $\mathcal{V}$ is n-open [16].
ii) $n\alpha$-OS, if $\mathcal{P} \subset N\text{-}int(n\text{-}cl(n\text{-}int(\mathcal{P})))$ [12].
iii) ng-closed, if $n\text{-}cl(\mathcal{P}) \subseteq \mathcal{V}$ while $\mathcal{P} \subseteq \mathcal{V}$ for a NOS $\mathcal{V}$.[1]
iv) $ng\alpha$-closed , if $n\alpha\text{-}cl(\mathcal{P}) \subseteq \mathcal{V}$ while $\mathcal{P} \subseteq \mathcal{V}$ for a nano α OS $\mathcal{V}$.[17].

Theorem 2.14. A set which is nano open is always a nano α-open.[12]

Theorem 2.15. A $n\alpha$-open set is always a $ng\alpha$-open set.[17]

3. $nI\alpha g$-CLOSED SETS

In this article $(\upsilon, \mathcal{N}, I)$ represents nano ideal topological space.

Definition 3.1. In $(\upsilon, \mathcal{N}, I)$, nano ideal α-generalized closed set (briefly $nI\alpha g$-closed set),

we mean if for a subset $\mathcal{P}$, $\mathcal{P}_n^* \subseteq V$ whenever $\mathcal{P} \subseteq V$ and V is a $n\alpha$-open set.
$\mathcal{P}$ is a $nI\alpha g$-open set if $\upsilon - P$ is a $nI\alpha g$-closed set.

Example 3.2. Consider the universal set $\upsilon = \{x, y, z, w\}$, the approximation space $\upsilon/R = \{\{x\}, \{z\}, \{y, w\}\}$, $X = \{x, y\} \subseteq \upsilon$ with the ideal $I = \{\phi, \{z\}, \{y\}, \{y, z\}\}$. $nI\alpha g$-closed sets are $\{\{y\}, \{z\}, \{x, z\}, \{y, z\}, \{z, w\}, \{x, y, z\},$
$\{x, z, w\}, \{y, z, w\}, \upsilon, \phi\}$.

Theorem 3.3 In a $(\upsilon, \mathcal{N}, I)$, the following implications are true and the reverse cases need not be true in general.

i) All NCS are $nI\alpha g$-closed.
ii) All n^*-CS are $nI\alpha g$-closed.
iii) Allng-CS are a $nI\alpha g$-CS.
iv) All $nI\alpha g$-CS are nIg-CS.
v) All $nI\alpha g$-CS are $ng\alpha$-closed only when $\mathcal{P}$ is n^*-dense in itself.

Proof.

i) Assume that υ be a nano α-CS and $\mathcal{P} \subseteq \upsilon$ be a NCS. Then we may infer that n-$cl(\mathcal{P}) = \mathcal{P} \subseteq \upsilon$, implies n-$cl(\mathcal{P}) \subseteq \upsilon$. Also n-$cl^*(\mathcal{P}) = \mathcal{P} \cup \mathcal{P}_n^* \subseteq n$-$cl(\mathcal{P}) \subseteq \upsilon$, leads $\mathcal{P} \subseteq \upsilon$ and $\mathcal{P} \cup \mathcal{P}_n^* \subseteq \upsilon$, implying $\mathcal{P}_n^* \subset \upsilon$ and υ is a nano α-OS. Therefore $\mathcal{P}$ is a $nI\alpha g$-CS.
ii) Assume that υ, a nano α-OS and $\mathcal{P}$, a n^*-CS. Therefore we get $\mathcal{P}_n^* \subseteq \mathcal{P} \subseteq \upsilon$, which implying $\mathcal{P}_n^* \subseteq \upsilon$ and $\mathcal{P} \subseteq \upsilon$ and υ is nano α-OS. Hence $\mathcal{P}$ is a $nI\alpha g$-CS.
iii) Consider a ng-CS $\mathcal{P}$ and a NOS υ and $\mathcal{P} \subset \upsilon$. As $\mathcal{P}$ is ng-CS, we have n-$cl(\mathcal{P}) \subseteq \upsilon$. Also we have $\mathcal{P}_n^* = n$-$cl(\mathcal{P}_n^*) \subseteq n$-$cl(\mathcal{P}) \subseteq \upsilon$, which implies $\mathcal{P}_n^* \subseteq \upsilon$ and $\mathcal{P} \subseteq \upsilon$. υ is n-OS. As every n-OS is a nano α-OS, proof follows.
iv) Consider υ as a NOS and $\mathcal{P}$ be a subset of υ. Referring Theorem 2.14, $\mathcal{P}$ may be a $n\alpha$-open set. Whenever $\mathcal{P}$ is considered to be a $nI\alpha g$-closed set, we get $\mathcal{P}_n^* \subseteq \upsilon$ and $\mathcal{P} \subseteq \upsilon$. By assumption υ is NOS. Hence $\mathcal{P}$ is a nIg-closed set.
v) Let $\mathcal{P}$ a $n\alpha$-OS υ. By Theorem 2.15 υ may be a $ng\alpha$-OS. Assume that $\mathcal{P}$ to be a $nI\alpha g$-CS. Therefore we have $\mathcal{P}_n^* \subseteq \upsilon$. Also $\mathcal{P}_n^* = n$-$cl(\mathcal{P})$. That is we lead that n-$cl(\mathcal{P}) \subseteq \upsilon$. Also $n\alpha$-$cl(\mathcal{P}) \subseteq n$-$cl(\mathcal{P}) \subseteq \upsilon$. υ is a nano α-OS. Hence $\mathcal{P}$ is a $ng\alpha$-CS.

The reverse implications of the results may not true in all occasions, examples below explains the fact.

Example 3.4. Consider the universal set $\upsilon = \{x, y, z, w\}$, approximation space $\upsilon/R = \{\{x\}, \{z\}, \{y, w\}\}$, $\chi = \{x, y\}$ and the ideal $I = \{\phi, \{z\}, \{y\}, \{y, z\}\}$.

i) $A = \{y\}$ is a $nI\alpha g$-CS and not a n-CS.
ii) $A = \{z, w\}$ is a $nI\alpha g$-CS and not a n^*-CS.
iii) $A = \{y\}$ is a $nI\alpha g$-CS and not a ng-CS.
iv) Consider the universal set $\upsilon = \{x, y, z, w, s\}$, approximation space $\upsilon/R = \{\{w\}, \{x, y\}, \{z, s\}\}$, $\chi = \{x, w\}$, $I = \{\phi, \{x\}, \{s\}, \{x, s\}\}$. In this example $\mathcal{P} = \{x, z\}$ is a nIg-CS and not a $nI\alpha g$-CS.

Theorem 3.5. Consider $(\upsilon, \mathcal{N}, I)$ to be a NITS. Whenever the subset $\mathcal{P} \in I$, then $\mathcal{P}$ is a $nI\alpha g$-CS in U.

Proof. Consider $\mathcal{P} \subseteq K$. K is $n\alpha$-OS. Since $\mathcal{P} \in I$, we get $\mathcal{P}_n^* = \phi$ always. Which implies $\mathcal{P}_n^* \subseteq \mathcal{P} \subseteq K$ and $\mathcal{P}_n^* \subseteq K$ and υ is a $n\alpha$-OS. Therefore $\mathcal{P} \in I$ is always a $nI\alpha g$-CS.

Theorem 3.6. In a $(\upsilon, \mathcal{N}, I)$, if $\mathcal{P}, \mathcal{Q} \subseteq U$ are $nI\alpha g$-closed sets, then $\mathcal{P} \cup \mathcal{Q}$ is also a $nI\alpha g$-CS.
Proof. As $\mathcal{P}$ and $\mathcal{Q}$ are $nI\alpha g$-CS, $\mathcal{P}_n^* \subseteq K$. $\mathcal{P} \subseteq K$ and K is a nano α-OS. Also $\mathcal{Q}_n^*$ subset K and $\mathcal{Q}$ subset K and K is a $n\alpha$-OS. As $\mathcal{P}, \mathcal{Q} \in K$. $\mathcal{P} \cup \mathcal{Q} \subseteq K$. Also $(\mathcal{P} \cup \mathcal{Q})_n^* = \mathcal{P}_n^* \cup \mathcal{Q}_n^*$ subset of K. K is $n\alpha$-OS. It means $\mathcal{P} \cup \mathcal{Q}$ is a $nI\alpha g$-CS.

Remark 3.7. $\mathcal{P} \cap \mathcal{Q}$ may not be a $nI\alpha g$-CS in all occasions.

Theorem 3.8. In a NITS, if a set is both $nI\alpha g$-closed and $n\alpha$-open set then it is a n^*-closed set.
Proof. Consider $\mathcal{P}$ to be a $nI\alpha g$-CS which is also a $n\alpha$-OS. Since $\mathcal{P}$ is $n\alpha$-open, we get $\mathcal{P} \subseteq \mathcal{P}$ and hence $\mathcal{P}_n^*$ subset $\mathcal{P}$. Hence $\mathcal{P}$ is a n^*-CS.

Theorem 3.9. In $(\upsilon, \mathcal{N}, I)$, the necessary and sufficient condition for any subset to be a $nI\alpha g$-CS is that every $n\alpha$-OS is a n^*-CS.
Proof. Necessary part. Let all subsets of υ is a $nI\alpha g$-closed sets and one of the subset K be a $n\alpha$-open set. By referring Theorem 3.8, the proof follows.
Sufficient part. Let every $n\alpha$-open subset of υ be a n^*-CS. Let K be one such set such that $\mathcal{P} \subseteq K \subseteq \upsilon$. Then we infers that $\mathcal{P}_n^* \subseteq K \subseteq \upsilon$. By definition $\mathcal{P}$ is a $nI\alpha g$-CS.

Theorem 3.10. Consider a NITS $(\upsilon, \mathcal{N}, I)$ and K be a non empty nano open(n-open set) subset of υ. Then the statements discussed below are equivalent to each other.

a) K is a $nI\alpha g$-CS.
b) $n\text{-}cl^*(K) \subset V$, V a $n\alpha$-open subset(n-open) of υ.
c) Each $x \in n\text{-}cl^*(K)$, there exists atleast one element in $n\alpha\text{-}cl(\{x\}) \cap K (n\text{-}cl(\{x\}) \cap K)$.
d) $n\text{-}cl^*(K) - K$ always contains an empty $n\alpha$-CS.
e) $K_n^* - K$ is always empty and a nano α-CS.

Proof. $a) \implies b)$. Consider a $nI\alpha g$-CS K of $(\upsilon, \mathcal{N}, I)$. Definition of $nI\alpha g$-CS infers that $K_n^* \subseteq V$ whenever $K \subseteq V$ and V is $n\alpha$-open in $(\upsilon, \mathcal{N}, I)$. Also $n\text{-}cl^*(K) = K_n^* \cup K \subseteq V$. V is a $n\alpha$-OS(n-OS). which proves hypothesis b).
$b) \implies c)$. Consider an element x in $n\text{-}cl^*(K)$. Let us assume the contrary that $n\alpha\text{-}cl(\{x\}) \cap K = \phi$. ($n\text{-}cl(\{x\}) \cap K = \phi$). So that we have $K \subseteq (\upsilon - (n\alpha\text{-}cl(\{x\})))$. By referring hypothesis b), it is possible that $n\text{-}cl^*(K) \subseteq (U - (n\alpha\text{-}cl(\{x\})))$. which contradicts to our assumption. Therefore $n\alpha\text{-}cl(\{x\}) \cap K \neq \phi$.
$c) \implies d)$. Consider a $n\alpha$-CS M which is non empty and let $M \subseteq n\text{-}cl^*(K) - K$. Let $x \in M$. As M is $n\alpha$-closed, we may write $M \subseteq (\upsilon - K)$. $K \subseteq (\upsilon - M)$. Therefore $n\alpha\text{-}cl(\{x\}) \cap K = \phi$, which contradicts to the hypothesis c). Hence $n\text{-}cl^*(K) - K$ always contains an empty $n\alpha$-CS.(n-CS)
$d) \implies e)$. Since $n\text{-}cl^*(K) = K \cup K_n^*$, we get $n\text{-}cl^*(K) - K = (K \cup K_n^*) - K = (K \cup K_n^*) \cap K^c = (K \cap K^c) \cup (K_n^* \cap K^c) = \phi \cup (K_n^* \cap K^c) = K_n^* - K$. Referring

hypothesis d) it is concluded that $K_n^* - K$ also contains no non empty nano α-CS.(n-CS).
$e) \Longrightarrow a)$. Consider a n-OS $\mathcal{P} \subseteq K$. K is a nano α-OS. Then $\upsilon - K \subseteq (\upsilon - \mathcal{P})$. Also $\mathcal{P}_n^* \cap (\upsilon - K) \subseteq \mathcal{P}_n^* \cap (\upsilon - \mathcal{P}) = \mathcal{P}_n^* \cap (\upsilon \cap \mathcal{P}^c) = \mathcal{P}_n^* - \mathcal{P}$. As $\mathcal{P}_n^*$ is n-closed, it is nano α-closed also. Since K is $n\alpha$-open, $\upsilon - K$ is nano α-closed. Hence $\mathcal{P}_n^* \cap (\upsilon - K)$ is a $n\alpha$-CS contained in $\mathcal{P}_n^* - \mathcal{P}$. Therefore $\mathcal{P}_n^* \cap (\upsilon - K) = \phi$. Hence $\mathcal{P}_n^* \subseteq K$ and $\mathcal{P} \subseteq K$, where K is a nano α-open set, which leads the proof.

Theorem 3.11. In a $(\upsilon, \mathcal{N}, I)$, when $\mathcal{P}$ is n^*-dense in itself, the if and only if condition for a subset $\mathcal{P}$ to be a $nI\alpha g$-CS is that $\mathcal{P}$ is ng-CS.
Proof. Necessity. Consider $\mathcal{P}$ to be $nI\alpha g$-CS. By the definition, $\mathcal{P}_n^* \subseteq V$ whenever $\mathcal{P} \subseteq V$ and V is $n\alpha$-open. When $I = \phi$, referring Theorem 2.12 $\mathcal{P}_n^* = n\text{-}cl(\mathcal{P})$. Therefore $n\text{-}cl(\mathcal{P}) \subseteq V$. Hence $\mathcal{P}$ is a ng-CS.
Sufficiency. Consider a ng-CS $\mathcal{P}$. Then $n\text{-}cl(\mathcal{P}) \subseteq V$. $\mathcal{P}_n^* \subseteq V$. Also $\mathcal{P} \subseteq n\text{-}cl(\mathcal{P}) \subseteq V$. V is n-open. By Theorem 2.14, $\mathcal{P}$ is $nI\alpha g$-CS.

Theorem 3.12. Consider a NITS $(\upsilon, \mathcal{N}, I)$ and $\mathcal{P} \subseteq \upsilon$. Then if and only if condition for $\mathcal{P}$ to be a $nI\alpha g$-CS is $\mathcal{P} = \mathcal{Q} - N$, where $\mathcal{Q}$ is a n^*-CS and N does not have no non empty $n\alpha$-CS.
Proof. Necessary. Assume $\mathcal{P}$ to be a $nI\alpha g$-CS. Referring Theorem 3.9 (e) $\mathcal{P}_n^* - \mathcal{P}$ does not have non empty $n\alpha$-CS. Let $N = \mathcal{P}_n^* - \mathcal{P}$. Whenever $\mathcal{Q} = n\text{-}cl^*(\mathcal{P})$ and $\mathcal{Q}$ is n^*-closed then $\mathcal{Q} - N = (\mathcal{P} \cup \mathcal{P}_n^*) - (\mathcal{P}_n^* - \mathcal{P}) = \mathcal{P}$. Hence the proof.
Sufficiency. Assume $\mathcal{P} = \mathcal{Q} - N$ with the given conditions that $\mathcal{Q}$ is a n^*-CS and N contains no non empty $n\alpha$-CS. Let $\mathcal{P} \subseteq K$ for some $n\alpha$-OS K, which leads that $\mathcal{Q} \cap (\chi - K) \subseteq N$. Also $\mathcal{P} \subseteq \mathcal{Q}$ implies $\mathcal{P}_n^* \subseteq \mathcal{Q}_n^* \subseteq \mathcal{Q}$ since $\mathcal{Q}$ is n^*-closed. Let $\mathcal{P}_n^* \cap (\chi - K) \subseteq \mathcal{Q}_n^* \cap (\chi - K) \subseteq \mathcal{Q} \cap (\chi - K) \subset N$. Referring the hypothesis $\mathcal{P}_n^* \cap (\chi - K) = \phi$. Which implies $\mathcal{P}_n^* \subseteq K$. Already $\mathcal{P} \subseteq K$ and K is nano α-open. Hence the proof.

Theorem 3.13. In $(\upsilon, \mathcal{N}, I)$, the condition for a subset $\mathcal{P}$ to be a $nI\alpha g$-OS is that $\jmath \subseteq n\text{-}int^*(\mathcal{P})$, whenever $\jmath \subseteq \mathcal{P}$ and $\jmath$ is a $n\alpha$-closed set and the reverse implication is also true.
Proof. Necessary. Assume a $nI\alpha g$-OS $\mathcal{P}$ and a $n\alpha$-CS $\jmath$ and $\jmath \subseteq \mathcal{P}$. So $(\chi - \mathcal{P}) \subseteq (\chi - \jmath)$. By referring Theorem 3.9 b), $n\text{-}cl^*(\chi - \mathcal{P}) \subseteq (\chi - \jmath)$ and $\jmath \subseteq (\chi - (n\text{-}cl^*(\chi - \mathcal{P})))$, which implies $\jmath \subseteq n\text{-}int^*(\mathcal{P})$.
Sufficiency. Let ρ be a $n\alpha$-OS and $(\chi - \mathcal{P}) \subseteq \rho$. Then $(\chi - \rho) \subseteq \mathcal{P}$. By hypothesis $(\chi - \rho) \subseteq n\text{-}int^*(\mathcal{P})$, therefore $n\text{-}cl^*(\chi - \mathcal{P}) \subseteq \upsilon$. By referring Theorem 3.9, $\mathcal{P}$ is a $nI\alpha g$-OS.

4. Normality via $nI\alpha g$-closed sets

Definition 4.1. $nI\alpha g$-normal space we mean, if for all pairs of $nI\alpha g$-CS $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{P} \cap \mathcal{Q} = \phi$, there corresponds atleast two NOS ρ and ω of $(\upsilon, \mathcal{N}, I)$ and $\rho \cap \omega = \phi$ satisfying $\mathcal{P} \subseteq \rho$ and $\mathcal{Q} \subseteq \omega$.

Theorem 4.2. In $(\upsilon, \mathcal{N}, I)$, the equivalent implications on $nI\alpha g$-normal-spaces are stated.

a) $(\upsilon, \mathcal{N}, I)$ is a $nI\alpha g$-normal-space.

b) For all $nI\alpha g$-closed set ω and a $nI\alpha g$-open set $\jmath$ such that $\omega \subseteq \jmath$, there corresponds a n-OS $V \subset \upsilon$ and $\omega \subseteq V \subseteq n\text{-}cl(V) \subseteq \jmath$.

Proof. a) $\Longrightarrow$ b). Assume ω be a $nI\alpha g$-CS and $\jmath$ be a $nI\alpha g$-OS and $\omega \subset \jmath$. Then $\upsilon - \jmath$ is a $nI\alpha g$-CS. Therefore $\omega \cap (\chi - \jmath) = \phi$. By hypothesis (a) of this theorem it is understood that for any two disjoint NOS $\mathcal{P}$ and $\mathcal{Q}$ such that $\omega \subseteq \mathcal{P}$ and $\chi - \jmath \subseteq \mathcal{Q}$ and $\mathcal{P} \cap \mathcal{Q} = \phi$. But $\mathcal{P} \subseteq (\chi - \mathcal{Q})$ implies $n\text{-}cl(\mathcal{P}) \subseteq (\chi - \mathcal{Q})$. Hence $\omega \subseteq \mathcal{P} \subseteq n\text{-}cl(\mathcal{P}) \subseteq (\chi - \mathcal{Q}) \subseteq \jmath$ which proves (b).

Proof. b) $\Longrightarrow$ a). Let ω and $\jmath$ are disjoint $nI\alpha g$-CS and $\omega \subseteq (\chi - \jmath)$. Reference on hypothesis b) of this theorem infers the existence of a n-open set $\mathcal{P}$ of $(\upsilon, \mathcal{N}, I)$ such that $\omega \subseteq \mathcal{P} \subseteq n\text{-}cl(\mathcal{P}) \subseteq (\chi - \jmath)$. Let $\mathcal{Q} = \upsilon - n\text{-}cl(\mathcal{P})$. Since $n\text{-}cl(M)$ is a NCS, $\mathcal{Q}$ is a n-open set. These $\mathcal{P}$ and $\mathcal{Q}$ are the NOS (n-open sets) that contains ω and $\jmath$. Which proves a).

Theorem 4.3. In $(\upsilon, \mathcal{N}, I)$, the equivalent implications on $nI\alpha g$-normal-spaces are as follows.

a) $(\upsilon, \mathcal{N}, I)$ is a $nI\alpha g$-normal-space.
b) For any two $nI\alpha g$-closed subsets $\mathcal{P}$ and $\mathcal{Q}$ of $(\upsilon, \mathcal{N}, I)$, there corresponds a NOS ρ of $(\upsilon, \mathcal{N}, I)$ satisfies $\mathcal{P} \subseteq \rho$, then $n\text{-}cl(\rho) \cap \mathcal{Q} = \phi$.
c) For any two $nI\alpha g$-CS $\mathcal{P}$ and $\mathcal{Q}$ and $\mathcal{P} \cap \mathcal{Q} = \phi$, there corresponds a NOS ρ satisfying $\mathcal{P} \subseteq \rho$ and a NOS ω satisfying $\mathcal{Q} \subseteq \omega$ then $n\text{-}cl(\rho) \cap n\text{-}cl(\omega)$ is an empty set.

Proof. a) $\Longrightarrow$ b). Consider a pair of $nI\alpha g$-CS $\mathcal{P}$ and $\mathcal{Q}$ and $\mathcal{P} \cap \mathcal{Q} = \phi$, then $\mathcal{P} \subseteq (\chi - \mathcal{Q})$, where $\chi - \mathcal{Q}$ is a $nI\alpha g$-OS. Referring Theorem 4.2, there corresponds a NOS ρ such that $\mathcal{P} \subseteq \rho \subseteq n\text{-}cl(\rho) \subseteq \chi - \mathcal{Q}$. Therefore $n\text{-}cl(\rho)$ and $\mathcal{Q}$ are disjoint sets. Hence ρ is the NOS satisfies b).

b)$\Longrightarrow$ c). b) of this theorem implies $n\text{-}cl(\rho)$ and $\mathcal{Q}$ are disjoint $nI\alpha g$-CS of the NITS υ. Therefore there exists a NOS ω containing $\mathcal{Q}$ such that $n\text{-}cl(\rho) \cap n\text{-}cl(\omega) = \phi$ which proves c).

c)$\Longrightarrow$ a). Hypothesis c) proves a).

5. Conclusion

A new class of generalised closed namely $nI\alpha g$-closed set is introduced in nano ideal topological spaces. A comparative study of the $nI\alpha g$-closed set with existing closed sets is endeavoured and the reverse implications are explained with counter examples. Characterisations theorems and heredity properties of $nI\alpha g$-closed sets are stated and proved. In addition to that a new type of normal space called $nI\alpha g$-normal space is introduced and its characterisation is studied.

6. Acknowledgments

The authors are grateful to the referees and editorial board for their expert suggestions and comments.

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