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One Parameter Discrete Ailamujia Distribution with Statistical Properties and Biodiversity and Abudance Data Applications

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Abstract.:The discretization of continuous distributions among researchers become important issue, so several discretization methods exists in the literature for obtaining discrete version of continuous distribution which can be pragmatic to discrete data. The present study proposes the discretization of continuous Ailamujia distribution. Subsequently various statistical properties has been studied including moment generating function, characteristic function, mode, reliability, probability generating function etc. Nature of density function and hazard function has been studied graphically. The technique of maximum likelihood estimation is used to estimate the unknown parameter of the said model. Biodiversity and abundance data applications are provided to flexibility and applicability of the new distribution in ecological studies.

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Key Words: Ailamujia distribution, discretization method, estimation, count data, ecology data.

1. Introduction

Count data come across on day to day basis and proceedings. Statistical analysis involving count data may take multiplex forms subject to the context of use. Simple count data are as - number of childrens of a couple, number of bacteria per unit, number of trades per month, number of plants in a particular field. Various count data may possess different characteristics and hence must have different count data models. Many count data mostly follows Binomial, Poisson, Geometric, Trucated possion, negative binomial, distribution [4]. These discrete distributions are frequently been used to analyze and model count data in scientific fields such as parasitology [2], veterinary medicine [18] and estimation of ore reserves [5].

From last few years discretization of continuous distribution interests the attention of researchers in statistical literature. As in numerous real life data we come from corner to corner samples of discrete type, where it is challenging or problematic to get samples of continuous type. For illustration, the data collected for testing the prevailing pandemic COVID-19 for the different people came out in the form of discrete type to examine whether a person is effected with the pandemic or not. It is problematic to apply a continuous distribution in such circumstances where the testing procedure is imperious for each one in similar way life testing or reliability experiments, it is often difficult to compute life length of a device on continuous scale e.g measuring reliability of an on / off switching device. Therefore, new discrete statistical distributions are required in many applied areas such as ecology, biology, seismology, etc.

In the statistical literature, there are several standard discrete distributions such as Poisson, geometric distributions which used to model count data. Similarly binomial, negative binomial, geometric, hyper geometric distributions where as these distributions have confined scope for modeling real life time data or having limited use as model for reliability, failure, times, counts etc. Moreover, many authors have developed different approaches for the discretization of continuous distribution. Some are given as Roy [21] investigated the statistical properties of the discrete normal distribution, Roy [22] obtained discrete Rayleigh distribution, Inusah et al. [9] defined a discrete analogue of the Laplace distribution, Krishna et al. [12] studied several properties of discrete Burr and discrete Pareto distributions, Al-Huniti ve Al-Dayjan [3] obtained discrete Burr type III distribution. Nekoukhou et al. [15] obtained discrete generalized exponential distribution Hussain and Ahmad [7] investigated the statistical properties of the discrete inverse Rayleigh distribution. Several others like Szablowski [23], Hussain and Ahmad [8], Nekoukhou et al. [16] and Alexander Kasyoki et al [1] etc suggested discretize analogue of continuous life time distributions and employ them on count data models Lv et al. [13] proposed a new continuous distribution known as Ailamujia distribution. The interval estimation and hypothesis testing based on small samples was studied by Pn et al. [17]. Reshi et al. [20] propsed the Bayes estimates of unkown parameter of Ailamujia distribution under different loss function. As a result of these studies, Ailamujia distribution is flexible and can be used with one parameter easily in real life applications. However, the number of discrete events can not explained with continuous distribution and new discrete and more flexible distributions are required especially for real data applications. The aim of present paper is to study discrete analogue of a continuous Ailamujia distribution. In order to obtain required probability mass function of the proposed model, we apply the procedure of infinite series method.

2. Discrete Analogue of Ailamujia Distribution

Let X follows a continuous distribution having the pdf $f_X(x)$ with range R, then the corresponding random variable Y has the pmf given as

$$P(Y=y)=P(y,\theta)=P(y)=\frac{f_X(y,\theta)}{\sum_{i=-\infty}^{\infty} f_X(i,\theta)}, \quad y\in\mathbb{Z} \tag{2. 1}$$

A random variable X is said to follows Ailamujia distribution if its pdf and cdf are, respectively, defined as

$$f(x,\alpha) = 4\alpha x e^{-2\alpha x}, \qquad \alpha > 0, x > 0 \tag{2. 2}$$

and

$$F(x,\alpha) = 1 - (1 + 2\alpha x)e^{-2\alpha x}, \quad \alpha > 0, x > 0c \tag{2. 3}$$

Discretization of (2. 2) is obtained using after using it in the discretization method given in (2. 1), and we obtained the probability mass function of discrete Ailamujia model given below

$$P(y) = \frac{(1-\theta)^2}{\theta} y\theta^y, \quad y = 1, 2...; \theta > 0 \tag{2. 4}$$

where $\theta = e^{-2\alpha x}$, we will called the pmf in (2. 4) as discrete Ailamujia distribution and analogous cdf of discrete Ailamujia distribution is acquired by using following procedure

$$\begin{aligned} F(y) &= P(Y) = 1 - P(Y > y) \\ &= 1 - \sum_{w=y+1}^{\infty} f(w) = 1 - \sum_{w=y+1}^{\infty} \frac{(1-\theta)^2}{\theta} w\theta^w \end{aligned}$$

After manipulating, we get

$$F(y) = 1 - [y(1-\theta) + 1]\theta^y, \quad y = 1, 2...; \theta > 0 \tag{2. 5}$$

Figures 1 and 2 depicts the behaviors of the pmf and cdf at various values of parameter ($\theta = 0.2, 0.8, 1.2, 2.2$)

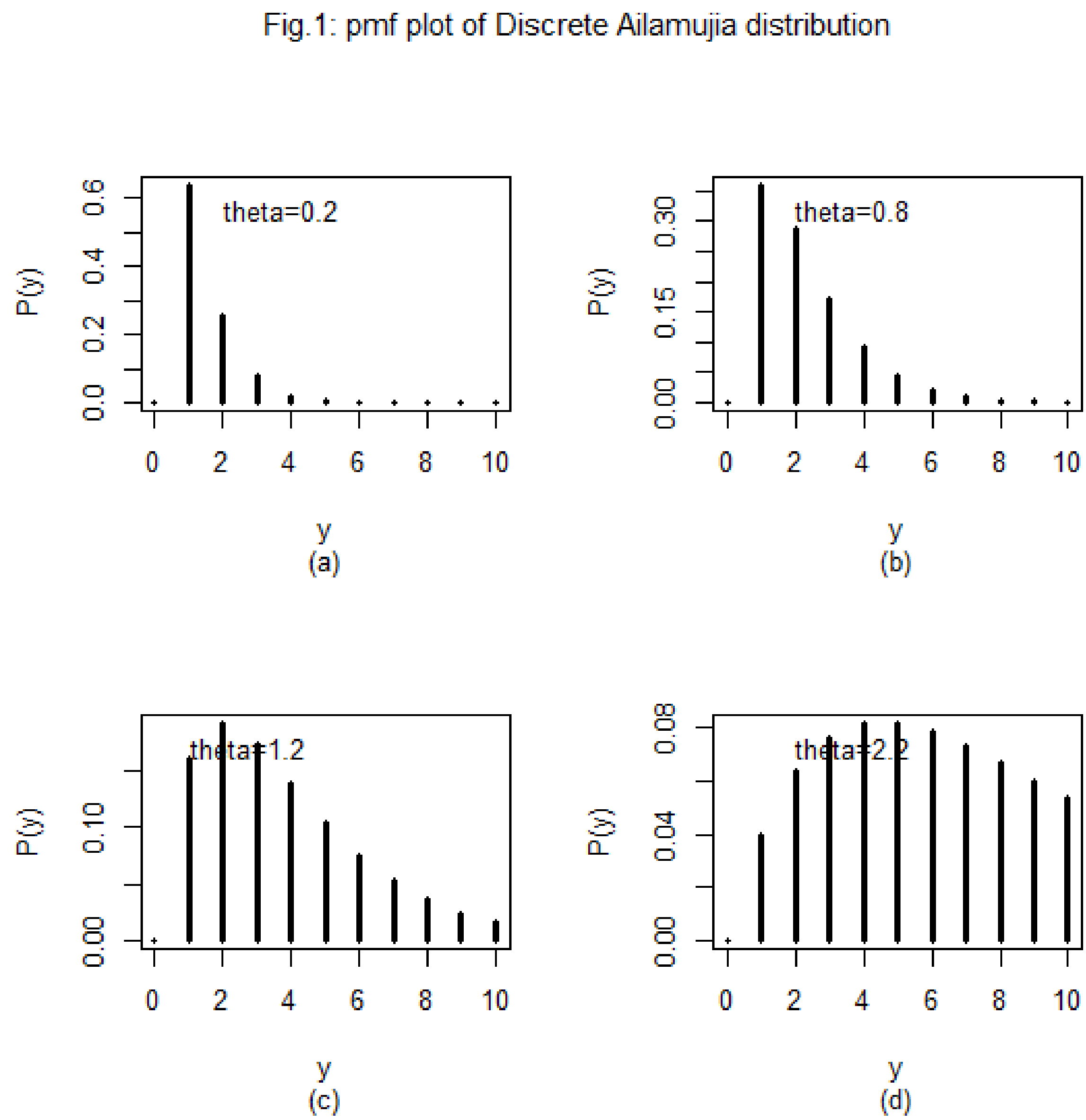

FIGURE 1. The plots for the pmf of the discrete Ailamujia distribution with several values of parameters.

3. STATISTICAL PROPERTIES OF DISCRETE AILAMUJIA DISTRIBUTION

3.1. **Moment generating function.** Let Y be a random variable which follows discrete Ailamujia distribution. Then, moment generating function of Y denoted by $M_Y(t)$ is obtained as follows:

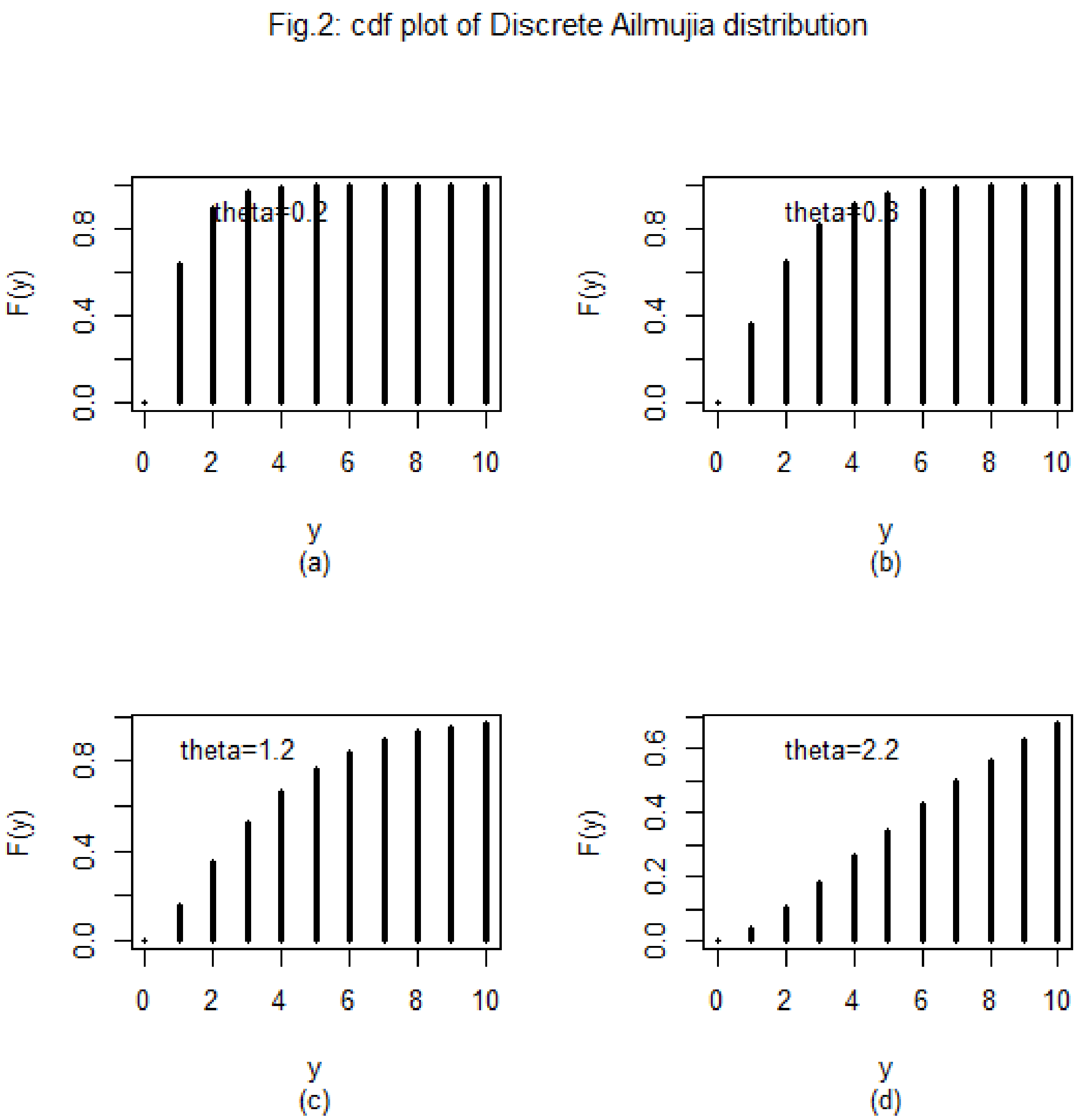

FIGURE 2. The plots for the cdf of the discrete Ailamujia distribution with several values of parameters.

$$
\begin{aligned}
M_y(t) =& E(e^{Yt}) = \sum_{y=1}^{\infty} e^{yt} P(y) \\
=& \sum_{y=1}^{\infty} \frac{(1-\theta)^2}{\theta} e^{ty} y \theta^y \\
=& \frac{(1-\theta)^2}{\theta} \sum_{y=1}^{\infty} y(\theta e^t)^y \\
=& \frac{(1-\theta)^2}{\theta} e^{\theta} [1-\theta e^{\theta}]^{-2}
\end{aligned}
$$

TABLE 1. Summary of discrete Ailamujia distribution under different values of parameters

θ	Mean	Variance	Skewness	Kurtosis
2.0	3	4	2.12	0
3.0	2.5	1.5	5.62	-3
4.0	2.3	0.88	11.5	-8
10	2.12	0.243	115.42	-80
12	2.10	0.238	122.56	-85

$$M_y(t) = \frac{(1-\theta)^2 e^t}{[1-\theta e^{\theta}]^2} \quad , \quad \forall t \neq log\frac{1}{\theta} \tag{3.1}$$

The first four moments about origin of the proposed distribution are given under:

$$\mu_1^{'} = \frac{1-2\theta}{1-\theta}, \qquad \mu_2^{'} = \frac{\theta^2+4\theta+1}{(1-\theta)^2}$$

$$\mu_3^{'} = \frac{4\theta^3+8\theta^2+11\theta+1}{(1-\theta)^3} \quad \mu_4^{'} = \frac{2\theta^4-42\theta^3-123\theta^2+21\theta+6}{(1-\theta)^4}$$

The moments about mean, variance, C.V, skewness, kurtosis of the discrete Ailamujia distribution are obtained as

$$\mu_2 = \frac{2\theta}{(1-\theta)^2}$$

$$\mu_3 = \frac{3\theta^2-\theta+2}{(1-\theta)^3}$$

$$\mu_4 = \frac{-11\theta^4-22\theta^3-163\theta^2+9\theta+5}{(1-\theta)^4}$$

$$\sigma = \frac{\sqrt{2\theta}}{1-\theta}$$

$$C.V = \frac{\sqrt{2\theta}}{1+\theta}$$

$$\sqrt{\beta_1} = \frac{3\theta^2-\theta+2}{8\sqrt{\theta}}$$

$$\sqrt{\beta_2} = \frac{-11\theta^4-22\theta^3-163\theta^2+9\theta+5}{4\theta^2}$$

where, C.V = coefficient of variation, β_1 =Skewness, β_2 =Kurtosis

From TABLE 1, we see that as the value of θ increases mean of the distribution decreases. Also from the data we see that the proposed model is positively skewed and has platykurtic curve.

3.2. **Characteristics function.** Let Y be a random variable which follows discrete Ailamujia distribution. Then, characteristic function of Y denoted by $\phi_Y(t)$ is given as

$$\begin{aligned}\phi_Y(t) =& E(e^{itY}) = \sum_{y=1}^{\infty} e^{itY} P(y)\\ =& \sum_{y=1}^{\infty} \frac{(1-\theta)^2}{\theta} e^{itY} y\theta^y\\ =& \frac{(1-\theta)^2}{\theta} \sum_{y=1}^{\infty} y(\theta e^{it})^y\\ =& \frac{(1-\theta)^2}{\theta} e^{it}[1-\theta e^{it}]^{-2}\end{aligned}$$

$$\phi_Y(t) = \frac{(1-\theta)^2 e^{it}}{[1-\theta e^{it}]^2} \quad , \forall it \neq log\frac{1}{\theta} \quad , \quad i = \sqrt{-1} \tag{3. 6}$$

3.3. **Probability generating function.** Let Y be a random variable which follows discrete Ailamujia distribution. Then, probability generating function of Y denoted by $P_Y(t)$is given as

$$\begin{aligned}P_Y(t) =& \sum_{y=0}^{\infty} t^y P(y)\\ =& \frac{(1-\theta)^2}{\theta} \sum_{y=0}^{\infty} t^y y\theta^y\\ =& \frac{(1-\theta)^2}{\theta} \sum_{y=0}^{\infty} y(\theta t)^y\\ =& \frac{(1-\theta)^2 t}{(1-\theta t)^2} \quad , \forall t \neq \frac{1}{\theta}\end{aligned}$$

3.4. **Survival and Hazard Rate Function.** Suppose Y be a discrete random variable with cdf $F(y), y = 1, 2, 3, ...$Then, its reliability function which is also called survival function is defined as

$$S(y) = P(Y > y) = 1 - F(y, \theta)$$

Survival function of discrete Ailamujia distribution is given as

$$S(y, \theta) = 1 - F(y, \theta) = [y(1-\theta) + 1]\theta^y \tag{3.2}$$

Plots for survival function of discrete Ailamujia distribution is presented in Figure 3.

Hazard Function: Hazard rate function of the random variable y is given as

$$h(y, \theta) = \frac{P(y)}{S(y, \theta)} \tag{3.3}$$

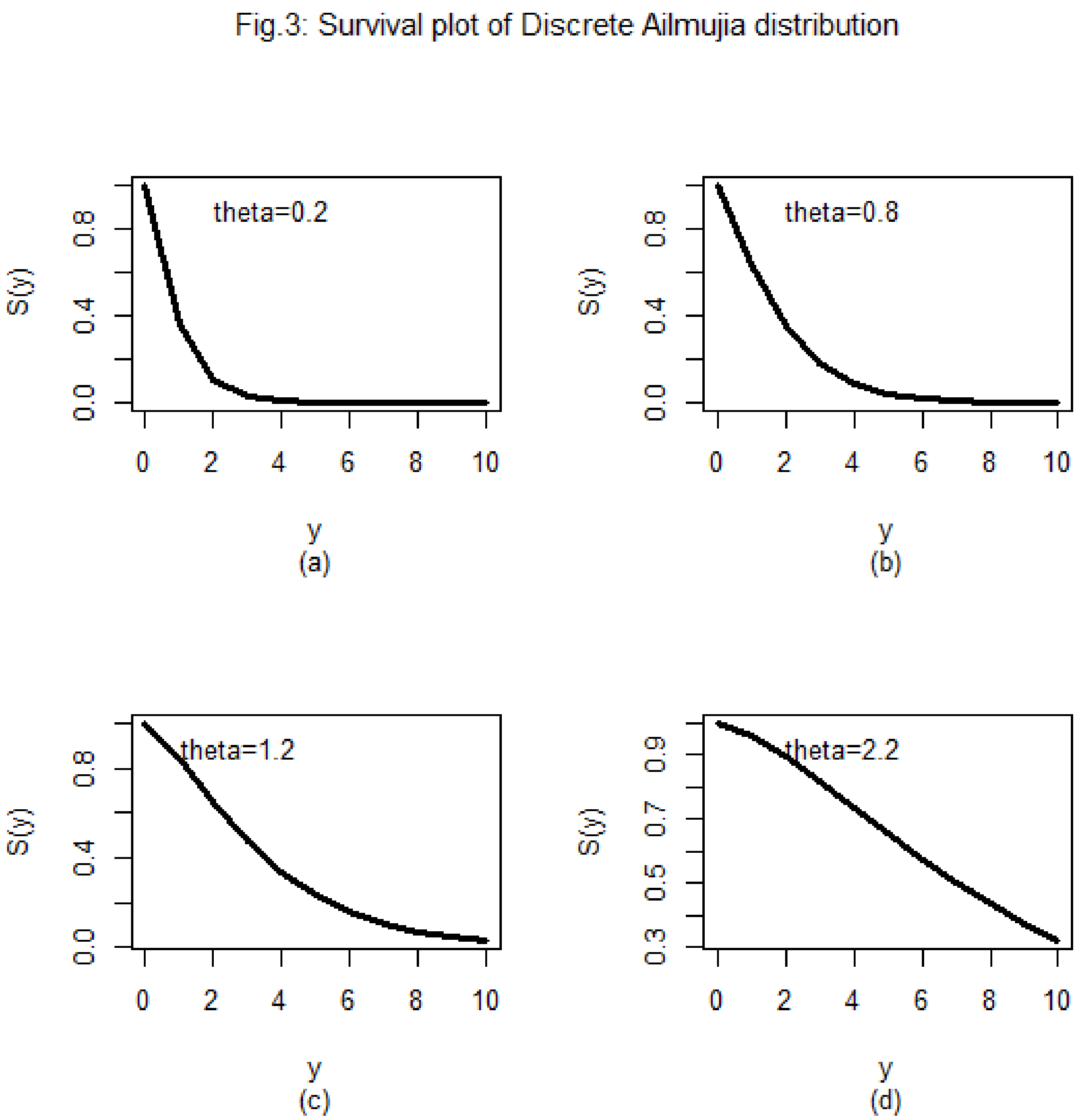

FIGURE 3. Plots of survival rate function of discrete Ailamujia distribution with several values of parameters.

Substituting (2.4) and (3.2), into (3.3), we get the hazard rate function of discrete Ailamujia distribution as

$$h(y,\theta) = \frac{\frac{(1-\theta)^2}{\theta} y\theta^y}{[y(1-\theta)+1]\theta^y} = \frac{(1-\theta)^2 y}{\theta[y(1-\theta)+1]}$$

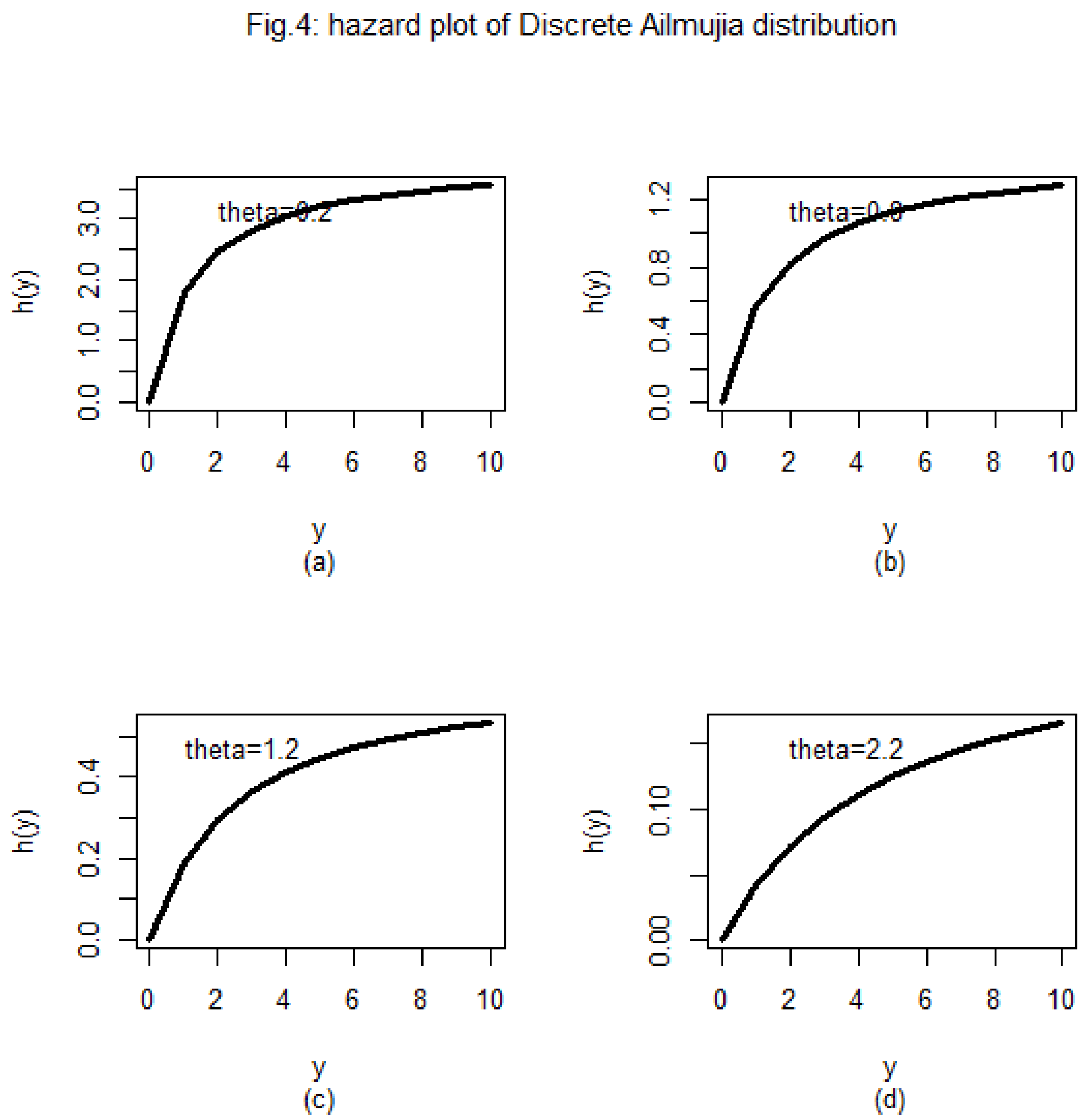

FIGURE 4. Plots of hazard rate function of discrete Ailamujia distribution with several values of parameters.

Plots for hazard rate function of discrete Ailamujia distribution are presented in Figure 4. As seen in Figure 4, the hazard rate function is monotone increasing shape.

4. ESTIMATION OF PARAMETER OF DISCRETE AILAMUJIA DISTRIBUTION

We apply two methods for estimating the parameter of proposed model namely method of moments and method of maximum likelihood.

4.1. Method of Moments.

To find the sample moments of discrete Ailamujia distribution, we equate population moments with sample moments

$$\mu = \mu_1' = \frac{1}{n}\sum_{i=1}^{n} yi$$

$$\mu = \bar{y} = \frac{1+\theta}{1-\theta}$$

$$\bar{y}(1-\theta) = (1+\theta) \implies \hat{\theta} = \frac{\bar{y}+1}{\bar{y}-1}$$

4.2. Maximum likelihood estimation.

Let $Y_1, Y_2, ...Y_n$ be a random sample of size n from discrete Ailamujia distribution. Then, its likelihood function is given by

$$l(\theta) = \prod_{i=1}^{n} P(y_i) = \prod_{i=1}^{n} \frac{(1-\theta)^2}{\theta} P(y_i)\theta^{y_i}$$

The log-likelihood is given as

$$logl(\theta) = = ln\left[\frac{(1-\theta)^2}{\theta}\right]^n + \sum_{i=1}^{n} lny_i + \sum_{i=1}^{n} y_i log\theta$$

$$=2nlog(1-\theta) - nlog\theta + \sum_{i=1}^{n} lny_i + \sum_{i=1}^{n} y_i log\theta$$

Differentiate w.r.t'θ'we get

$$\frac{\partial log(\theta)}{\partial\theta} = \frac{2n}{\theta-1} - \frac{n}{\theta} + \frac{1}{\theta}\sum_{i=1}^{n} y_i$$

Substituting$\frac{\partial log(\theta)}{\partial\theta}$=0,we get the required estimator ofθ

$$\frac{2n}{\theta-1} - \frac{n}{\theta} + \frac{1}{\theta}\sum_{i=1}^{n} y_i = 0 \implies \frac{n(\theta+1)}{\theta(\theta-1)} + \frac{n}{\theta}\bar{y} = 0$$

After simplification it yields as

$$\hat{\theta} = \frac{\bar{y}-1}{\bar{y}+1}$$

5. APPLICATION

Discrete Ailamujia distribution has been fitted numerous data sets to test its goodness of fit over other candidates model viz zero truncated Poisson (ZTP), zero truncated generalized Poisson (ZTGP) and zero truncated Poisson gamma (ZTPG) distributions. Data I: The first data set is count data set presented in Table 2 shows the number of mites per leaf, the European red mite belongs to a group of plant-feeding mites, called spider mites stated by Garman [6].

TABLE 2. The number of mites per leaf, the European red mite belongs to a group of plant-feeding mites

Y	1	2	3	4	5	6	7
f	38	17	10	9	3	2	1

TABLE 3. Summary of the table 2.

Mean	Variance	Median	Ist Quartile	3rd Quartile
2.15	2.103	2.00	1.00	3.00

Tables 6 portray the goodness of fit test of the first and second data sets, respectively. We calculate the expected frequencies for fitting discrete Ailamujia, ZTP, ZTGP and ZTPG distributions, using R studio Pearson?s chi-square test is used for testing the goodness of fit.

TABLE 4. Goodness of fit test result for the number of mites per leaf data

Y	Observed Frequency	Discrete Ailamujia	ZTP	ZTPG	ZTGP
1	38	32.258	28.7	30.31	30.7
2	17	23.538	25.7	25.15	22.2
3	10	12.892	15.3	12.34	12.9
4	9	6.274	6.9	7.23	5.9
5	3	2.962	2.5	3.22	3.0
6	2	1.453	0.7	1.22	1.7
7	1	0.633	0.2	0.62	1.1
	80.00	80.00	80.00	80.00	80.0
	χ^2	2.067	9.827	5.631	2.467
	d.f	3	2	2	3
	P-value	0.566	0.091	0.059	0.4813

Table 6, reveals ZTGP distribution is not good fit at all, whereas ZTP and ZTGP fits good. However, discrete Ailamujia distribution fits better than the ZPG and ZTGP distributions. Thus, the null hypothesis that data emanate from discrete Ailamujia distributions is powerfully accepted. Tables 7 shows the maximum likelihood estimates, $?2??ogL$, AIC and BIC values from which it is seen that AIC and BIC values of discrete Ailamujia is

TABLE 5. AIC, BIC and log likelihood values for the fitted distributions for the number of mites per leaf data

Criterion	Discrete Ailamujia	ZTP	ZPGD		ZTGP	
Maximum Likelihood Estimate	0.36507	1.7916	1.3842	0.9265	0.2646	0.9986
(Standard Error)	(0.0303)	(0.1705)	(0.2732)	(0.1323)	(0.0548)	(0.1805)
- l	118.779	122.794	118.928		118.9245	
AIC	241.559	247.589	241.856		242.849	
BIC	243.941	249.639	246.221		246.6131	

smaller than other existing models.

Data II: The second data set presented in Table 6 is the animal abundance data of Keith and Meslow[10] regarding the distribution of snowshoe hares captured over 7 days. Alberta snowshoe investigations carried out near the town of Rochester, approximately 60 miles north of Edmonton. The data presented here are from intensive livetrapping on six different study areas during 1962-67.

TABLE 6. The animal abundance data of Keith and Meslow[10]

Y	1	2	3	4	5
f	184	55	14	4	4

TABLE 7. : Summary of the table 8.

Mean	Variance	Median	Ist Quartile	3rd Quartile
1.425	0.629	1.00	1.00	2.00

TABLE 8. The goodness of fit test results of the animal abundance data of Keith and Meslow[10]

Y	Observed Frequency	Discrete Ailamujia	ZTP	ZTPG	ZTGP
1	184	177.643	170.6	172.74	172.7
2	55	62.175	72.5	63.335	60.1
3	14	15.320	15.4	16.734	18.3
4	4	3.808	2.2	5.843	6.8
5	4	1.833	0.3	2.325	3.1
	261	261	261	261	261
	χ^2	1.105	6.216	1.492	2.241
	d.f	2	1	2	2
	P-value	0.575	0.013	0.474	0.5239

TABLE 9. AIC, BIC and log likelihhod values for the fitted distributions for the animal abundance data

Criterion	Discrete Ailamujia	ZTP	ZPGD		ZTGP	
Maximum Likelihood Estimate	0.175	0.425	0.6132	1.7561	0.7984	6.6177
(Standard Error)	(0.0151)	(0.0738)	(0.0534)	(1.012)	(0.0280)	(0.7277)
- l	228.400	335.243	234.347		263.6471	
AIC	458.801	672.486	472.694		528.432	
BIC	462.366	676.0505	474.337		534.516	

TABLE 10. AIC, BIC and log likelihood values for simulated data

Criterion	Discrete Ailamujia	ZTP	ZPGD		ZTGP	
Maximum Likelihood Estimate	1.133	2.322	2.4314	3.5221	0.8864	4.3643
(Standard Error)	(0.0263)	(0.1022)	(0.9542)	(1.752)	(0.02674)	(0.8653)
- l	133.41	148.567	139.633		144.063	
AIC	268.82	299.134	283.266		290.126	
BIC	268.82	299.134	284.266		290.126	

Table 10 reveals ZTGP distribution is not good fit at all, whereas ZTP and ZTGP fits good. However, discrete Ailamujia distribution fits better than the ZPG and ZTGP distributions. Thus, the null hypothesis that data emanate from discrete Ailamujia distributions is powerfully accepted. Tables 11 shows the maximum likelihood estimates,?2??ogL, AIC and BIC values from which it is seen that AIC and BIC values of discrete Ailamujia is smaller than other existing models.

6. SIMULATION STUDY

In our simulation study, we choose a sample size of $n = 100$ to represent medium data set. Random numbers are generated and iterated 5000 times using inverse probability transformation method. On the basis of sample, we estimate the value parameter. On the basis of generated sample, we use R code and check the goodness of fit of the Discrete Ailamujia Distribution and compare it with other candidates model viz zero truncated Poisson (ZTP), zero truncated generalized Poisson (ZTGP) and zero truncated Poisson gamma (ZTPG) distributions. Results are shown in Table 12.

From table 12 it is seen that AIC and BIC values of discrete Ailamujia is smaller than other existing models. Thus we can say that discrete Ailamujia performs better fit than other existing models.

7. CONCLUSION

In this paper a discrete analogue of continuous Ailamujia distribution is proposed and several structural properties of the proposed model is discussed. The proposed model is compared with other candidate?s model as discussed in various tables. From Tables 6 and 10 it is evident that proposed model displays its appropriateness by showing higher P-values. Moreover, from Tables 7 and 11 it is shown that the proposed model shows smaller values AIC, BIC and AICC. These lesser values of the model selection statistics advocate that the proposed model is the superlative probability model among other competing models.

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