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Fractional Integration and Solution of Generalized kinetic equation considering Generalized Lommel-Wright function

Saima Naheed*[1], Shahid Mubeen[1] and Gauhar Rahman[2]
[1] Department of Mathematics, University of Sargodha, Sargodha, Pakistan,
[2] Department of Mathematics, Hazara University, Mansehra, Pakistan
Email: saima.naheed@uos.edu.pk*, shahid.mubeen@uos.edu.pk,
gauhar55uom@gmail.com

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Abstract.: In this article, we initially built the generalized form of the Lommel-Wright function and then evaluated the Saigo hypergeometric fractional integrals of the newly built special function. We developed a generalized form of the fractional kinetic equation using the introduced special function. The solution of the generalized fractional kinetic equation in terms of the Mittag-Leffler functions is established via Laplace transform. Some special cases are also discussed.

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1. Introduction

In mathematical analysis and its applications, the hypergeometric function plays a vital role. Various special functions which are used in different branches of science are special cases of hypergeometric functions. Numerous extensions of special functions have introduced by many authors (see [1, 4, 5]).

In recent years, study on fractional differential equations is very dynamic and extensive all around the world. Some of its applications in different fields are covered in [24, 42]. A hybrid analytical solution to examine the fractional model of the nonlinear wave-like equation is explored by Kumar *et al.* in [22]. Explicit analytical solutions of incommensurate fractional differential equation systems based solutions are discussed by Huseynov *et al.* [15]. Certain Integral operators involving the Gauss hypergeometric functions are discussed in [30, 31]. A brief systematic history of the generalized fractional calculus operators and their applications is being profoundly analyzed in [19, 40]. A concise description of generalized fractional calculus operators together with their applications is available in [17, 26, 32].

We start with some basic definitions. Diaz and Pariguan [10] introduced the k-gamma function defined by

$$\Gamma_k(\omega) = lim_{n\to\infty} \frac{n!k^n(nk)^{\frac{\omega}{k}-1}}{(\omega)_{n,k}} \tag{1. 1}$$

with k-Pochhammer symbol $(\omega)_{n,k}$ given by

$$(\omega)_{n,k} = \omega(\omega+k)(\omega+2k)....(\omega+(n-1)k),\ x\in\mathbb{C}, k\in\mathbb{R}, n\in\mathbb{N}^+. \tag{1. 2}$$

The classical Euler Gamma function and Gamma k-function are related as

$$\Gamma_k(\omega) = k^{\frac{\omega}{k}-1}\Gamma(\frac{\omega}{k}). \tag{1. 3}$$

The Lommel-Wright function [9] is defined as

$$J^{c,m}_{a,b}(z) = (\frac{z}{2})^{a+2b}\sum_{n=0}^{\infty}\frac{(-1)^n(\frac{z}{2})^{2n}}{(\Gamma(b+n+1))^m\Gamma(a+b+nc+1)}$$
$$= (\frac{z}{2})^{a+2b}\ {}_1\psi_{m+1}[(1,1);\underbrace{(b+1,1),....(b+1,1)}_{m-times},(a+b+1,c);\frac{-z^2}{4}], \tag{1. 4}$$

where

$$z\in\mathbb{C}\backslash(-\infty,0],\ c>0,\ m\in\mathbb{N},\ a,b\in\mathbb{C}$$

and ${}_u\psi_v$ represents the Wright hypergeometric function [11]

$${}_u\psi_v[(a_1,A_1),.....(a_u,A_u);(b_1,B_1),.....(b_v,B_v);z] = \sum_{n=0}^{\infty}\frac{\Gamma(a_1,nA_1),....\Gamma(a_u,nA_u)z^n}{\Gamma(b_1,nB_1).....\Gamma(b_v,nB_v)n!},$$
$$A_i>0(i=1,2,..u);\ B_i>0(i=1,2,..v);$$
$$1+\sum_{i=1}^{v}B_i-\sum_{i=1}^{u}A_i\geq 0,$$

for reasonably bounded values of $|z|$.
For $m=1$, Lommel-Wright function (1. 4) diminishes to Bessel-Maitland function [28]

$$J^{c,1}_{a,b}(z) = J^{c}_{a,b}(z) = (\frac{z}{2})^{a+2b}\sum_{n=0}^{\infty}\frac{(-1)^n(\frac{z}{2})^{2n}}{\Gamma(b+n+1)\Gamma(a+b+nc+1)}. \tag{1. 5}$$

If we choose $m=1$, $c=1$ and $b=o$ in (1. 4), it provides the following form of the Bessel function [26]

$$J^{1,1}_{a,1}(z) = J_a(z) = \sum_{n=0}^{\infty}\frac{(-1)^n(\frac{z}{2})^{a+2n}}{\Gamma(b+n+1)n!}. \tag{1. 6}$$

Konovska [20] discussed the convergence of series involving generalized Lommel-Wright function.
Presently, we define the Lommel-Wright k-function as follows

$$J^{c,m}_{a,b,k}(z) = (\frac{z}{2})^{\frac{a+2b}{k}}\sum_{n=0}^{\infty}\frac{(-1)^n(\frac{z}{2})^{2n}}{(\Gamma_k(b+nk+k))^m\Gamma_k(a+b+nc+k)}, \tag{1. 7}$$

where

$$z \in \mathbb{C}|(-\infty, 0],\ c > 0,\ m \in \mathbb{N},\ k \in \mathbb{R},\ a, b \in \mathbb{C}.$$

The generalized Bessel-Maitland k-function is defined as

$$J^{c}_{a,b,k}(z) = (\frac{z}{2})^{\frac{a+2b}{k}} \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{z}{2})^{2n}}{\Gamma_k(b + nk + k)\Gamma_k(a + b + nc + k)}. \tag{1. 8}$$

2. Fractional integration of generalized Lommel-Wright k-function

This section of the paper contains fractional integration of the generalized Lommel-Wright k-function. For this purpose, we use the accompanying Saigo hypergeometric fractional integral operators [30], which are defined as follows:
For $y > 0$, $\varpi, \varrho, \varsigma \in \mathbb{C}$ and $Re(\varpi) > 0$

$$(I^{\varpi,\varrho,\varsigma}_{0,y} f(t))y = \frac{y^{-\varpi-\varrho}}{\Gamma(\varpi)} \int_0^y (y - t)^{\varpi-1}\ {}_2F_1(\varpi + \varrho, -\varsigma; \varpi; 1 - \frac{t}{y}) f(t)dt \tag{2. 9}$$

and

$$(Q^{\varpi,\varrho,\varsigma}_{y,\infty} f(t))y = \frac{y^{-\varpi-\varrho}}{\Gamma(\varpi)} \int_y^\infty (t - y)^{\varpi-1}\ {}_2F_1(\varpi + \varrho, -\varsigma; \varpi; 1 - \frac{y}{t}) f(t)dt. \tag{2. 10}$$

where ${}_2F_1$ is the Gauss hypergeometric function. Specifically for $\varrho = -\varpi$, the operators (2. 9)and (2. 10) become Riemann-Liouville and the Weyl fractional integral operators [26]

$$(R^{\varpi}_{0,y} f(t))y = (I^{\varpi,-\varpi,\varsigma}_{0,y} f(t))y = \frac{1}{\Gamma(\varpi)} \int_0^y (y - t)^{\varpi-1} f(t)dt \tag{2. 11}$$

and

$$(W^{\varpi}_{y,\infty} f(t))y = (Q^{\varpi,-\varpi,\varsigma}_{y,\infty} f(t))y = \frac{1}{\Gamma(\varpi)} \int_y^\infty (t - y)^{\varpi-1} f(t)dt \tag{2. 12}$$

and for $\varrho = 0$ the operators (2. 9)and (2. 10) reduce to the Erd$\acute{e}$lyi-Kober fractional integral operators [26] as given

$$(E^{\varpi,\varsigma}_{0,y} f(t))y = (I^{\varpi,0,\varsigma}_{0,y} f(t))y = \frac{y^{-\varpi-\varsigma}}{\Gamma(\varpi)} \int_0^y (y - t)^{\varpi-1} t^{\varpi-1} f(t)dt \tag{2. 13}$$

and

$$(K^{\varpi,\varsigma}_{y,\infty} f(t))y = (Q^{\varpi,0,\varsigma}_{y,\infty} f(t))y = \frac{y^{\varsigma}}{\Gamma(\varpi)} \int_y^\infty (t - y)^{\varpi-1} t^{-\varpi-\varsigma} f(t)dt. \tag{2. 14}$$

The lemmas stated below are supportive for establishing our major results. These are presented by Kilbas and Sebastin [18].

Lemma 2.1. *Let $\varpi, \varrho, \varsigma \in \mathbb{C}$ and $Re(\varpi) > 0$, $Re(\varepsilon) > max[0, Re(\varrho - \varsigma)]$, then*

$$(I^{\varpi,\varrho,\varsigma}_{0,y} t^{\varepsilon-1})(y) = \frac{\Gamma(\varepsilon)\Gamma(\varepsilon + \varsigma - \varrho)}{\Gamma(\varepsilon - \varrho)\Gamma(\varepsilon + \varpi + \varsigma)} y^{\varepsilon-\varrho-1}. \tag{2. 15}$$

Lemma 2.2. *Let $\varpi, \varrho, \varsigma \in \mathbb{C}$ and $Re(\varpi) > 0$, $Re(\varepsilon) < 1 + min[Re(\varrho), Re(\varsigma)]$, then*

$$(Q^{\varpi,\varrho,\varsigma}_{y,\infty} t^{\varepsilon-1})(y) = \frac{\Gamma(\varrho - \varepsilon + 1)\Gamma(\varsigma - \varepsilon + 1)}{\Gamma(1 - \varepsilon)\Gamma(\varpi + \varrho + \varsigma - \varepsilon + 1)} y^{\varepsilon-\varrho-1}. \tag{2. 16}$$

Now , we will apply these operators on Lommel-Wright k-function in the upcoming theorems.

Theorem 2.3. *Let* $\varpi, \varrho, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $k \in \mathbb{R}$,$m \in \mathbb{N}$, $c > 0$, *min* $\{Re(a), Re(b), Re(c)\} > 0$ *such that* $Re(\varepsilon) > max[0, Re(\varrho - \varsigma)]$, *then*

$$(I_{0,y}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_{a,b,k}^{c,m}(t))(y) = (\frac{y}{2})^{(\frac{a+2b}{k})} y^{\varepsilon-\varrho-1} \tag{2. 17}$$

$$\times {}_3\psi_{m+3}\left[\begin{array}{c} (1,1), (\frac{a+2b}{k}+\varepsilon+\varsigma-\varrho, 2), (\frac{a+2b}{k}+\varepsilon, 2) \\ \underbrace{(b+k,k)....(b+k,k)}_{m-times}, (a+b+k, c), (\frac{a+2b}{k}+\varepsilon-\varrho, 2), (\frac{a+2b}{k}+\varepsilon+\varpi+\varsigma, 2) \end{array} \middle| \frac{-y^2}{4}\right].$$

Proof. Using definition (1. 7) to L.H.S. of equation (2. 17) and denoting it by $\Im$, we have

$$\Im = \left(I_{0,y}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} (\frac{t}{2})^{\frac{a+2b}{k}} \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{t}{2})^{2n}}{(\Gamma_k(b+nk+k))^m \Gamma_k(a+b+nc+k)}\right)(y), \tag{2. 18}$$

now changing the order of integration and summation

$$\Im = (\frac{1}{2})^{\frac{a+2b}{k}} \sum_{n=0}^{\infty} \frac{(-1)^n}{(\Gamma_k(b+nk+k))^m \Gamma_k(a+b+nc+k)} (I_{0,y}^{\varpi,\varrho,\varsigma} t^{\frac{a+2b}{k}+2n+\varepsilon-1})(y), \tag{2. 19}$$

using lemma 2.1 in equation (2. 19), we obtain

$$\Im = (\frac{1}{2})^{\frac{a+2b}{k}} \sum_{n=0}^{\infty} \frac{(-1)^n}{(\Gamma_k(b+nk+k))^m \Gamma_k(a+b+nc+k)}$$
$$\times \frac{\Gamma(\frac{a+2b}{k}+\varepsilon+2n)\Gamma(\frac{a+2b}{k}+\varepsilon+\varsigma-\varrho+2n)}{\Gamma(\frac{a+2b}{k}+\varepsilon-\varrho+2n)\Gamma(\frac{a+2b}{k}+\varepsilon+\varpi+\varsigma+2n)} y^{\frac{a+2b}{k}+2n-\varrho+\varepsilon-1}, \tag{2. 20}$$

on simplification and using equation (1. 4), we get the required result as given

$$\Im = (\frac{y}{2})^{(\frac{a+2b}{k})} y^{\varepsilon-\varrho-1} \tag{2. 21}$$

$$\times {}_3\psi_{m+3}\left[\begin{array}{c} (1,1), (\frac{a+2b}{k}+\varepsilon+\varsigma-\varrho, 2), (\frac{a+2b}{k}+\varepsilon, 2) \\ \underbrace{(b+k,k)....(b+k,k)}_{m-times}, (a+b+k, c), (\frac{a+2b}{k}+\varepsilon-\varrho, 2), (\frac{a+2b}{k}+\varepsilon+\varpi+\varsigma, 2) \end{array} \middle| \frac{-y^2}{4}\right].$$

□

Theorem 2.4. *Let* $\varpi, \varrho, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $k \in \mathbb{R}$,$m \in \mathbb{N}$, $c > 0$, *min* $\{Re(a), Re(b), Re(c)\} > 0$ *such that* $Re(\varepsilon) > 1 + min[Re(\varrho), Re(\varsigma)]$, *then*

$$(Q_{y,\infty}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_{a,b,k}^{c,m}(\frac{1}{t}))(y) = (\frac{2}{y})^{(\frac{a+2b}{k})} y^{\varepsilon-\varrho-1} \tag{2. 22}$$

$$\times {}_3\psi_{m+3}\left[\begin{array}{c} (1,1), (\frac{a+2b}{k}+\varrho-\varepsilon+1, 2), (\frac{a+2b}{k}-\varepsilon+\varsigma+1, 2) \\ \underbrace{(b+k,k)....(b+k,k)}_{m-times}, (a+b+k, c), (\frac{a+2b}{k}-\varepsilon+1, 2), (\frac{a+2b}{k}+\varepsilon+\varpi+\varsigma+1, 2) \end{array} \middle| \frac{-4}{y^2}\right].$$

Proof. The proof is same like Theorem 2.3. □

Special Cases

Here, we will discuss the special cases depending upon the different values of parameters. If we replace $-\varpi$ by ϱ in Theorems 2.3 and 2.4 and using equations (2. 11), (2. 12), we have the following corollaries.

Corollary 2.5. *Let* $\varpi, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $k \in \mathbb{R}$,$m \in \mathbb{N}$, $c > 0$, *min* $\{Re(a), Re(b), Re(c)\} > 0$ *such that* $min\{Re(\varpi), Re(\varepsilon)\} > 0$, *, then*

$$(R_{0,y}^{\varpi} t^{\varepsilon-1} J_{a,b,k}^{c,m}(t))(y) = (\frac{y}{2})^{(\frac{a+2b}{k})} y^{\varepsilon+\varpi-1}$$

$$\times_3 \psi_{m+3} \left[\begin{array}{c} (1,1), (\frac{a+2b}{k} + \varepsilon + \varsigma + \varpi, 2), (\frac{a+2b}{k} + \varepsilon, 2), (\frac{a+2b}{k} + \varepsilon, 2) \\ \underbrace{(b+k,k)....(b+k,k)}_{m-times}, (a+b+k,c), (\frac{a+2b}{k} + \varepsilon + \varpi, 2), (\frac{a+2b}{k} + \varepsilon + \varpi + \varsigma, 2) \end{array} \middle| \frac{-y^2}{4} \right].$$

Corollary 2.6. *Let* $\varpi, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $k \in \mathbb{R}$,$m \in \mathbb{N}$, $c > 0$, *min* $\{Re(a), Re(b), Re(c)\} > 0$ *such that* $min\{Re(\varpi), Re(\varepsilon)\} > 0$, *then*

$$(W_{y,\infty}^{\varpi} t^{\varepsilon-1} J_{a,b,k}^{c,m}(t))(y) = (\frac{2}{y})^{(\frac{a+2b}{k}} y^{\varepsilon+\varpi-1}$$

$$\times_3 \psi_{m+3} \left[\begin{array}{c} (1,1), (\frac{a+2b}{k} - \varepsilon - \varpi + 1, 2), (\frac{a+2b}{k} - \varepsilon + \varsigma + 1, 2) \\ \underbrace{(b+k,k)....(b+k,k)}_{m-times}, (a+b+k,c), (\frac{a+2b}{k} - \varepsilon + 1, 2)(\frac{a+2b}{k} - \varepsilon + \varsigma + 1, 2) \end{array} \middle| \frac{-4}{y^2} \right].$$

Furthermore, if we substitute $\varrho = 0$ in Theorems 2.3 and 2.4, we have the following corollaries.

Corollary 2.7. *Let* $\varpi, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $k \in \mathbb{R}$,$m \in \mathbb{N}$, $c > 0$, *min* $\{Re(a), Re(b), Re(c)\} > 0$ *such that* $Re(\varepsilon) > Re(\varsigma)]$, *then*

$$(E_{0,y}^{\varpi,\varsigma} t^{\varepsilon-1} J_{a,b,k}^{c,m}(t))(y) = (\frac{y}{2})^{(\frac{a+2b}{k})} y^{\varepsilon-1}$$

$$\times_3 \psi_{m+3} \left[\begin{array}{c} (1,1), (\frac{a+2b}{k} + \varepsilon + \varsigma, 2), (\frac{a+2b}{k} + \varepsilon, 2) \\ \underbrace{(b+k,k)....(b+k,k)}_{m-times}, (a+b+k,c), (\frac{a+2b}{k} + \varepsilon, 2), (\frac{a+2b}{k} + \varepsilon + \varpi + \varsigma, 2) \end{array} \middle| \frac{-y^2}{4} \right].$$

Corollary 2.8. *Let* $\varpi, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $k \in \mathbb{R}$, $m \in \mathbb{N}$, $c > 0$, *min* $\{Re(a), Re(b), Re(c)\} > 0$ *such that* $Re(\varepsilon) < 1 + Re(\varsigma)]$, *then*

$$(K_{y,\infty}^{\varpi,\varsigma} t^{\varepsilon-1} J_{a,b,k}^{c,m}(\frac{1}{t}))(y) = (\frac{2}{y})^{(\frac{a+2b}{k})} y^{\varepsilon-1}$$

$$\times_3 \psi_{m+3} \left[\begin{array}{c} (1,1), (\frac{a+2b}{k} - \varepsilon + 1, 2), (\frac{a+2b}{k} + \varepsilon, 2) \\ \underbrace{(b+k,k)....(b+k,k)}_{m-times}, (a+b+k,c), (\frac{a+2b}{k} - \varepsilon + 1, 2), (\frac{a+2b}{k} + \varpi - \varepsilon + \varsigma + 1, 2) \end{array} \middle| \frac{-4}{y^2} \right].$$

For $m = 1$, the generalized Lommel-Wright k-function reduces to Bessel-Maitland k-function as defined in (1. 8). Theorems (2.3) and (2.4) lead to the following corollaries.

Corollary 2.9. *Let* $\varpi, \varrho, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $k \in \mathbb{R}$, $c > 0$, *min* $\{Re(a), Re(b), Re(c)\} > 0$ *such that* $Re(\varepsilon) > max[0, Re(\varrho - \varsigma)]$, *then*

$$(I_{0,y}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_{a,b,k}^{c}(t))(y) = (\frac{y}{2})^{(\frac{a+2b}{k})} y^{\varepsilon-\varrho-1}$$

$$\times_3 \psi_4 \left[\begin{array}{c} (1,1), (\frac{a+2b}{k} + \varepsilon + \varsigma - \varrho, 2), (\frac{a+2b}{k} + \varepsilon, 2) \\ (b+k,k), (a+b+k,c), (\frac{a+2b}{k} + \varepsilon - \varrho, 2), (\frac{a+2b}{k} + \varepsilon + \varpi + \varsigma, 2) \end{array} \middle| \frac{-y^2}{4} \right].$$

Corollary 2.10. *Let $\varpi, \varrho, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $k \in \mathbb{R}$, $c > 0$, min $\{Re(a), Re(b), Re(c)\} > 0$ such that $Re(\varepsilon) > 1 + min[Re(\varrho), Re(\varsigma)]$, then*

$$(I_{y,\infty}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_{a,b,k}^{c}(\frac{1}{t}))(y) = (\frac{2}{y})^{(\frac{a+2b}{k})} y^{\varepsilon-\varrho-1}$$
$$\times {}_3\psi_4 \left[\begin{matrix} (1,1), (\frac{a+2b}{k} - \varepsilon + \varrho + 1, 2), (\frac{a+2b}{k} - \varepsilon + \varsigma + 1, 2) \\ (b+k,k), (a+b+k,c), (\frac{a+2b}{k} - \varepsilon + 1, 2), (\frac{a+2b}{k} - \varepsilon + \varpi + \varrho + \varsigma + 1, 2) \end{matrix} \middle| \frac{-4}{y^2} \right].$$

The generalized Lommel-Wright k-function reduces to generalized Lommel-Wright function for $k = 1$ and the expressions (2. 17) and (2. 22) reduced to the subsequent corollaries.

Corollary 2.11. *Let $\varpi, \varrho, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $m \in \mathbb{N}$, $c > 0$, min $\{Re(a), Re(b), Re(c)\} > 0$ such that $Re(\varepsilon) > max[0, Re(\varrho - \varsigma)]$, then*

$$(I_{0,y}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_{a,b}^{c,m}(t))(y) = (\frac{y}{2})^{(a+2b)} y^{\varepsilon-\varrho-1}$$
$$\times {}_3\psi_{m+3} \left[\begin{matrix} (1,1), (a+2b+\varepsilon+\varsigma-\varrho, 2), (a+2b+\varepsilon, 2) \\ \underbrace{(b+1,1)....(b+1,1)}_{m-times}, (a+b+1,c), (a+2b+\varepsilon-\varrho, 2)(a+2b+\varepsilon+\varpi+\varsigma, 2) \end{matrix} \middle| \frac{-y^2}{4} \right].$$

Corollary 2.12. *Let $\varpi, \varrho, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $m \in \mathbb{N}$, $c > 0$, min $\{Re(a), Re(b), Re(c)\} > 0$ such that $Re(\varepsilon) > 1 + min[Re(\varrho), Re(\varsigma)]$,, then*

$$(I_{y,\infty}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_{a,b}^{c,m}(\frac{1}{t}))(y) = (\frac{y}{2})^{(a+2b)} y^{\varepsilon-\varrho-1}$$
$$\times {}_3\psi_{m+3} \left[\begin{matrix} (1,1), (a+2b-\varepsilon+\varrho+1, 2), (a+2b-\varepsilon+\varsigma+1, 2) \\ \underbrace{(b+1,1)....(b+1,1)}_{m-times}, (a+b+1,c), (a+2b-\varepsilon+1, 2)(a+2b-\varepsilon+\varpi+\varrho+\varsigma+1, 2) \end{matrix} \middle| \frac{-4}{y^2} \right].$$

If we take $k, m = 1$ in generalized Lommel Wright k-function we have Bessel-Maitland function (1. 5) and the expressions (2. 17) and (2. 22) take the following form.

Corollary 2.13. *Let $\varpi, \varrho, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $c > 0$, min $\{Re(a), Re(b), Re(c)\} > 0$ such that $Re(\varepsilon) > max[0, Re(\varrho - \varsigma)]$, then*

$$(I_{0,y}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_{a,b}^{c}(t))(y) = (\frac{y}{2})^{(a+2b)} y^{\varepsilon-\varrho-1}$$
$$\times {}_3\psi_4 \left[\begin{matrix} (1,1), (a+2b+\varepsilon+\varsigma-\varrho, 2), (a+2b+\varepsilon, 2) \\ (b+1,1), (a+b+1,c), (a+2b+\varepsilon-\varrho, 2)(a+2b+\varepsilon+\varpi+\varsigma, 2) \end{matrix} \middle| \frac{-y^2}{4} \right].$$

Corollary 2.14. *Let $\varpi, \varrho, \varsigma, \varepsilon, a, b \in \mathbb{C}$, $c > 0$, min $\{Re(a), Re(b), Re(c)\} > 0$ such that $Re(\varepsilon) > 1 + min[Re(\varrho), Re(\varsigma)]$, then*

$$(I_{y,\infty}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_{a,b}^{c}(\frac{1}{t}))(y) = (\frac{2}{y})^{(a+2b)} y^{\varepsilon-\varrho-1}$$
$$\times {}_3\psi_4 \left[\begin{matrix} (1,1), (a+2b+\varepsilon+\varsigma-\varrho+1, 2), (a+2b-\varepsilon+\varsigma+1, 2) \\ (b+1,1), (a+b+1,c), (a+2b-\varepsilon+1, 2)(a+2b-\varepsilon+\varpi+\varrho+\varsigma+1, 2) \end{matrix} \middle| \frac{-4}{y^2} \right].$$

If we choose $m = 1$,$c = 1$, $b = o$ and $k = 1$ in generalized Lommel-Wright k-function, we have the function (1. 6). Then Theorems (2.3) and (2.4) reduce to the following corollaries.

Corollary 2.15. *Let* $\varpi, \varrho, \varsigma, \varepsilon, a, \in \mathbb{C}$*, min* $\{Re(a)\} > 0$ *such that* $Re(\varepsilon) > max[0, Re(\varrho - \varsigma)]$*, then*

$$(I_{0,y}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_a(t))(y) = (\frac{y}{2})^a y^{\varepsilon-\varrho-1}$$
$$\times {}_2\psi_3 \left[\begin{array}{c} (a+\varepsilon+\varsigma-\varrho, 2), (a+\varepsilon, 2) \\ (a+1,1), (a+\varepsilon-\varrho, 2)(a+\varepsilon+\varpi+\varsigma, 2) \end{array} \Bigg| \frac{-y^2}{4} \right].$$

Corollary 2.16. *Let* $\varpi, \varrho, \varsigma, \varepsilon, a, \in \mathbb{C}$*, min* $\{Re(a)\} > 0$ *such that* $Re(\varepsilon) > 1 + min[Re(\varrho), Re(\varsigma)]$*, then*

$$(I_{y,\infty}^{\varpi,\varrho,\varsigma} t^{\varepsilon-1} J_a(\frac{1}{t}))(y) = (\frac{2}{y})^a y^{\varepsilon-\varrho-1}$$
$$\times {}_2\psi_3 \left[\begin{array}{c} (a-\varepsilon+\varrho+1, 2), (a-\varepsilon+\varsigma+1, 2) \\ (a+1,1), (a-\varepsilon+1, 2)(a-\varepsilon+\varpi+\varrho+\varsigma+1, 2) \end{array} \Bigg| \frac{-4}{y^2} \right].$$

3. Generalized fractional kinetic equation

Fractional differential equations have acquired significant attention in various fields of applied science, not only in mathematics but also in physics, dynamical systems, control systems and engineering, where they are used to develop mathematical models of numerous physical phenomenon. The kinetic equations portray the continuity of motion of a substance. Kinetic equations of fractional order including Bessel function, Aleph function and the thermonuclear functions are briefly discussed in [8,14,23,25]. Chaurasia *et al.* have described the computable solutions of the fractional kinetic equation in astrophysics [6, 7]. Solutions of fractional kinetic equations and fractional reaction-diffusion equation are developed in [12–14, 34, 35].

The modification and speculation of fractional kinetic equations including a number of fractional order operators have been studied by different authors (see, [3, 29, 36, 37]). Solution of fractional kinetic equations associated with the (p, q)-Mathieu-type series is explored in [41]. Factional kinetic equations involving generalized k-Bessel function and k-Struve function are investigated in [27, 38]. Agarwal *et al.* established the solution of fractional kinetic equation by considering k-Mittag-Leffler function [2].

In view of the immense importance and useful applications of the kinetic equation in certain astrophysical problems, we establish a more generalized form of the fractional kinetic equation and find its solution.

If an arbitrary reaction is depicted by a function $\Upsilon = \Upsilon(t)$, then the rate of change of the reaction is associated with the demolition rate and production rate of Υ, was set up by Haubold and Mathai in [14], is described by the differential equation

$$\frac{d\Upsilon}{dt} = -d(\Upsilon_t) + P(\Upsilon_t), \tag{3. 23}$$

where $\Upsilon(t)$, $d(\Upsilon)$ and $P(\Upsilon)$ are the rate of reaction, destruction and production respectively. Whereas Υ_t represents the function given by $\Upsilon_t(t^*) = \Upsilon(t - t^*)$, $t^* > 0$. A particular case of (3. 23), when spatial fluctuations and inhomogeneities in the quantity $\Upsilon(t)$ are ignored, is discussed in [14, 21] and given by the equation

$$\frac{d\Upsilon_j}{dt} = -c_j \Upsilon_j(t), \tag{3. 24}$$

where initial condition $\Upsilon_j(t=0)=\Upsilon_0$ is the number of density of species j at time $t=0$ and $c_j>0$. Standard kinetic Eq. has solution as given in [21] as

$$\Upsilon_j(t)=\Upsilon_0 e^{-c_j t}. \tag{3. 25}$$

The elimination of index j and integrating equation (3. 24) provides the following equation

$$\Upsilon(t)-\Upsilon_0=-(c_0)_0D_t^{-1}\Upsilon(t), \tag{3. 26}$$

where c is a constant and ${}_0D_t^{-1}$ is the particular case of the Riemann-Liouville fractional integral operator ${}_0D_t^{-\nu}$ which is characterized as [33]

$${}_0D_t^{-\nu}f(t)=\frac{1}{\Gamma(\nu)}\int_0^t(t-y)^{\nu-1}f(y)dy,\ (t>0,R(\nu)>0). \tag{3. 27}$$

Haubold and Mathai [14] established the fractional generalization of the standard kinetic equation (3. 26), which is

$$\Upsilon(t)-\Upsilon_0=-(c_0)_0D_t^{-\nu}\Upsilon(t), \tag{3. 28}$$

and achieved the solution of (3. 28) as follows

$$\Upsilon(t)=\Upsilon_0\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(\nu n+1)}(c_0t)^{\nu n}. \tag{3. 29}$$

By taking $\nu=1$ in (3. 29), we get the exponential solution of the standard kinetic equation (3. 25). Two parameter Mittag-Leffler function [43] is defined as

$$E_{\zeta,\eta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\zeta n+\eta)}, \tag{3. 30}$$

where $\zeta,\eta\in C,\ \Re(\zeta),\ \Re(\eta)>0$. Let $f(t)$ be a real or complex valued function of variable t then Laplace transform of $f(t)$ is defined as in [39]

$$L\{f(t);p\}=\int_0^{\infty}f(t)e^{-pt}dt. \tag{3. 31}$$

where p is real or complex. Srivastava and Saxena [40] defined the Laplace transform of the Riemann-Liouville fractional integral operator as follows

$$L\{{}_0D_t^{-\nu};p\}=p^{-\nu}L\{f(t);p\}. \tag{3. 32}$$

4. Solution of fractional kinetic equation involving generalized Lommel-Wright k-function

Taking into account the immense significance of the kinetic equation in many astrophysical problems, we built up a new generalized form of the fractional kinetic equation considering generalized Lommel-Wright k-function.

Theorem 4.1. *For $d,\ \nu,\ a,\ c>0$,$b\in\mathbb{C}$, $k,\ m\in\mathbb{N}$ and $|d^\nu|<|p^\nu|$, then the equation*

$$\Upsilon(t)-\Upsilon_0J_{a,b,k}^{c,m}(t)=-d^\nu\ {}_0D_t^{-\nu}\Upsilon(t) \tag{4. 33}$$

has solution of the form

$$\Upsilon(t)=\Upsilon_0\sum_{n=0}^{\infty}\frac{(-1)^n(\Gamma_k(2b+a+2nk+k))}{(\Gamma_k(b+nk+k))^m\Gamma_k(a+b+nc+k)}(\frac{t}{2})^{(\frac{a+2b}{k})+2n}$$
$$\times E_{\nu,\frac{a+2b}{k}+2n+1}(-d^\nu t^\nu), \quad (4.\ 34)$$

where $E_{\zeta,\eta}$ *is presented in (3. 30).*

Proof. Operating Laplace transform to equation (4. 33), we obtain

$$L\{\Upsilon(t);p\}=\Upsilon_0 L\{J^{c,m}_{a,b,k}(t);p\}-d^\nu L\{{}_0D_t^{-\nu}\Upsilon(t);p\} \quad (4.\ 35)$$

$$L\{\Upsilon(t);p\}=\Upsilon_0(\int_0^\infty e^{-pt}\sum_{n=0}^{\infty}\frac{(-1)^n(\frac{t}{2})^{2n+2b+a}}{(\Gamma_k(b+nk+k))^m\Gamma_k(a+b+nc+k)}dt)$$
$$-d^\nu L\{{}_0D_t^{-\nu}\Upsilon(t);p\} \quad (4.\ 36)$$

$$\Upsilon(p)+d^\nu p^{-\nu}\Upsilon(p)=\Upsilon_0\textstyle\sum_{n=0}^{\infty}\frac{(-1)^n 2^{-(\frac{a+2b}{k}+2n)}}{(\Gamma_k(b+nk+k))^m\Gamma_k(a+b+nc+k)}$$
$$-\textstyle\int_0^\infty e^{-pt}t^{\frac{a+2b}{k}+2n}dt. \quad (4.\ 37)$$

Now by utilizing the following result

$$L\{t^\sigma\}=\frac{\Gamma(\sigma+1)}{s^{\sigma+1}}=\frac{\sigma!}{s\sigma+1}, s>0,$$

we have

$$(1+d^\nu p^{-\nu})\Upsilon(p)=\Upsilon_0\textstyle\sum_{n=0}^{\infty}\frac{(-1)^n 2^{-(\frac{a+2b}{k}+2n)}}{(\Gamma_k(b+nk+k))^m\Gamma_k(a+b+nc+k)}$$
$$\times\frac{\Gamma(\frac{a+2b}{k}+2n+1)}{p^{(\frac{a+2b}{k}+2n+1)}}. \quad (4.\ 38)$$

This further implies that

$$\Upsilon(p)=\Upsilon_0\textstyle\sum_{n=0}^{\infty}\frac{(-1)^n 2^{-(\frac{a+2b}{k}+2n)}}{(\Gamma_k(b+nk+k))^m\Gamma_k(a+b+nc+k)}$$
$$\times\frac{\Gamma(\frac{a+2b}{k}+2n+1)}{p^{(\frac{a+2b}{k}+2n+1)}}\frac{1}{(1+d^\nu p^{-\nu})}, \quad (4.\ 39)$$

on simplification, we get

$$\Upsilon(p)=\Upsilon_0\textstyle\sum_{n=0}^{\infty}\frac{(-1)^n 2^{-(\frac{a+2b}{k}+2n)}\Gamma(\frac{a+2b}{k}+2n+1)}{(\Gamma_k(b+nk+k))^m\Gamma_k(a+b+nc+k)}$$
$$\{\textstyle\sum_{q=0}^{\infty}(-1)^q d^{\nu q}p^{(\frac{a+2b}{k}+2n+\nu q+1)}\}, \quad (4.\ 40)$$

operating inverse Laplace transform to (4. 40) and by utilizing

$$L^{-1}\{s^{-\sigma};t\}=\frac{t^{\sigma-1}}{\Gamma(\sigma)},$$

we get

$$L^{-1}\{\Upsilon(p)\}=\Upsilon_0\textstyle\sum_{n=0}^{\infty}\frac{(-1)^n 2^{-(\frac{a+2b}{k}+2n)}\Gamma(\frac{a+2b}{k}+2n+1)}{(\Gamma_k(b+nk+k))^m\Gamma_k(a+b+nc+k)}$$
$$\times L^{-1}\{\textstyle\sum_{q=0}^{\infty}(-1)^q d^{\nu q}p^{(\frac{a+2b}{k}+2n+\nu q+1)}\}, \quad (4.\ 41)$$

$$\Upsilon(t) = \Upsilon_0 \sum_{n=0}^{\infty} \frac{(-1)^n 2^{-(\frac{a+2b}{k}+2n)} \Gamma(\frac{a+2b}{k}+2n+1)}{(\Gamma_k(b+nk+k))^m \Gamma_k(a+b+nc+k)}$$
$$\times \left(\sum_{q=0}^{\infty} (-1)^q d^{\nu q} \frac{t^{\frac{a+2b}{k}+2n+\nu q}}{\Gamma(\frac{(a+2b)}{k}+2n+\nu q+1)} \right) \quad (4.\ 42)$$
$$= \Upsilon_0 \sum_{n=0}^{\infty} \frac{(-1)^n \frac{t}{2}^{(\frac{a+2b}{k}+2n)} \Gamma(\frac{a+2b}{k}+2n+1)}{(\Gamma_k(b+nk+k))^m \Gamma_k(a+b+nc+k)}$$
$$\times \left(\sum_{q=0}^{\infty} \frac{(-d^\nu t^\nu)^q}{\Gamma(\frac{(a+2b)}{k}+2n+\nu q+1)} \right) \quad (4.\ 43)$$

now using equation (3. 30) in (4. 43), we obtain

$$L^{-1}\{\Upsilon(p)\} = \Upsilon_0 \sum_{n=0}^{\infty} \frac{(-1)^n \Gamma(\frac{a+2b}{k}+2n+1)}{(\Gamma_k(b+nk+k))^m \Gamma_k(a+b+nc+k)} \left(\frac{t}{2}\right)^{(\frac{a+2b}{k})+2n}$$
$$\times E_{\nu, \frac{a+2b}{k}+2n+1}(-d^\nu t^\nu). \quad (4.\ 44)$$

Hence the proof. □

Theorem 4.2. *For $d,\ \nu,\ a,\ c > 0$,$b \in \mathbb{C}$, $k,\ m \in \mathbb{N}$ and $|d^\nu| < |p^\nu|$, then the equation*

$$\Upsilon(t) - \Upsilon_0 J^{c,m}_{a,b,k}(d^\nu t^\nu) = -d^\nu\ {}_0D_t^{-\nu}\Upsilon(t) \quad (4.\ 45)$$

has solution of the form

$$\Upsilon(t) = \Upsilon_0 \sum_{n=0}^{\infty} \frac{(-1)^n (\Gamma_k(2b+a+2nk+k))}{(\Gamma_k(b+nk+k))^m \Gamma_k(a+b+nc+k)} \left(\frac{d^\nu}{2}\right)^{(\frac{a+2b}{k})+2n} t^{\nu(\frac{a+2b}{k}+2n)}$$
$$\times E_{\nu,\nu(\frac{a+2b}{k}+2n)+1}(-d^\nu t^\nu), \quad (4.\ 46)$$

where $E_{\zeta,\eta}$ is presented in (3. 30).

Theorem 4.3. *For $d,\ \nu,\ a,\ c > 0$,$b \in \mathbb{C}$,$g \neq d$ $k,\ m \in \mathbb{N}$ and $|d^\nu| < |p^\nu|$, then the equation*

$$\Upsilon(t) - \Upsilon_0 J^{c,m}_{a,b,k}(d^\nu t^\nu) = -g^\nu\ {}_0D_t^{-\nu}\Upsilon(t) \quad (4.\ 47)$$

has solution of the form

$$\Upsilon(t) = \Upsilon_0 \sum_{n=0}^{\infty} \frac{(-1)^n (\Gamma_k(2b+a+2nk+k))}{(\Gamma_k(b+nk+k))^m \Gamma_k(a+b+nc+k)} \left(\frac{d^\nu}{2}\right)^{(\frac{a+2b}{k})+2n} t^{\nu(\frac{a+2b}{k}+2n)}$$
$$\times E_{\nu,\nu(\frac{a+2b}{k}+2n)+1}(-g^\nu t^\nu), \quad (4.\ 48)$$

where $E_{\zeta,\eta}$ is presented in 3. 30 .

Proof. The proofs of Theorems 4.2 and 4.3 are similar to Theorem 4.1. □

Remark 4.4. *In particular, if we choose $k = 1$ in Theorems 4.1, 4.2 and 4.3, we turn up [16, Theorems 3.1, 3.4 and 3.7] respectively.*

Furthermore, for $m = 1$ and in view of equation (1. 8), Theorems 4.1, 4.2 and 4.3 lead to the corollaries discussed below.

Corollary 4.5. *For $d,\ \nu,\ a,\ c > 0$,$b \in \mathbb{C}$, $k \in \mathbb{N}$ and $|d^\nu| < |p^\nu|$, then the equation*

$$\Upsilon(t) - \Upsilon_0 J^{c}_{a,b,k}(t) = -d^\nu\ {}_0D_t^{-\nu}\Upsilon(t) \quad (4.\ 49)$$

has the following solution

$$\Upsilon(t) = \Upsilon_0 \sum_{n=0}^{\infty} \frac{(-1)^n (\Gamma_k(2b+a+2nk+k))}{\Gamma_k(b+nk+k)\Gamma_k(a+b+nc+k)} \left(\frac{t}{2}\right)^{(\frac{a+2b}{k})+2n}$$
$$\times E_{\nu, \frac{a+2b}{k}+2n+1}(-d^\nu t^\nu), \quad (4.\ 50)$$

where $E_{\zeta,\eta}$ is presented in (3. 30).

Corollary 4.6. *For $d,\ \nu,\ a,\ c>0$,$b\in\mathbb{C}$, $k\in\mathbb{N}$ and $|d^{\nu}|<|p^{\nu}|$, then the equation*

$$\Upsilon(t)-\Upsilon_0 J^{c}_{a,b,k}(d^{\nu}t^{\nu})=-d^{\nu}\ {}_0D_t^{-\nu}\Upsilon(t) \tag{4. 51}$$

has solution of the form

$$\Upsilon(t)=\Upsilon_0\sum_{n=0}^{\infty}\frac{(-1)^n(\Gamma_k(2b+a+2nk+k))}{\Gamma_k(b+nk+k)\Gamma_k(a+b+nc+k)}(\frac{d^{\nu}}{2})^{(\frac{a+2b}{k})+2n}t^{\nu(\frac{a+2b}{k}+2n)}\times E_{\nu,\nu(\frac{a+2b}{k}+2n)+1}(-d^{\nu}t^{\nu}), \tag{4. 52}$$

where $E_{\zeta,\eta}$ is presented in (3. 30).

Corollary 4.7. *For $d,\ \nu,\ a,\ c>0$,$b\in\mathbb{C}$,$g\neq d$ $k\in\mathbb{N}$ and $|d^{\nu}|<|p^{\nu}|$, then the equation*

$$\Upsilon(t)-\Upsilon_0 J^{c}_{a,b,k}(d^{\nu}t^{\nu})=-g^{\nu}\ {}_0D_t^{-\nu}\Upsilon(t) \tag{4. 53}$$

has solution of the form

$$\Upsilon(t)=\Upsilon_0\sum_{n=0}^{\infty}\frac{(-1)^n(\Gamma_k(2b+a+2nk+k))}{\Gamma_k(b+nk+k)\Gamma_k(a+b+nc+k)}(\frac{d^{\nu}}{2})^{(\frac{a+2b}{k})+2n}t^{\nu(\frac{a+2b}{k}+2n)}\times E_{\nu,\nu(\frac{a+2b}{k}+2n)+1}(-g^{\nu}t^{\nu}), \tag{4. 54}$$

where $E_{\zeta,\eta}$ is presented in (3. 30).

Remark 4.8. *In particular, if we choose $k=1$ in Corollaries 4.5, 4.6 and 4.7, we arrive at [16, Corollaries 3.2, 3.5 and 3.8] respectively.*

If we opt $m=1$, $k=1$, $c=1$ and $b=o$ in Theorems (4.1), (4.2) and (4.3), we obtain the following corollaries

Corollary 4.9. *For $d,\ \nu,\ a,\ c>0$,$b\in\mathbb{C}$, $k\in\mathbb{N}$ and $|d^{\nu}|<|p^{\nu}|$, then the equation*

$$\Upsilon(t)-\Upsilon_0 J_a(t)=-d^{\nu}\ {}_0D_t^{-\nu}\Upsilon(t) \tag{4. 55}$$

has the following solution

$$\Upsilon(t)=\Upsilon_0\sum_{n=0}^{\infty}\frac{(-1)^n(\Gamma(a+2n+1))}{\Gamma(a+n+1)n!}(\frac{t}{2})^{a+2n}\times E_{\nu,a+2n+1}(-d^{\nu}t^{\nu}), \tag{4. 56}$$

where $E_{\zeta,\eta}$ is presented in (3. 30).

Corollary 4.10. *For $d,\ \nu,\ a>0$,$b\in\mathbb{C}$, and $|d^{\nu}|<|p^{\nu}|$, then the equation*

$$\Upsilon(t)-\Upsilon_0 J_a(d^{\nu}t^{\nu})=-d^{\nu}\ {}_0D_t^{-\nu}\Upsilon(t) \tag{4. 57}$$

has solution of the form

$$\Upsilon(t)=\Upsilon_0\sum_{n=0}^{\infty}\frac{(-1)^n(\Gamma(+a+2n+1))}{\Gamma(a+n+1)n!}(\frac{d^{\nu}}{2})^{a+2n}t^{\nu(a+2n)}\times E_{\nu,\nu(a+2n+1)}(-d^{\nu}t^{\nu}), \tag{4. 58}$$

where $E_{\zeta,\eta}$ is presented in (3. 30).

Corollary 4.11. *For $d,\ \nu,\ a>0$, $g\neq d$ and $|d^{\nu}|<|p^{\nu}|$, then the equation*

$$\Upsilon(t)-\Upsilon_0 J_a(d^{\nu}t^{\nu})=-g^{\nu}\ {}_0D_t^{-\nu}\Upsilon(t) \tag{4. 59}$$

has solution of the form

$$\Upsilon(t)=\Upsilon_0\sum_{n=0}^{\infty}\frac{(-1)^n(\Gamma(a+2n+1))}{\Gamma(a+n+1)n!}(\frac{d^{\nu}}{2})^{a+2n}t^{\nu(a+2n)}\times E_{\nu,\nu(a+2n+1)}(-g^{\nu}t^{\nu}), \tag{4. 60}$$

where $E_{\zeta,\eta}$ is presented in (3. 30).

5. Conclusion

In the current paper, we introduced Lommel-Wright k-function and discussed its fractional integrals. We also established a new fractional generalization of the classical kinetic equation involving Lommel-Wright k-function and developed a solution for the same using Laplace transforms. As this special function is firmly identified with numerous other special functions, therefore, a variety of well known and new fractional kinetic equations can be derived.

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