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# Decision Support System Based on Spherical 2-tuple Linguistic Fuzzy Aggregation Operators and their Application in Green Supplier Selection

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**Abstract.** In this manuscript, we give the idea of Spherical 2-tuple linguistic fuzzy set (S2TLFS) for the multi criteria decision making (MCDM) problem with the information. We utilized some operation to define some Spherical 2-tuple linguistic fuzzy (S2TLF) aggregation operators (AOs). We discussed some properties of the developed operators. Then, to solve an MCDM problem using the Spherical 2-tuple linguistic information, we proposed an approach, and utilized these operators. Lastly, a numerical example of the green supplier selection for chemical processing industry is given to show the advantage of the defined approach and to show its practicability and performance.

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## 1. Introduction

Universally, there is some confusion in the decision-making (DM) process when describing data information. To address this drawback, the idea of fuzzy set (FS) was first developed by Zadeh [50]. In FS, only the member grade of a number in the given set is shown by Zadeh, and tested in several fields for example, fuzzy decision taking issues [5, 6]. Yet the negative membership rating is not discussed. The FS concept failed to overcome the uncertainty in the daily problem because of the negative grade. Thus, Atanassov [1] established intuitionistic fuzzy set (IFS) definition, which have both a membership and a nonmembership grades, IFS has the advantage of having two membership grades that decrease the fluidity. Garg [8] showed widespread improved collaborative AOs to solve the IF-set decision-making problems. Apart from this, various researchers (Shen and Wang [36]; Wang and Peng [42]) integrated the concept of aggregation method into the various applications and provided their DM methods with the IF set and expanded there. Zulkifli [53] proposed an integrated interval-valued intuitionistic fuzzy vague set and their linguistic variables.

In some cases, the value of $\mu+\nu \geq 1$(membership and nonmembership) unlike the cases capture in IFSs. The Pythagorean fuzzy set (PFS) is then defined by Yager [46, 48, 49], which have membership and nonmembership grades, and satisfy the condition $\mu^2+\nu^2 \geq 1$. To deal with this sort of situation, Yager [46] provide an example: a DM makes his positive term for an alternative is $\frac{\sqrt{3}}{2}$ and negative term is $\frac{1}{2}$. Now, their number is higher than 1, so they can't be set to intuitionistic fuzzy, but they can be set to Pythagorean fuzzy because $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \leq 1$. This shows that PFS is more capable of managing uncertainty in real-life problems than intuitionistic fuzzy. Now a few days the idea of picture fuzzy encourages researchers to think and applied the implementation of the Pythagorean fuzzy system in many areas of study. Rahman et al. [34] defined the geometric AOs to the IVPFS setting for group decision problem. Liang & Xu [27] developed the idea of hesitant PFS and tested on TOPSIS approach for the selection model of energy projects. Garg [11] implemented a series of AOs with the Pythagorean fuzzy information, using the idea of immediate probabilities. Garg [9] defined the generalized Pythagorean fuzzy geometric AOs using the t-norm and conorm operations for MCGDM problem. Ren et al. [35] implemented TODIM method to find the best solution for DM problems, where the information will be in the form of PFNs. Wei [41] implemented many AOs, like as weighted average interaction and geometric operators. Wei & Lu [43] built the power AOs to handle MADM problems using PF information. Xu et al. [44] introduced generalized OWA induced PF operators information. Xue et al. [45] proposed the LINMAP approach to use Pythagorean fuzzy information to identify the best investment firm in railway projects. Yager [47] has begun weighted average, linear, ordered linear AOs for Pythagorean fuzzy details. To solve a MCDM problem with the unknown weight information, Garg [10] implemented a score function where attribute preferences occur in the form of IVPFSs. Hamacher operations [16] are the good alternatives to the algebraic product and algebraic sum, correspondingly [51]. Many researchers have addressed the Hamacher aggregation operators and their implementations in the last few years [26, 37]. Pei, L. et al. [29] defined local adjustment strategy-driven probabilistic linguistic group decision-making method and its application for fog-haze influence factors evaluation. Garg, H. [12] developed linguistic interval-valued Pythagorean fuzzy sets and their application to MAGDM process.

Herrera and Martinez [19] introduced the idea of a 2-tuple linguistic processing model using the model of symbolic translation, and demonstrated that the 2-tuple linguistic information processing system can effectively avoid loss and distortion. Herrera et al. [20] suggest a MAGDM model to handle non-homogeneous information. Wang [38] built a 2-tuple fuzzy linguistic processing model to choose the appropriate agile production system. The TOPSIS approach is developed by Wei [39] with 2-tuple linguistic knowledge for MAGDM question. Chang and Wen [7] introduced the efficient approach for DFMEA by combining the 2-tuple and the OWA operator. For the 2-tuple linguistic knowledge, mean

Bonferroni operators are extended by Jiang and Wei [21]. Liu et al. [25] defined the dependent interval 2-tuple linguistic AOs for MAGDM. Wang et al. [40] developed a MAGDM problem approach using 2-tuple linguistic information intervals and integrated AOs Choquet. To research the application of MADM to the supplier selection Liu [30] has specified the Muirhead mean 2-tuple linguistic operator. Zhang et al. [52] proposes a consensus reaching model for the 2-tuple linguistic MADM, with the incomplete weight information. Merigo and Gil-Lafuente [28] proposed the concept of induced 2-tuple linguistic generalized aggregation operators and discussed their application in decision-making. Khan et al. [24] defined some analysis of Robot selection based on 2-tuple picture fuzzy linguistic AOs.

The idea of the Spherical fuzz set (SFS) was first time developed by S. Ashraf et al. [29], and also developed the Spherical fuzzy AOs for MADM problem. Gundogdu et al. [23] extended the TOPSIS approach for SFS and solved a numerical example of MAGDM problem. The extended form of PFS is essentially SFS. In the Spherical fuzzy set, all the membership degrees are gratifying the condition $0 \leq (\mu_{\mathbb{N}}(r))^2 + (\eta_{\mathbb{N}}(r))^2 + (\nu_{\mathbb{N}}(r))^2 \leq 1$ instead of $0 \leq \mu_{\mathbb{N}}(r) + \eta_{\mathbb{N}}(r) + \nu_{\mathbb{N}}(r) \leq 1$ as in picture fuzzy set. Huanhuan et al. [22] specified SFS, combining the concept of linguistic term set and SFS. Abdullah et al. [3] proposed an analysis of decision support system using 2-tuple Spherical fuzzy linguistic information. Qiyas et al. [31, 32] defined sine trigonometric Spherical fuzzy AOs and their application in decision support system. Qiyas et al [33] defined Spherical uncertain linguistic Hamacher AOs and discussed their application on achieving consistent opinion fusion in group decision making.

However, all the above approaches are unsuitable to describe the degree of positive membership, degree of neutral membership, degree of negative membership and degree of refusal membership of an element to a linguistic label, which can reflect the decision-maker confidence level when they are making an evaluation. In order to overcome this limit, we shall propose the concept of Spherical 2-tuple linguistic set to solve this problem based on the picture fuzzy sets and 2-tuple linguistic information processing model. Thus, how to aggregate these Spherical 2-tuple linguistic numbers is an interesting topic. To solve this issue, in this paper, we shall develop some Spherical 2-tuple linguistic information aggregation operators on the basis of the traditional arithmetic and geometric operations.

In order to do so, the remainder of this paper is set out as follows; In Sec. II, We discussed briefly the basic knowledge of the SFS and the 2-tuple linguistic model. In Sec. III, we discussed some Spherical 2-tuple linguistic averaging and geometric AOs, and study basic properties of the developed operators. We introduced an algorithm for multi attribute DM problems in Sec. IV utilizing the S2TLWA and S2TLWG operators. Inside Sec. V, makes some discussions on the implementation and the contrast of the established methodology to the current method, and finally write the conclusion in Sec. VI.

## 2. Preliminaries

### 2.1. **2-Tuple linguistic term set.**

Definition 2.1. [17, 18] Let $\acute{S} = (s_1, ..., \tau)$ are the linguistic term set, and $\tau$ denote the odd cardinality, such as $s_\tau$, $\tau$ are the possible value of the linguistic variable and positive integer, correspondingly. If, $\tau$ is considered as $3, ...$ e.g., when $\tau = 5$, then the linguistic term set $\acute{S}$ is described as $\{s_1 =$ Poor, $s_2 =$ Slightly poor, $s_3 =$ Fair, $s_4 =$ Slightly good , $s_5 =$ Good$\}$.

If $s_\kappa, s_t \in \acute{S}$ , then we have the following characteristic;

(1). The ordered set: $s_\kappa \prec s_t, \Leftrightarrow \kappa \prec t$;

(2). The negation operator: Neg $(s_\kappa) = s_{\tau-\kappa}$;

(3). Maximum $(s_\kappa, s_t) = s_\kappa$, iff $s_\kappa \geq s_t$;

(4). Minimum $(s_\kappa, s_t) = s_\kappa$, iff $s_\kappa \leq s_t$.

Utilizing the idea of symbolic translation, Herrera & Martinez [17, 18] developed the 2-tuple linguistic model. This model are utilized to presenting the linguistic assessment information with the 2-tuple $(s_i, \chi_i)$, where $s_i$ and $\chi_i$ are the linguistic label and symbolic translation from the linguistic term set $\acute{S}$ and $\chi \in [-0.5, 0.5]$, respectively.

Definition 2.2. [24] Let $\varsigma$ be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set $\acute{S}$, for example, the result of a symbolic aggregation operation, $\check{e} \in [1, \tau]$, where $\tau$ be the cardinality of $\acute{S}$. Let $i =$round$(\varsigma)$ and $\chi = \varsigma - i$ be two values, such as, $i \in [1, \tau]$ and $\varsigma \in [-0.5, 0.5]$, then $\chi$ is known is symbolic translation.

Definition 2.3. [24] Let $\acute{S} = (s_1, ..., \tau)$ are the finite linguistic term set and $\varsigma \in [1, \tau]$ are the value of the aggregation result of linguistic symbolic. Then, the function $\Lambda$ are used to obtain the 2-tuple linguistic information equivalent to numerical value $\varsigma$, and defined as:

$$\Lambda \quad : \quad [1, \tau] \to \acute{S} \times [-0.5, 0.5), \qquad (1)$$

$$\Lambda(\varsigma) \quad = \quad \begin{cases} s_i, i = \text{round}(\varsigma) \\ \chi = \varsigma - i, \chi \in [-0.5, 0.5), \end{cases} \qquad (2)$$

where round $(.)$ denote the usual round operation, $s_i$ denote the closest index label to $\varsigma$ and $\chi$ denote the symbolic translation value.

Definition 2.4. [24] Let $\acute{S} = (s_1, ..., \tau)$ are the finite linguistic term set and $(s_i, \chi_i)$ are the 2-tuple. Then, there exist a mapping $\Lambda^{-1}$, where from the 2-tuple $(s_i, \chi_i)$ it returns to equivalent numerical value $\varsigma \in [1, \tau] \subset R$, like as;

$$\Lambda^{-1} \quad : \quad \acute{S} \times [-0.5, 0.5) \to [1, \tau] \qquad (3)$$

$$\Lambda^{-1}(s_i, \chi) \quad = \quad i + \chi = \varsigma \qquad (4)$$

From Definitions $(2.1)$ and $(2.1)$, we note that the conversion of a linguistic word to a 2-tuple language consists of adding a value $0$ as a symbolic translation:

$$\Lambda(s_i) = (s_i, 0) \qquad (5)$$

Definition 2.5. [23] Let $\mathbb{R} \neq \phi$, be a universal set. Then, $\Re$ is known as SFS, and defined as;

$$\Re = \{\langle \mu_\Re(r), \eta_\Re(r), \nu_\Re(r) |\ r \in \mathbb{R} \rangle\}. \qquad (6)$$

Where $\mu_\Re(r)$, $\eta_\Re(r)$, $\nu_\Re(r) : \mathbb{R} \to [0, 1]$ are the positive grade, neutral grade and negative grade of each $r \in \mathbb{R}$, respectively. Furthermore, $\mu_\Re(r)$, $\eta_\Re(r)$ and $\nu_\Re(r)$ satisfy that $0 \leq \mu^2_\Re(r) + \eta^2_\Re(r) + \nu^2_\Re(r) \leq 1\ \forall\ r \in \mathbb{R}$. $\chi_\Re(r) = \sqrt{1 - (\mu^2_\Re(r) + \eta^2_\Re(r) + \nu^2_\Re(r))}$ is called refusal grade of $r \in \mathbb{R}$, and a triple components $\langle \mu_\Re, \eta_\Re, \nu_\Re \rangle$ are called the SFN and each SF number are denoted by $\Re = \langle \mu_\Re, \eta_\Re, \nu_\Re \rangle$, where $\mu_\Re, \eta_\Re$ and $\nu_\Re \in [0, 1]$, have the condition

$$0 \leq \mu^2_\Re + \eta^2_\Re + \nu^2_\Re \leq 1. \qquad (7)$$

Definition 2.6. [23] Let $\Re_1 = \langle \mu_{\Re_1}(r), \eta_{\Re_1}(r), \nu_{\Re_1}(r)\rangle$ and $\Re_2 = \langle \mu_{\Re_2}(r), \eta_{\Re_2}(r), \nu_{\Re_2}(r)\rangle$ are two SFNs define on the universal set $\mathbb{R} \neq \phi$. Then, some operational laws on SFNs are defined as;

(1). $\Re_1 \subseteq \Re_2$ if

$$\mu_{\Re_1}(r) \leq \mu_{\Re_2}(r), \eta_{\Re_1}(r) \leq \eta_{\Re_2}(r) \text{ and } \nu_{\Re_1}(r) \geq \nu_{\Re_2}(r), \forall r \in \mathbb{R},$$

(2). Union

$$\Re_1 \cup \Re_2 = \left\{ \begin{array}{c} (r, \max(\mu_{\Re_1}(r), \mu_{\Re_2}(r)),\ \min(\eta_{\Re_1}(r), \eta_{\Re_2}(r)), \\ \min(\nu_{\Re_1}(r), \nu_{\Re_2}(r)) | r \in \mathbb{R} \end{array} \right\};$$

(4). Intersection

$$\Re_1 \cap \Re_2 = \left\{ \begin{array}{c} (r, \min(\mu_{\Re_1}(r), \mu_{\Re_2}(r)),\ \max(\eta_{\Re_1}(r), \eta_{\Re_2}(r)), \\ \max(\nu_{\Re_1}(r), \nu_{\Re_2}(r)) | r \in \mathbb{R} \end{array} \right\};$$

(5). Compliment

$$\Re_1^{\nu} = \{(r, \nu_{\Re_1}(r), \eta_{\Re_1}(r), \mu_{\Re_1}(r) | r \in \mathbb{R}\}.$$

Definition 2.6. [23] Let $\Re_1 = \langle \mu_{\Re_1}, \eta_{\Re_1}, \nu_{\Re_1}\rangle$ and $\Re_2 = \langle \mu_{\Re_2}, \eta_{\Re_2}, \nu_{\Re_2}\rangle$ are two SFNs define on the universal set of $\mathbb{R} \neq \phi$. Then, some operational laws on SFNs are described as, where $\lambda \geq 0$.

(1) $\Re_1 \oplus \Re_2 = \left\{\sqrt{\mu_{\Re_1}^2 + \mu_{\Re_2}^2 - \mu_{\Re_1}^2.\mu_{\Re_2}^2},\ \eta_{\Re_1}.\eta_{\Re_2},\ \nu_{\Re_1}.\nu_{\Re_2}\right\};$

(2) $\Re_1 \otimes \Re_2 = \left\{\mu_{\Re_1}.\mu_{\Re_2}, \sqrt{\eta_{\Re_1}^2 + \eta_{\Re_2}^2 - \eta_{\Re_1}^2.\eta_{\Re_2}^2},, \sqrt{\nu_{\Re_1}^2 + \nu_{\Re_2}^2 - \nu_{\Re_1}^2.\nu_{\Re_2}^2}\right\};$

(3) $\lambda \otimes \Re_1 = \left\{\sqrt{1 - (1 - \mu_{\Re_1}^2)^{\lambda}},\ (\eta_{\Re_1})^{\lambda},\ (\nu_{\Re_1})^{\lambda}\right\};$

(4) $\Re_1^{\lambda} = \left\{(\mu_{\Re_1})^{\lambda}, \sqrt{1 - (1 - \eta_{\Re_1}^2)^{\lambda}}, \sqrt{1 - (1 - \nu_{\Re_1}^2)^{\lambda}}\right\};$

(5) $\Re_1^{\nu} = \langle \nu_{\Re_1}, \eta_{\Re_1}, \mu_{\Re_1}\rangle;$

2.2. **Spherical 2-tuple linguistic fuzzy set (S2TLFS).** In this subsection, we defined the idea of the S2TLFS and some operation based on the SFS and 2-tuple linguistic information.

Definition 2.7. A S2TLFS $\Re$ in $\mathbb{R} \neq \phi$, is defined as;

$$\Re = \left\{\left\langle(s_{\theta(r)}, \rho), \mu_{\Re}(r), \eta_{\Re}(r), \nu_{\Re}(r) |\ r \in \mathbb{R}\right\rangle\right\}. \quad (8)$$

where $s_{\theta(r)} \in \acute{S}, \rho \in [-0.5, 0.5)$, $\mu_{\Re}(r), \eta_{\Re}(r), \nu_{\Re}(r) : \mathbb{R} \rightarrow [0,1]$ with the condition $0 \leq \mu_{\Re}^2(r) + \eta_{\Re}^2(r) + \nu_{\Re}^2(r) \leq 1, \forall\ r \in \mathbb{R}$. And the numbers $\mu_{\Re}(r)$, $\eta_{\Re}(r)$ and $\nu_{\Re}(r)$ represent the positive grade, neutral grade and negative grade of the number $r$ to linguistic variable $(\mathrm{s}_{\theta(r)}, \rho_i)$. This term $\chi_{\Re}(r)$ is known as refusal degree of $r$ to $(s_i, \rho_i)$, and defined as

$$\chi_{\Re}(r) = \sqrt{1 - (\mu_{\Re}^2(r) + \eta_{\Re}^2(r) + \nu_{\Re}^2(r))} \quad (9)$$

For convenience, we said $\widetilde{\alpha} = \{((s_{\theta(r)}, \rho_i), \mu(\alpha), \eta(\alpha), \nu(\alpha))\}$, a S2TLFN, where $\mu(\alpha), \eta(\alpha), \nu(\alpha) \in [0,1]$, $0 \leq \mu_{\Re}^2 + \eta_{\Re}^2 + \nu_{\Re}^2 \leq 1$, $s_{\theta(\alpha)} \in \acute{S}$ and $\rho \in [-0.5, 0.5)$.

Definition 2.8. Let $\widetilde{\Re} = \{(s_{\theta(\Re)}, \rho), \mu_{\Re}, \eta_{\Re}, \nu_{\Re}\}$ be a S2TLFN. Then, the score index of S2TLFN are defined as;

$$S\nu^*(\widetilde{\Re}) = \Lambda\left\{\Lambda^{-1}(s_{\theta(\Re)}, \rho)\frac{2 + (\mu_{\Re})^2 - (\eta_{\Re})^2 - (\nu_{\Re})^2}{3}\right\}, \Re^{-1}(S\nu^*(\widetilde{\Re})) \in [1, t] \quad (10)$$

Definition 2.9. Let $\widetilde{\Re} = \{(s_{\theta(\Re)}, \rho), \mu_{\Re}, \eta_{\Re}, \nu_{\Re}\}$, a S2TLFN. Then, the accuracy index of S2TLFN are defined as;

$$H\nu^*(\widetilde{\Re}) = \Lambda\left\{\Lambda^{-1}(s_{\theta(\Re)}, \rho).\frac{(\mu_{\Re})^2 + (\eta_{\Re})^2 + (\nu_{\Re})^2}{3}\right\}, \Lambda^{-1}(H\nu^*(\widetilde{\Re})) \in [1, t] \quad (11)$$

Definition 2.10. $\widetilde{\Re}_1 = \{(s_{\theta(\Re_1)}, \rho), \mu_{\Re_1}, \eta_{\Re_1}, \nu_{\Re_1}\}$ and $\widetilde{\Re}_2 = \{(s_{\theta(\Re_2)}, \rho), \mu_{\Re_2}, \eta_{\Re_2}, \nu_{\Re_2}\}$ are the two S2TLFNs. Then, if

(1) $S\nu^*(\widetilde{\Re}_1) < S\nu^*(\widetilde{\Re}_2)$, then $\widetilde{\Re}_1 < \widetilde{\Re}_2$, if

(2) $S\nu^*(\widetilde{\Re}_1) = S\nu^*(\widetilde{\Re}_2)$, then

(3) If $H\nu^*(\widetilde{\Re}_1) < H\nu^*(\widetilde{\Re}_2)$, then $\widetilde{\Re}_1 < \widetilde{\Re}_2$, if

(4) $H\nu^*(\widetilde{\Re}_1) = H\nu^*(\widetilde{\Re}_2)$, then $\widetilde{\Re}_1$ and $\widetilde{\Re}_2$ have the same information.

Definition 2.11. Let $\widetilde{\Re}_1 = \{(s_{\theta(\Re_1)}, \rho_1), \mu_{\Re_1}, \eta_{\Re_1}, \nu_{\Re_1}\}$ and $\widetilde{\Re}_2 = \{(s_{\theta(\Re_2)}, \rho_2), \mu_{\Re_2}, \eta_{\Re_2}, \nu_{\Re_2}\}$ be the S2TLFNs. Then,

$$\begin{aligned}
\widetilde{\Re}_1 \oplus \widetilde{\Re}_2 &= \left\{\begin{array}{c} \Lambda\left(\Lambda^{-1}(s_{\theta(\Re_1)}, \rho_1) + \Lambda^{-1}(s_{\theta(\Re_1)}, \rho_2)\right), \\ \sqrt{\mu_{\Re_1}^2 + \mu_{\Re_2}^2 - \mu_{\Re_1}^2.\mu_{\Re_2}^2},\ \eta_{\Re_1}.\eta_{\Re_2},\ \nu_{\Re_1}.\nu_{\Re_2} \end{array}\right\}; \\
\widetilde{\Re}_1 \otimes \widetilde{\Re}_2 &= \left\{\begin{array}{c} \Lambda\left(\Lambda^{-1}(s_{\theta(\Re_1)}, \rho_1) + \Lambda^{-1}(s_{\theta(\Re_1)}, \rho_2)\right), \mu_{\Re_1}.\mu_{\Re_2}, \\ \sqrt{\eta_{\Re_1}^2 + \eta_{\Re_2}^2 - \eta_{\Re_1}^2.\eta_{\Re_2}^2},\ \sqrt{\nu_{\Re_1}^2 + \nu_{\Re_2}^2 - \nu_{\Re_1}^2.\nu_{\Re_2}^2} \end{array}\right\}; \\
\lambda\widetilde{\Re}_1 &= \left\{\Lambda\left(\lambda\Lambda^{-1}(s_{\theta(\Re_1)}, \rho_1)\right), \sqrt{1 - (1 - \mu_{\Re_1}^2)^{\lambda}},\ (\eta_{\Re_1})^{\lambda},\ (\nu_{\Re_1})^{\lambda}\right\}; \\
\left(\widetilde{\Re}_1\right)^{\lambda} &= \left\{\Lambda\left(\Lambda^{-1}(s_{\theta(\Re_1)}, \rho_1)\right)^{\lambda}, (\mu_{\Re_1})^{\lambda}, \sqrt{1 - (1 - \eta_{\Re_1}^2)^{\lambda}},\ \sqrt{1 - (1 - \nu_{\Re_1}^2)^{\lambda}}\right\}.
\end{aligned}$$

## 3. Spherical 2-Tuple Linguistic Fuzzy Arithmetic Aggregation Operators

In this portion, we developed some average aggregation operators, based on S2TLFNs.

Definition 3.1. Let $\widetilde{\Re}_j = \{(r_j, \chi_j), \mu_j, \eta_j, \nu_j)\}(j = 1, ..., n)$ be the family of S2TLF numbers. Then, the S2TLF weighted average (S2TLFWA) operator is a mapping of $\Omega^n \to \Omega$, and

$$S2TLFW\mu_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) = \bigoplus_{j=1}^{n}\left(\Theta_j\widetilde{\Re}_j\right), \quad (12)$$

the weight of $\widetilde{\Re}_j$ is $\Theta = (\Theta_1, ..., \Theta_n)^T$, such that $\Theta_j > 0$, $\sum_{j=1}^{n}\Theta_j = 1$.

Theorem 3.1. The aggregated value obtained by utilizing the S2TLFWA operator is also a S2TLF fuzzy numbers, where

$$\begin{aligned} & S2TLFW\mu_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) = \bigoplus_{j=1}^{n}\left(\Theta_j\widetilde{\Re}_j\right) \\ = & \left\{\Lambda\left(\sum_{j=1}^{n}\Theta_j\Lambda^{-1}(r_j,\chi_j)\right), \left(\sqrt{1-\prod_{j=1}^{n}\left(1-\mu_j^2\right)^{\Theta_j}}, \prod_{j=1}^{n}(\eta_j)^{\Theta_j}, \prod_{j=1}^{n}(\nu_j)^{\Theta_j}\right)\right\} \end{aligned} \tag{13}$$

and $\Theta = (\Theta_1, ..., \Theta_n)^T$ is the weighting of $\widetilde{\Re}_j$, as $\Theta_j > 0$, $\sum_{j=1}^{n}\Theta_j = 1$.

Proof. To prove Equ. (13), we used the mathematical induction principle,

(1). When $n = 2$, we obtain

$$\begin{aligned} \Theta_1\widetilde{\Re}_1 &= \left\{\Lambda\left(\Theta_1\Lambda^{-1}(r_1,\alpha_1)\right), \left(\sqrt{1-(1-\mu_1^2)^{\Theta_1}}, (\eta_1)^{\Theta_1}, (\nu_1)^{\Theta_1}\right)\right\}. \\ \Theta_2\widetilde{\Re}_2 &= \left\{\Lambda\left(\Theta_2\Lambda^{-1}(r_2,\alpha_2)\right), \left(\sqrt{1-(1-\mu_2^2)^{\Theta_2}}, (\eta_2)^{\Theta_2}, (\nu_2)^{\Theta_2}\right)\right\}. \end{aligned}$$

Then,

$$\begin{aligned} & S2TLFW\mu(\widetilde{\Re}_1, \widetilde{\Re}_2) = (\Theta_1\widetilde{\Re}_1 \oplus \Theta_2\widetilde{\Re}_2) \\ = & \left\{\begin{array}{c} \Lambda\left(\Theta_1\Lambda^{-1}(r_1,\alpha_1) + \Theta_2\Lambda^{-1}(r_2,\alpha_2)\right), \\ \sqrt{2-(1-\mu_1^2)^{\Theta_1} - (1-\mu_2^2)^{\Theta_2} - (1-(1-\mu_1^2)^{\Theta_1})(1-(1-\mu_2^2)^{\Theta_2})}, \\ (\eta_1)^{\Theta_1}(\eta_2)^{\Theta_2}, (\nu_1)^{\Theta_1}(\nu_2)^{\Theta_2} \end{array}\right\} \\ = & \left\{\begin{array}{c} \Lambda\left(\Theta_1\Lambda^{-1}(r_1,\chi_1) + \Theta_2\Lambda^{-1}(r_2,\alpha_2)\right), \\ \sqrt{1-(1-\mu_1^2)^{\Theta_1}(1-\mu_2^2)^{\Theta_2}}, (\eta_1)^{\Theta_1}(\eta_2)^{\Theta_2}, (\nu_1)^{\Theta_1}(\nu_2)^{\Theta_2}) \end{array}\right\} \end{aligned}$$

(2). Assume that Equ. (13), true for $n = \kappa$, that is;

$$\begin{aligned} & S2TLFW\mu_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_\kappa) = \bigoplus_{j=1}^{\kappa}\left(\Theta_j\widetilde{\Re}_j\right) \\ = & \left\{\Lambda\left(\sum_{j=1}^{\kappa}\Theta_j\Lambda^{-1}(r_j,\chi_j)\right), \left(\sqrt{1-\prod_{j=1}^{\kappa}\left(1-\mu_j^2\right)^{\Theta_j}}, \prod_{j=1}^{\kappa}(\eta_j)^{\Theta_j}, \prod_{j=1}^{\kappa}(\nu_j)^{\Theta_j}\right)\right\} \end{aligned}$$

and prove Equ. (13), for $n=\kappa+1$, then

$$S2TLFW\mu_{\Theta}(\widetilde{\Re}_1,...,\widetilde{\Re}_{\kappa+1}) = \Theta_1\widetilde{\Re}_1 \oplus \Theta_2\widetilde{\Re}_2 \oplus ... \oplus \Theta_{\kappa}\widetilde{\Re}_{\kappa} \oplus \Theta_{\kappa+1}\widetilde{\Re}_{\kappa+1}$$

$$= \left\{ \begin{array}{c} \Lambda\left(\sum_{j=1}^{\kappa}\Theta_j\Lambda^{-1}(r_j,\chi_j) + \Theta_{\kappa+1}\Lambda^{-1}(r_{\kappa+1},\chi_{\kappa+1})\right), \\ \sqrt{\left(\begin{array}{c} 1-\prod_{j=1}^{\kappa}\left(1-\mu_j^2\right)^{\Theta_j} + (1-(1-\mu_{\kappa+1}^2)^{\Theta_{\kappa+1}}) \\ -\left(1-\prod_{j=1}^{\kappa}\left(1-\mu_j^2\right)^{\Theta_j}\right)(1-(1-\mu_{\kappa+1}^2)^{\Theta_{\kappa+1}}) \end{array}\right)}, \prod_{j=1}^{\kappa}(\eta_j)^{\Theta_j}, \prod_{j=1}^{\kappa}(\nu_j)^{\Theta_j} \end{array} \right\}$$

$$= \left\{\Lambda\left(\sum_{j=1}^{\kappa+1}\Theta_j\Lambda^{-1}(r_j,\chi_j)\right), \left(\sqrt{1-\prod_{j=1}^{\kappa+1}\left(1-\mu_j^2\right)^{\Theta_j}}, \prod_{j=1}^{\kappa+1}(\eta_j)^{\Theta_j}, \prod_{j=1}^{\kappa+1}(\nu_j)^{\Theta_j}\right)\right\}$$

which represent that the aggregated value is also S2TLFN. Hence, Equ. (13), is true for all $n$.

The below properties are satisfied by S2TLFWA operator.

**Property 1. (Idempotency)**. If $\widetilde{\Re}_j = \widetilde{\Re}$ for all $j$, then

$$S2TLFW\mu_{\Theta}(\widetilde{\Re}_1,...,\widetilde{\Re}_n) = \widetilde{\Re}. \qquad (14)$$

**Property 2.(Boundedness)**. Let $\widetilde{\Re}_j(j=1,...,n)$ be the collection of S2TLFNs, and $\widetilde{\Re}^+ = \max_j \widetilde{\Re}_j, \widetilde{\Re}^- = \min_j \widetilde{\Re}_j$, then

$$\widetilde{\Re}^- \leq S2TLFW\mu_{\Theta}(\widetilde{\Re}_1,...,\widetilde{\Re}_n) \leq \widetilde{\Re}^+. \qquad (15)$$

**Property 3. (Monotonicity)**. Let $\widetilde{\Re}_j$ and $\widetilde{\Re}_j^{/}(j=1,...,n)$ be the collection of S2TLFNs, if $\widetilde{\Re}_j \leq \widetilde{\Re}_j^{/}, \forall j$, then

$$S2TLFW\mu_{\Theta}(\widetilde{\Re}_1,...,\widetilde{\Re}_n) \leq S2TLFW\mu_{\Theta}(\widetilde{\Re}_1^{/},...,\widetilde{\Re}_n^{/}). \qquad (16)$$

Definition 3.2. Let $\widetilde{\Re}_j = \{(r_j,\chi_j),\mu_j,\eta_j,\nu_j\}(j=1,...,n)$ be the collection of S2TLF numbers. Then, S2TLF ordered weighted average (S2TLFOWA) operator with the dimension $n$ is a function $S2TLFOW\mu : \Omega^n \to \Omega$, where the associated weights are $\Theta = (\Theta_1,...,\Theta_n)^T$, and $\Theta_j > 0$, $\sum_{j=1}^{n}\Theta_j = 1$. Then,

$$S2TLFOW\mu_{\Theta}(\widetilde{\Re}_1,...,\widetilde{\Re}_n) = \bigoplus_{j=1}^{n}\left(\Theta_j\widetilde{\Re}_{\sigma(j)}\right)$$

$$= \left\{\Lambda\left(\sum_{j=1}^{n}\Theta_j\Lambda^{-1}(r_{\sigma(j)},\chi_{\sigma(j)})\right), \left(\begin{array}{c} \sqrt{1-\prod_{j=1}^{n}\left(1-\mu_{\sigma(j)}^2\right)^{\Theta_j}}, \\ \prod_{j=1}^{n}\left(\eta_{\sigma(j)}\right)^{\Theta_j}, \prod_{j=1}^{n}\left(\nu_{\sigma(j)}\right)^{\Theta_j} \end{array}\right)\right\} \qquad (17)$$

and $\sigma(1),...,\sigma(n)$ are the permutation of $(1,...,n)$, and $\widetilde{\Re}_{\sigma(j-1)} \geq \widetilde{\Re}_{\sigma(j)} \forall j = 2,...,n$.

**Property 1. (Idempotency)**. If $\widetilde{\Re}_j = \widetilde{\Re}\,\forall j$, then

$$S2TLFOW\mu_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) = \widetilde{\Re}. \tag{18}$$

**Property 2. (Boundedness)**. Let $\widetilde{\Re}_j(j = 1, ..., n)$ are the collection of S2TLFNs, and $\widetilde{\Re}^{+} = \max_j \widetilde{\Re}_j, \widetilde{\Re}^{-} = \min_j \widetilde{\Re}_j$. Then,

$$\widetilde{\Re}^{-} \leq S2TLFOW\mu_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) \leq \widetilde{\Re}^{+}. \tag{19}$$

**Property 3. (Monotonicity)**. Let $\widetilde{\Re}_j(j = 1, ..., n)$ and $\widetilde{\Re}^{/}_j(j = 1, ..., n)$ be the collection of S2TLFNs, if $\widetilde{\Re}_j \leq \widetilde{\Re}^{/}_j, \forall j$. Then,

$$S2TLFOW\mu_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) \leq S2TLFOW\mu_{\Theta}(\widetilde{\Re}^{/}_1, ..., \widetilde{\Re}^{/}_n). \tag{20}$$

Definition 3.3. Let $\widetilde{\Re}_j = \{(r_j, \chi_j), \mu_j, \eta_j, \nu_j\}(j = 1, ..., n)$ are the set of S2TLFNs. Then, the S2TLF hybrid average (S2TLFHA) operator with the dimension $n$ is a function $S2TLFH\mu : \Omega^n \rightarrow \Omega$, that as

$$S2TLFH\mu_{\Theta,\omega}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) = \bigoplus_{j=1}^{n}\left(\Theta_j \overset{*}{\widetilde{\Re}}_{\sigma(j)}\right)$$

$$= \left\{ \Lambda\left(\sum_{j=1}^{n}\Theta_j\Lambda^{-1}(\overset{*}{r}_{\sigma(j)}, \overset{*}{\chi}_{\sigma(j)})\right), \left( \begin{array}{c} \sqrt{1-\prod_{j=1}^{n}\left(1-\overset{*}{\mu}{}^{2}_{\sigma(j)}\right)^{\Theta_j}}, \\ \prod_{j=1}^{n}\left(\overset{*}{\eta}_{\sigma(j)}\right)^{\Theta_j}, \prod_{j=1}^{n}\left(\overset{*}{\nu}_{\sigma(j)}\right)^{\Theta_j} \end{array} \right) \right\} \tag{21}$$

where $\Theta = (\Theta_1, ..., \Theta_n)^T$ are the associated weighting vector, as $\Theta_j > 0, \sum_{j=1}^{n}\Theta_j = 1$, and $\overset{*}{\widetilde{\Re}}_{\sigma(j)}$ is the $j^{th}$ biggest number of the S2TLF arguments $\overset{*}{\widetilde{\Re}}_{\sigma(j)}\left(\overset{*}{\widetilde{\Re}}_{\sigma(j)} = n\omega_j\widetilde{\Re}_j, j = 1, ..., n\right)$, $(\omega = \omega_1, ..., \omega_n)$ is the weight vector of S2TLF arguments $\widetilde{\Re}_j$, with $\omega_j > 0, \sum_{j=1}^{n}\omega_j = 1$, and $n$ show the balancing coefficient.

3.1. **Spherical 2-Tuple Linguistic Fuzzy Geometric Aggregation Operators.** In this subsection, we used the information of S2TLF numbers, and developed some geometric AOs.

Definition 3.4. Let $\widetilde{\Re}_j = \{(r_j, \chi_j), \mu_j, \eta_j, \nu_j\}(j = 1, ..., n)$ be the collection of S2TLF numbers. The S2TLF weighted geometric (S2TLFWG) operator is a function $\Omega^n \rightarrow \Omega$, and

$$S2TLFWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) = \bigotimes_{j=1}^{n}\left(\widetilde{\Re}_j\right)^{\Theta_j}, \tag{22}$$

and $\Theta = (\Theta_1, ..., \Theta_n)^T$ is the weighting of $\widetilde{\Re}_j$, where $\Theta_j > 0, \sum_{j=1}^{n}\Theta_j = 1$.

Theorem 3.2. The aggregated value obtained by utilizing the S2TLFWG operator is also a S2TLF number, such that

$$S2TLFWG_{\Theta}(\widetilde{\Re}_1,...,\widetilde{\Re}_n)=\bigotimes_{j=1}^{n}\left(\widetilde{\Re}_j\right)^{\Theta_j}$$
$$=\left\{\Lambda\left(\prod_{j=1}^{n}\left(\Lambda^{-1}(r_j,\chi_j)^{\Theta_j}\right)\right),\left(\begin{array}{c}\prod_{j=1}^{n}(\mu_j)^{\Theta_j},\sqrt{1-\prod_{j=1}^{n}\left(1-\eta_j^2\right)^{\Theta_j}},\\ \sqrt{1-\prod_{j=1}^{n}\left(1-\nu_j^2\right)^{\Theta_j}}\end{array}\right)\right\}, \quad (23)$$

and $\Theta=(\Theta_1,...,\Theta_n)^T$ is the weighting vector of $\widetilde{\Re}_j$, where $\Theta_j>0$, $\sum_{j=1}^{n}\Theta_j=1$.

Proof. Using the mathematical induction principal to prove Equ. (23).

(1). When $n=2$, we have

$$\left(\widetilde{\Re}_1\right)^{\Theta_1}=\left\{\Lambda\left(\Lambda^{-1}(r_1,\chi_1)\right)^{\Theta_1},\mu_1^{\Theta_1},\sqrt{1-(1-\eta_1^2)^{\Theta_1}},\sqrt{1-(1-\nu_1^2)^{\Theta_1}}\right\}.$$
$$\left(\widetilde{\Re}_2\right)^{\Theta_2}=\left\{\Lambda\left(\Lambda^{-1}(r_2,\chi_2)\right)^{\Theta_2},\mu_2^{\Theta_2},\sqrt{1-(1-\eta_2^2)^{\Theta_2}},\sqrt{1-(1-\nu_2^2)^{\Theta_2}}\right\}.$$

Then

$$S2TLFWG(\widetilde{\Re}_1,\widetilde{\Re}_2)=(\widetilde{\Re}_1)^{\Theta_1}\otimes(\widetilde{\Re}_2)^{\Theta_2}$$
$$=\left\{\begin{array}{c}\Lambda\left(\left(\Lambda^{-1}(r_1,\chi_1)\right)^{\Theta_1}+\left(\Lambda^{-1}(r_2,\chi_2)\right)^{\Theta_2}\right),{\mu_1}^{\Theta_1}{\mu_2}^{\Theta_2},\\ \sqrt{2-(1-\eta_1^2)^{\Theta_1}-(1-\eta_2^2)^{\Theta_2}-(1-(1-\eta_1^2)^{\Theta_1})(1-(1-\eta_2^2)^{\Theta_2})},\\ \sqrt{2-(1-\nu_1^2)^{\Theta_1}-(1-\nu_2^2)^{\Theta_2}-(1-(1-\nu_1^2)^{\Theta_1})(1-(1-\nu_2^2)^{\Theta_2})}\end{array}\right\}$$
$$=\left\{\begin{array}{c}\Lambda\left(\Lambda^{-1}(r_1,\chi_1)\right)^{\Theta_1}+\left(\Lambda^{-1}(r_2,\chi_2)\right)^{\Theta_2}),\\ {\mu_1}^{\Theta_1}{\mu_2}^{\Theta_2},\sqrt{1-(1-\eta_1^2)^{\Theta_1}(1-\eta_2^2)^{\Theta_2}},\sqrt{1-(1-\nu_1^2)^{\Theta_1}(1-\nu_2^2)^{\Theta_2}}\end{array}\right\}$$

(2). Assume that Equ. (23), hold for $n=\kappa$, i.e.,

$$S2TLFWG_{\Theta}(\widetilde{\Re}_1,...,\widetilde{\Re}_\kappa)=\bigotimes_{\kappa=1}^{\kappa}\left(\widetilde{\Re}_j\right)^{\Theta_j}$$
$$=\left\{\Lambda\left(\prod_{j=1}^{\kappa}\left(\Lambda^{-1}(r_j,\chi_j)\right)^{\Theta_j}\right),\left(\begin{array}{c}\prod_{j=1}^{\kappa}(\mu_j)^{\Theta_j},\sqrt{1-\prod_{j=1}^{\kappa}\left(1-\eta_j^2\right)^{\Theta_j}},\\ \sqrt{1-\prod_{j=1}^{\kappa}\left(1-\nu_j^2\right)^{\Theta_j}}\end{array}\right)\right\}$$

Now for $n = \kappa + 1$, Equ. (23), become

$$S2TLFWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_{\kappa+1}) = \left(\widetilde{\Re}_1\right)^{\Theta_1} \otimes \left(\widetilde{\Re}_2\right)^{\Theta_2} \otimes ... \otimes \left(\widetilde{\Re}_{\kappa}\right)^{\Theta_{\kappa}} \otimes \left(\widetilde{\Re}_{\kappa+1}\right)^{\Theta_{\kappa+1}}$$

$$= \left\{ \begin{array}{c} \Lambda\left(\prod_{j=1}^{\kappa}\left(\Lambda^{-1}(r_j,\chi_j)\right)^{\Theta_j} . \left(\Lambda^{-1}(r_{\kappa+1},\chi_{\kappa+1})\right)^{\Theta_{\kappa+1}}\right), \\ \left( \begin{array}{c} \prod_{j=1}^{\kappa+1}(\mu_j)^{\Theta_j}, \sqrt{\begin{array}{c} 1-\prod_{j=1}^{\kappa}(1-\eta_j^2)^{\Theta_j} + (1-(1-\eta_{\kappa+1}^2)^{\Theta_{\kappa+1}}) \\ -\left(1-\prod_{j=1}^{\kappa}\left(1-\eta_j^2\right)^{\Theta_j}\right)(1-(1-\eta_{\kappa+1}^2)^{\Theta_{\kappa+1}}) \end{array}}, \\ \sqrt{\begin{array}{c} 1-\prod_{j=1}^{\kappa}(1-\nu_j^2)^{\Theta_j} + (1-(1-\nu_{\kappa+1}^2)^{\Theta_{\kappa+1}}) \\ -\left(1-\prod_{j=1}^{\kappa}\left(1-\nu_j^2\right)^{\Theta_j}\right)(1-(1-\nu_{\kappa+1}^2)^{\Theta_{\kappa+1}}) \end{array}} \end{array} \right) \end{array} \right\}$$

$$= \left\{ \Lambda\left(\sum_{j=1}^{\kappa+1}\left(\Lambda^{-1}(r_j,\chi_j)\right)^{\Theta_j}\right), \left( \begin{array}{c} \prod_{j=1}^{\kappa+1}(\mu_j)^{\Theta_j}, \sqrt{1-\prod_{j=1}^{\kappa+1}\left(1-\eta_j^2\right)^{\Theta_j}}, \\ \sqrt{1-\prod_{j=1}^{\kappa+1}\left(1-\nu_j^2\right)^{\Theta_j}} \end{array} \right) \right\}$$

which denote the aggregated value is also a S2TLFN. Hence, Equ. (23), holds for $n$.
**Property 1. (Idempotency)**. If $\widetilde{\Re}_j = \widetilde{\Re}\ \forall j$, then

$$S2TLFWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) = \widetilde{\Re}. \tag{24}$$

**Property 2. (Boundedness)**. Let $\widetilde{\Re}_j (j = 1, ..., n)$ be the collection of S2TLFNs, and $\widetilde{\Re}^+ = \max_j \widetilde{\Re}_j, \widetilde{\Re}^- = \min_j \widetilde{\Re}_j$. Then,

$$\widetilde{\Re}^- \leq S2TLFWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) \leq \widetilde{\Re}^+. \tag{25}$$

**Property 3. (Monotonicity)**. Let $\widetilde{\Re}_j (j = 1, ..., n)$ and $\widetilde{\Re}_j^{/} (j = 1, ..., n)$ be the collection of S2TLFNs, if $\widetilde{\Re}_j \leq \widetilde{\Re}_j^{/}, \forall j$. Then,

$$S2TLFWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) \leq S2TLFWG_{\Theta}(\widetilde{\Re}_1^{/}, ..., \widetilde{\Re}_n^{/}). \tag{26}$$

Definition 3.5. Let $\widetilde{\Re}_j = \{(r_j, \chi_j), \mu_j, \eta_j, \nu_j\} (j = 1, ..., n)$ be the collection of S2TLF numbers. Then, the S2TLF ordered weighted geometric (S2TLFOWG) operator with the dimension $n$ is a function $S2TLFOWG : \Omega^n \to \Omega$, such that $\Theta = (\Theta_1, ..., \Theta_n)^T$ be the associated weighting vector, and $\Theta_j > 0$, $\sum_{j=1}^{n} \Theta_j = 1$. Then,

$$S2TLFWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_\kappa) = \bigotimes_{j=1}^{n} \left(\widetilde{\Re}_{\sigma(j)}\right)^{\Theta_j}$$

$$= \left\{ \begin{array}{c} \Lambda\left(\sum_{j=1}^{n} \left(\Lambda^{-1}(r_{\sigma(j)}, \chi_{\sigma(j)})\right)^{\Theta_j}\right), \\ \left( \begin{array}{c} \prod_{j=1}^{n} \left(\mu_{\sigma(j)}\right)^{\Theta_j}, \sqrt{1 - \prod_{j=1}^{n} \left(1 - \eta_{\sigma(j)}^2\right)^{\Theta_j}}, \\ \sqrt{1 - \prod_{j=1}^{n} \left(1 - \nu_{\sigma(j)}^2\right)^{\Theta_j}} \end{array} \right) \end{array} \right\}, \tag{27}$$

and $\sigma(1), ..., \sigma(n)$ are the permutation of $(1, ..., n)$, and $\widetilde{\Re}_{\sigma(j-1)} \geq \widetilde{\Re}_{\sigma(j)}$ $\forall j = 2, ..., n$.

**Property 1. (Idempotency)**. If $\widetilde{\Re}_j = \widetilde{\Re}$ for all $j$, then

$$S2TLFOWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) = \widetilde{\Re}. \tag{28}$$

**Property 2. (Boundedness)**. Let $\widetilde{\Re}_j (j = 1, ..., n)$ be the collection of S2TLFNs, and $\widetilde{\Re}^+ = \max_j \widetilde{\Re}_j, \widetilde{\Re}^- = \min_j \widetilde{\Re}_j$. Then,

$$\widetilde{\Re}^- \leq S2TLFOWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) \leq \widetilde{\Re}^+. \tag{29}$$

**Property 3. (Monotonicity)**. Let $\widetilde{\Re}_j (j = 1, ..., n)$ and $\widetilde{\Re}_j^{/} (j = 1, ..., n)$ be the collection of S2TFLNs, if $\widetilde{\Re}_j \leq \widetilde{\Re}_j^{/}, \forall j$. Then,

$$S2TLFOWG_{\Theta}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) \leq S2TLFOWG_{\Theta}(\widetilde{\Re}_1^{/}, ..., \widetilde{\Re}_n^{/}). \tag{30}$$

Definition 3.6. Let $\widetilde{\Re}_j = \{(r_j, \chi_j), \mu_j, \eta_j, \nu_j\} (j = 1, ..., n)$ be the collection of S2TLF numbers. Then, the S2TLF hybrid geometric (S2TLFHG) operator with the dimension $n$ is a function $S2TLFHG : \Omega^n \rightarrow \Omega$, where

$$S2TLFHG_{\Theta,\omega}(\widetilde{\Re}_1, ..., \widetilde{\Re}_n) = \bigotimes_{j=1}^{n} \left(\overset{*}{\widetilde{\Re}}_{\sigma(j)}\right)^{\Theta_j}$$

$$= \left\{ \begin{array}{c} \Lambda\left(\sum_{j=1}^{n} (\Lambda^{-1}(\overset{*}{r}_{\sigma(j)}, \overset{*}{\chi}_{\sigma(j)}))^{\Theta_j}\right), \\ \left( \begin{array}{c} \prod_{j=1}^{n} \left(\overset{*}{\mu}_{\sigma(j)}\right)^{\Theta_j}, \sqrt{1 - \prod_{j=1}^{n} \left(1 - \overset{*}{\eta}{}^2_{\sigma(j)}\right)^{\Theta_j}}, \\ \sqrt{1 - \prod_{j=1}^{n} \left(1 - \overset{*}{\nu}{}^2_{\sigma(j)}\right)^{\Theta_j}} \end{array} \right) \end{array} \right\}, \tag{31}$$

where $\Theta = (\Theta_1, ..., \Theta_n)^T$ be the associated weighting, and $\Theta_j > 0$, $\sum_{j=1}^{n} \Theta_j = 1$, and $\overset{*}{\widetilde{\Re}}_{\sigma(j)}$ is the $j^{th}$ biggest number of S2TLF arguments $\overset{*}{\widetilde{\Re}}_{\sigma(j)}$ $\left(\overset{*}{\widetilde{\Re}}_{\sigma(j)} = n\omega_j \widetilde{\Re}_j, (j = 1, ..., n)\right)$, $(\omega = \omega_1, ..., \omega_n)$ is the weights of S2TLF arguments $\widetilde{\Re}_j$, with $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$, and $n$ show the balancing coefficient.

## 4. Approach of Spherical 2-tuple linguistic fuzzy information for MCDM Problem

Using the developed two operators (S2TLFWA or S2TLFWG) in this portion, we developed an algorithm for MCDM problem, with the S2TLFNs information. Let the discrete set of alternatives are $\Upsilon = (\Upsilon_1, ..., \Upsilon_m)$, and the attributes set are $\mathbb{N} = (\mathbb{N}_1, ..., \mathbb{N}_n)$, where $\Theta = (\Theta_1, ..., \Theta_n)^T$ is the weights of the criteria set $\mathbb{N}_j$, and $\Theta_j \in [0, 1]$, $\sum_{j=1}^{n} \Theta_j = 1$. Let $\mathbb{Z} = (\widetilde{r}_{ij})_{m \times n} = \langle (r_{ij}, \chi_{ij}), (\mu_{ij}, \eta_{ij}, \nu_{ij}) \rangle_{m \times n}$ are the S2TLF decision matrix, where $\widetilde{r}_{ij}$, take the form of the S2TLFNs, and $\mu_{ij}, \eta_{ij}, \nu_{ij}$ show the positive, neutral and negative grades correspondingly, that the alternative $\Upsilon_i$ satisfies the attribute $\mathbb{N}_j$ given by the decision maker. Where $\mu_{ij}, \eta_{ij}, \nu_{ij} \in [0, 1]$, $\mu_{ij}^2, \eta_{ij}^2, \nu_{ij}^2 \leq 1, \pi_{ij} = \sqrt{1 - \left(\mu_{ij}^2 + \eta_{ij}^2 + \nu_{ij}^2\right)}$, $s_{ij} \in \acute{S}, \rho_{ij} \in [-0.5, 0.5), i = 1, ..., m; j = 1, ..., n$. Now, we used the S2TLF information and apply the S2TLFWA or S2TLFWG operator for the MCDM problem.

**Step 1.** Calculate the overall values $\widetilde{\Re}_i (i = 1, ..., m)$ of the alternative $\Upsilon_j$, utilized the information of the given matrix $\mathbb{Z}$, and the S2TLFWA or S2TLFWG operator.

$$\widetilde{\Re}_i = S2TLFW\mu_\Theta(r_{i1}, ..., r_{in}) = \bigoplus_{j=1}^{n} (\Theta_j r_{ij})$$

$$= \left\{ \Lambda \left( \sum_{j=1}^{n} \Theta_j \Lambda^{-1}(r_j, \chi_j) \right), \left( \sqrt{1 - \prod_{j=1}^{n} \left(1 - \mu_j^2\right)^{\Theta_j}}, \prod_{j=1}^{n} (\eta_j)^{\Theta_j}, \prod_{j=1}^{n} (\nu_j)^{\Theta_j} \right) \right\}. \tag{32}$$

Or

$$\widetilde{\Re}_i = S2TLFWG_\Theta(r_{i1}, ..., r_{in}) = \bigotimes_{j=1}^{n} (r_{ij})^{\Theta_j}$$

$$= \left\{ \Lambda \left( \sum_{j=1}^{n} \Lambda^{-1}(r_j, \chi_j)^{\Theta_j} \right), \left( \prod_{j=1}^{n} \mu_{ij}^{\Theta_j}, \sqrt{1 - \prod_{j=1}^{n} \left(1 - \eta_j^2\right)^{\Theta_j}}, \sqrt{1 - \prod_{j=1}^{n} \left(1 - \nu_j^2\right)^{\Theta_j}} \right) \right\}. \tag{33}$$

**Step 2.** Determine the scores $S\nu(\widetilde{\Re}_i)(i = 1, ..., m)$ of the overall S2TLFNs $\widetilde{\Re}_i$.

**Step 3.** By the scores $S\nu(\widetilde{\Re}_i)(i = 1, ..., m)$, choose the one.

## 5. Practical Example of Green Supplier Selection

In this portion, we are using a practical MCDM green supplier selection problem for the chemical processing industry. Suppose we have a chemical processing industry which changes the chemical structure of natural materials so that products of value can be derived in other industries or in every day life. Chemicals, primarily minerals, metals and hydrocarbons, are produced from these raw materials using step by step process. More treatment, like as mixing and blending, is often needed to convert them into final products (such as adhesives, paints, medicines and cosmetics). The selection of raw materials or natural materials by suppliers is known to be very important to the chemical processing industry. For the supplier selection (Alternatives), we have eight attributes (1) Cost; (2) Quality; (3) Delivery; (4) Service; (5) Technique capability; (6) Green product; (7) Pollution control; and (8) Environmental management. We collect data from the procurement department which is responsible for the entire procurement process in a given chemical processing industry. To collect the data, we invite the five suppliers (Alternatives). The five suppliers (Alternatives) are evaluated by utilizing the Spherical 2-tuple linguistic fuzzy numbers with the weight vector are $\Theta = (0.11, 0.13, 0.13, 0.10, 0.16, 0.10, 0.15, 0.12)^T$. Applying the developed algorithm for the best supplier selection. The decision lays down that all the above steps must be followed.

Table 1. Spherical 2-tuple linguistic fuzzy decision matrix

| | $\mathbb{N}_1$ | $\mathbb{N}_2$ | $\mathbb{N}_3$ |
|---|---|---|---|
| $\Upsilon_1$ | $\langle(s_5,0),(0.3,0.8,0.5)\rangle$ | $\langle(s_4,0),(0.3,0.1,0.9)\rangle$ | $\langle(s_4,0),(0.4,0.8,0.4)\rangle$ |
| $\Upsilon_2$ | $\langle(s_2,0),(0.4,0.2,0.4)\rangle$ | $\langle(s_6,0),(0.5,0.3,0.7)\rangle$ | $\langle(s_3,0),(0.2,0.3,0.7)\rangle$ |
| $\Upsilon_3$ | $\langle(s_8,0),(0.6,0.1,0.6)\rangle$ | $\langle(s_1,0),(0.7,0.6,0.2)\rangle$ | $\langle(s_5,0),(0.6,0.3,0.6)\rangle$ |
| $\Upsilon_4$ | $\langle(s_3,0),(0.8,0.4,0.3)\rangle$ | $\langle(s_6,0),(0.2,0.6,0.7)\rangle$ | $\langle(s_1,0),(0.9,0.1,0.3)\rangle$ |
| $\Upsilon_5$ | $\langle(s_5,0),(0.3,0.9,0.1)\rangle$ | $\langle(s_3,0),(0.5,0.4,0.2)\rangle$ | $\langle(s_7,0),(0.8,0.5,0.2)\rangle$ |

| | $\mathbb{N}_4$ | $\mathbb{N}_5$ | $\mathbb{N}_6$ |
|---|---|---|---|
| $\Upsilon_1$ | $\langle(s_2,0),(0.5,0.7,0.1)\rangle$ | $\langle(s_4,0),(0.6,0.3,0.4)\rangle$ | $\langle(s_7,0),(0.8,0.3,0.6)\rangle$ |
| $\Upsilon_2$ | $\langle(s_1,0),(0.3,0.8,0.6)\rangle$ | $\langle(s_3,0),(0.2,0.9,0.3)\rangle$ | $\langle(s_2,0),(0.1,0.7,0.4)\rangle$ |
| $\Upsilon_3$ | $\langle(s_4,0),(0.6,0.3,0.4)\rangle$ | $\langle(s_8,0),(0.8,0.5,0.2)\rangle$ | $\langle(s_3,0),(0.3,0.6,0.6)\rangle$ |
| $\Upsilon_4$ | $\langle(s_5,0),(0.4,0.6,0.2)\rangle$ | $\langle(s_7,0),(0.2,0.5,0.8)\rangle$ | $\langle(s_1,0),(0.9,0.1,0.3)\rangle$ |
| $\Upsilon_5$ | $\langle(s_7,0),(0.8,0.5,0.2)\rangle$ | $\langle(s_4,0),(0.4,0.3,0.2)\rangle$ | $\langle(s_3,0),(0.3,0.9,0.1)\rangle$ |

| | $\mathbb{N}_7$ | $\mathbb{N}_8$ |
|---|---|---|
| $\Upsilon_1$ | $\langle(s_3,0),(0.1,0.5,0.7)\rangle$ | $\langle(s_2,0),(0.4,0.3,0.6)\rangle$ |
| $\Upsilon_2$ | $\langle(s_4,0),(0.5,0.3,0.6)\rangle$ | $\langle(s_3,0),(0.2,0.9,0.3)\rangle$ |
| $\Upsilon_3$ | $\langle(s_4,0),(0.6,0.3,0.4)\rangle$ | $\langle(s_6,0),(0.3,0.6,0.4)\rangle$ |
| $\Upsilon_4$ | $\langle(s_4,0),(0.4,0.5,0.2)\rangle$ | $\langle(s_8,0),(0.7,0.1,0.3)\rangle$ |
| $\Upsilon_5$ | $\langle(s_7,0),(0.8,0.5,0.2)\rangle$ | $\langle(s_5,0),(0.4,0.6,0.2)\rangle$ |

To select the best supplier, utilized the following steps;

Table 2. The aggregated values of the alternatives (Suppliers) using the S2TLFWA (S2TLWG) Operators

| | $S2TLW\mu$ | $S2TLWG$ |
|---|---|---|
| $\Upsilon_1$ | $\langle(s_4, -0.18), (0.48649, 0.30377, 0.47151)\rangle$ | $\langle(s_3, 0.43), (0.35459, 0.54419, 0.62644)\rangle$ |
| $\Upsilon_2$ | $\langle(s_3, 0.11), (0.34522, 0.46853, 0.47237)\rangle$ | $\langle(s_3, -0.33), (0.27106, 0.70156, 0.54320)\rangle$ |
| $\Upsilon_3$ | $\langle(s_5, -0.04), (0.59794, 0.3677, 0.37551)\rangle$ | $\langle(s_4, 0.12), (0.45187, 0.46121, 0.45160)\rangle$ |
| $\Upsilon_4$ | $\langle(s_5, -0.08), (0.61007, 0.36083, 0.35407)\rangle$ | $\langle(s_4, -0.43), (0.36527, 0.56201, 0.51241)\rangle$ |
| $\Upsilon_5$ | $\langle(s_5, 0.14), (0.62447, 0.51758, 0.17290)\rangle$ | $\langle(s_4, -0.08), (0.50444, 0.65014, 0.18378)\rangle$ |

Table 3. Alternatives (Suppliers) score values

| | $S2TLFW\mu$ operator | $S2TLFWG$ operator |
|---|---|---|
| $\Upsilon_1$ | $(s_3, 0.32)$ | $(s_1, 0.23)$ |
| $\Upsilon_2$ | $(s_2, -0.16)$ | $(s_1, 0.31)$ |
| $\Upsilon_3$ | $(s_3, 0.43)$ | $(s_2, 0.45)$ |
| $\Upsilon_4$ | $(s_3, 0.41)$ | $(s_2, 0.37)$ |
| $\Upsilon_5$ | $(s_3, 0.38)$ | $(s_2, 0.34)$ |

Table 4. Ordering of the Alternatives (Suppliers)

| Operator | Ordering |
|---|---|
| S2TLFWA | $\Upsilon_3 > \Upsilon_4 > \Upsilon_5 > \Upsilon_1 > \Upsilon_2$ |
| S2TLFWG | $\Upsilon_3 > \Upsilon_4 > \Upsilon_5 > \Upsilon_2 > \Upsilon_1$ |

**Comparative analysis**

To determine the effectiveness of the introduced technique under the S2TLNs, we studied an example and evaluated the selection of the best alternative based on defined method. Table 4 displays the order of ordering of the alternatives obtained by using the established approach. In Table 5, we show that the ordering of alternatives between the three approaches is reasonably matched. Which also shows that the current method is validated. The best alternative is $\Upsilon_3$ using the suggested approach, which is the same as the best alternative obtained by sample Induced 2-tuple linguistic generalized operator [28] and SLFNs [29] approach. From the analysis, we note that the ranking of the alternatives obtained by our proposed approach is stable and accurate compared to the SLF approach. The method proposed is absolutely outstanding because it can completely escape any loss of information that has previously occurred in the linguistic information.

Table 5. Alternatives ordering

| Method | Ordering |
|---|---|
| 2-TILGOWA operator [28] | $\Upsilon_3 > \Upsilon_2 > \Upsilon_4 > \Upsilon_5 > \Upsilon_1$ |
| SLFSs approach [29] | $\Upsilon_3 > \Upsilon_5 > \Upsilon_2 > \Upsilon_4 > \Upsilon_1$ |
| S2TLFWA operator | $\Upsilon_3 > \Upsilon_4 > \Upsilon_5 > \Upsilon_1 > \Upsilon_2$ |
| S2TLFWG operator | $\Upsilon_3 > \Upsilon_4 > \Upsilon_5 > \Upsilon_2 > \Upsilon_1$ |

## 6. CONCLUSION

In this article, we analyze the multiple attribute DM problem under the Spherical 2-tuple linguistic fuzzy set. We first introduced such S2TLF operational laws. Then, using these operational laws, we proposed the some aggregation operators like as; S2TLFWA, S2TLFWG, S2TLFOWA, S2TLFOWG, S2TLFHA and S2TLFHG operators. We're analyzing many properties of the proposed AOs. The prominent characteristic of these proposed operators is studied. We used the developed operators and write an approach to solve the problem of MCDM. Finally, an example of green supplier selection in the chemical processing industry is given to demonstrate the defined method to find its practicability and efficacy. We also compare our proposed method with other existing methods.

In the future, we investigate the use of S2TLFNs in many other areas of study, such as; Novel similarity measure based on the transformed right-angled triangles between intuitionistic fuzzy sets and its applications; A new possibility degree measure for interval-valued q-rung orthopair fuzzy sets in decision-making; T-spherical fuzzy power aggregation operators and their applications in multi-attribute decision making; Three-way decisions making using covering based Fractional orthotriple fuzzy Rough set model.

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