Punjab University Journal of Mathematics (2023),55(5-6),222-240
https://doi.org/10.52280/pujm.2023.55(5-6)04

Instability of Gravitating Object Under Expansion-free Condition in Rastall Theory

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Received: 31 March, 2023 / Accepted: 09 June, 2023 / Published online: 15 June,2023

Abstract.: In this paper, a perturbation scheme is applied to discuss the instability model of a gravitating source under expansion free condition upto Newtonian (N) and Post-Newtonian (pN) approximations in Rastall theory. For this purpose, we established field equations which are set of partial differential equations with the help of suitable metric and fluid. The linear perturbation scheme is used on these partial differential equations to formulate a collapse equation. We derived dynamical equations by applying N and pN approximations. These equations represent that instability of gravitating object is independent of adiabatic index Γ, while the instability of the gravitating object can be determined with the help of anisotropic pressure, energy density, Rastall parameter λ, and some constraints at N and pN approximations.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Instability, Perturbation, Newtonian Approximation, Rastall Theory.

1. INTRODUCTION

The problem of dynamical instability of gravitating source is an interesting study in the field of General Relativity (GR). Gravitational collapse is happen when a massive star does not maintain its equilibrium between outward forces and inward gravitational pull. During the gravitational collapse a source passes through many stages of evolution. The instability of spherically symmetric object with perfect fluid and non-adiabatic gravitating source has been studied in [11, 20]. Herrera and Santos [21, 22] explored the results of spherically symmetric gravitational collapsing object containing anisotropic fluid. The consequences of limiting case of quasi-static approximation and dissipation of radiation transportation has been discussed [35]-[38]. Sharif and Azam [35] worked to study the instability of gravitating source at N and pN order. The authors in [23, 28] discussed the dynamics of gravitating source under expansion-free condition . During the gravitational collapse a high energetic explosion occur [29, 15], the expansion free condition is a reasonable condition to describe this kind of explosion [18, 19]. The dust model with expansion-free condition have some disadvantages and there is a negative energy density distribution, therefor, the Skripkin model is not compatible with Darmois [12] junction conditions. During the explosion at center of the object a vacuum cavity is generated, in this situation the spherical symmetric distribution of fluid remains constant [23]. A model [33] has been established in which energy density remains constant and also this model useful for non-dissipative fluid under expansion-free condition.

In the resent decades, some modified gravity theories have been studied to explore the more mysterious features of the cosmos. These modifications have been made in Einstein's General theory of Relativity. The modified gravity theories describe the accelerated expansion of universe. In this regard Rastall [32] proposed an interesting modification to General Relativity (GR). Rastall challenged the law of conservation of energy momentum tensor (EMT) ($\nabla_\alpha T^{\alpha\beta} = 0$) in a curved space-time. He claimed that this law of conservation of EMT does not always hold in curved space-time. According to Rastall, the covariant divergence of energy momentum tensor is directly proportional to derivative of Ricci scalar i.e $\nabla_\alpha T^{\alpha\beta} \propto R^\beta$. Rastall introduced a coupling parameter in his theory which is known as Rastall parameter. If this parameter is equal to zero, Rastall theory of gravity provides the results back to GR.

New astrophysical models in Rastall theory have been investigated to explore the evolution of stars. Recently, Abbas et al. [5] explored the models of collapse and expansion of non-static anisotropic gravitating object in Rastall theory. The effects of Rastall parameter in the evolution of expansion and collapse models has been discussed. Tahir et al. [39] studied the collapse of non-static configuration of dissipative source in Rastall theory. They used M$\ddot{u}$ller-Israel-Stewart approach to formulate a heat transport equation and discus the consequences of heat flux on the collapsing source. Dorrani [13] studied the collapse of spherical symmetric gravitating source with homogeneous distibution in the field of Rastall theory. He found that, the collapse becomes to stop when a scale factor attains its minimum value, after this stage a bounce is occurred. The collapse of gravitating source has been studied in Rastall gravity by taking a Vaidya space-time [40]. A dynamical evolution of collapsing source has been studied by assuming a linear equation of state for the matter, a naked singularity is the result of this collapse. Capozziello et al. [9, 10] explored the collapse of gravitating object using dust fluid and they obtained certain instability limits for a collapsing source by using perturbation scheme.

Levia and Steinhof [26] discussed the spinning gravitating objects in the effective field theory in the post-Newtonian scheme in the context of the binary inspiral problem. They obtained the equations of motion of the spin via a proper variation of the action, and also derived the Hamiltonians. By applying the effective field theory for spin to derive all spin dependent potentials up to next-to-leading order to quadratic level in spin, namely up to the third post-Newtonian order for rapidly rotating compact objects. Boschung et al. [8] studied the instability of the gravitating regular sphaleron solutions of the SU(2) Einstein-Yang-Mills-Higgs system. The frequency spectrum of a class of radial perturbations have been studied. It has been derived by using the a variational principle that there exist always unstable modes. $\ddot{O}$vg$\ddot{u}$n et al. [31] studied the shadow and energy emission rate of a spherically symmetric non-commutative black hole in Rastall theory. The non-commutative and non-vanishing parameter affects the formation of event horizon, also the visibility of obtained shadow effected by the non-commutative parameter in Rastall theory. This obtained shadow my obey the unstable circular orbit condition, which is crucial for the physical validity of the black hole image model. Gurtug and his collaborators [16] explored the Singular and Nonsingular Colliding Wave Solutions in Einstein-Maxwell-Scalar theory. Black hole solutions have been studied in the field of dRGT massive gravity coupled with non linear electrodynamics [25].

Abbas and his collaborator [4] studied the dynamical instability of adiabatic fluid in Einstein Gauss-Bonnet (EGB) theory. Jhingan and Ghosh [24] explored the exact solutions for the collapse by assuming dust fluid in EGB theory. Sunil et. al [27] found the solutions of spherically symmetric collapse of a source in EGB theory. Abbas and Tahir [3, 2] discussed the dynamical solutions for perfect fluid dissipative collapse of a source in EGB gravity. The dynamical solution of a collapsing object under expansion-free condition has been studied in $f(R,T)$ theory [30]. Bamba et al. [7] discussed the solution of gravitating source in $f(R)$ theory and they found that this collapse leads to formation of a curvature singularity. Arbuzova and Dolgov [6] studied the solutions for various viable $f(R)$ paradigms for the accelerated expansion of the universe with time dependent mass density.

The higher order curvature terms are added to generalize the Einstein-Hilbert action which come from the diffeomorphism property of the action. These higher order curvature terms contribute a lot in the formation of geometry and evolution of collapsing source. Therefore, the EGB, $f(R)$, $f(R,T)$, Rastall or Lanczos-Lovelock gravity theories have interesting features in the higher order curvature gravity theories. An interior space-time curvature of a source (star) increases during the continuous collapse of source and turns into very large at the last state of collapse of source. Recently, Rastall theory of gravity attained the much interest of the research community to study its impact on the gravitating source. In this regard, some studies inculcate the compatibility of the results obtained in the Rastall theory with the observational constraints[14, 34, 1]. This is the reason for us that we worked in the modified theory of gravity like Rastall theory of gravity to study the instability of the gravitating object under expansion-free condition [23]. For this purpose, we assumed the spherically symmetric configuration of a source. A perturbation scheme is to be considered for the metric as well as fluid material variables to establish the dynamical equations for collapsing source up to N and pN order. This paper is managed as: Section 2 deals with the field equations and matter distribution. We present perturb configuration of the field equations in section 3. Section 4 describes the N and PN approximations. In section 5, the dynamical equations are formulated. Section 6 contains conclusion of the paper. The last section contains an appendix.

2. Field Equations and Interior Matter Distribution

According to the Rastall theory, the law of conservation of EMT has the following form

$$\nabla_a T^a_b = \lambda R_{,b} \tag{2. 1}$$

where λ is Rastall coupling parameter. The field equations in Rastall theory of gravity are given in the following form

$$G_{ab} + \kappa\lambda g_{ab} R = \kappa T_{ab}, \tag{2. 2}$$

where κ is gravitational coupling constant, G_{ab} is Einstein tensor, g_{ab} is a metric tensor, R is Ricci scalar and T_{ab} is an energy momentum tensor. We used the following metric to formulate the geometry of the spherically symmetric object,

$$ds^2 = -A^2 dt^2 + B^2 dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2. 3}$$

where $A = A(r,t)$, $B = B(r,t)$ and $R = R(r,t)$.
A non-static anisotropic fluid is given in the following equation

$$T_{ab} = (\mu + p_\perp)V_a V_b + (p_r - p_\perp)\chi_a\chi_b + p_\perp g_{ab}, \tag{2. 4}$$

where μ denotes density, p_r and $p_\perp$ are radial and tangential pressures, χ^a is unit normal four vector and V^a is four velocity which satisfy

$$V^a = A^{-1}\delta^a_0,\ \chi^a = B^{-1}\delta^a_1, \tag{2. 5}$$

and

$$\chi^a\chi_a = 1,\ V^a V_a = -1\ and\ \chi^a V_a = 0. \tag{2. 6}$$

Four acceleration a_α and expansion scalar Θ are defined as

$$\Theta = V^\alpha_{;\alpha},\ a_\alpha = V_{\alpha;\beta}V^\beta. \tag{2. 7}$$

Shear tensor σ_{ab} of the fluid is given below

$$\sigma_{ab} = V_{(a;b)} + a_{(a}V_{b)} - \frac{1}{3}\Theta\big(g_{ab} + V_a V_b\big). \tag{2. 8}$$

Eqs.(2. 3), (2. 5) and (2. 7) provide us the following non-zero component of four-acceleration,

$$a_1 = \frac{A'}{A},\ a^\alpha a_\alpha = \left(\frac{A'}{AB}\right)^2, \tag{2. 9}$$

and expansion scalar takes the following form

$$\Theta = \frac{1}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{R}}{R}\right), \tag{2. 10}$$

where $'$ and $.$ represent the partial derivative with respect to r and t, respectively. From Eqs.(2. 8) and (2. 5), we get the following non zero components of shear tensor,

$$\sigma_{11} = \frac{2}{3}B^2\sigma,\ \sigma_{22} = \sigma_{33}sin^{-2}\theta = -\frac{1}{3}R^2\sigma, \tag{2. 11}$$

and shear scalar is given below

$$\sigma^{ab}\sigma_{ab} = \frac{2}{3}\sigma^2, \tag{2. 12}$$

where

$$\sigma = \frac{1}{A}\bigg(\frac{\dot{B}}{B} - \frac{\dot{R}}{R}\bigg). \tag{2. 13}$$

We obtained the following field equations with the help of Eqs.(2. 2)-(2. 4),

$$\kappa\mu A^2 = -\frac{1}{B^3R^2}[A^2R'(BR' - 2B'R) - B^3(A^2 + \dot{R}^2) - 2BR(B\dot{B}\dot{R} - A^2R'')]$$
$$-\kappa\frac{2\lambda}{AB^3R^2}[A^3(B(2RR'' + R'^2 - B^2) - 2B'RR') + A^2R(B(A''R + A'R) - A'B'R)$$
$$-ABR(2B^2\ddot{R} + R\ddot{B}) - AB^2\dot{R}(\dot{R}B + 2\dot{B}R) + \dot{A}B^2R(2B\dot{R} + R\dot{B})], \tag{2. 14}$$

$$\kappa p_r B^2 = \frac{1}{A^3R^2}[A^2(AB^2 - (2A'R + AR')R') - B^2((\dot{A}\dot{R} - A\ddot{R})2R - A\dot{R}^2)]$$
$$+\frac{2\kappa\lambda}{A^3BR^2}[A^3(B(2RR'' + R'^2 - B^2) - 2B'RR') + A^2R(B(A''R + A'R) - A'B'R)$$
$$-ABR(2B^2\ddot{R} + \ddot{B}R) - AB^2\dot{R}(B\dot{R} + 2\dot{B}R) + \dot{A}B^2R(2B\dot{R} + \dot{B}R)], \tag{2. 15}$$

$$\kappa p_\perp R^2 = \frac{R}{(AB)^3}[R((\ddot{B}A - \dot{A}\dot{B})B^2$$
$$-A^2(A''B + A'B')) + AB((\dot{B}\dot{R} + B\ddot{R})B - A(A'R' + AR'')) + A^3B'R' - B^3\dot{A}\dot{R}]$$
$$+\frac{2\kappa\lambda}{A^3B^3}[A^3(B(2RR'' + R'^2 - B^2) - 2B'RR') + A^2R(B(A''R + A'R) - RA'B')$$
$$-ABR(2B^2\ddot{R} + R\ddot{B}) - AB^2\dot{R}(\dot{R}B + 2\dot{B}R) + \dot{A}B^2R(2B\dot{R} + C\dot{B})], \tag{2. 16}$$

$$0 = \frac{2}{ABR}(A'B\dot{R} - AB\dot{R}' + A\dot{B}R'), \tag{2. 17}$$

From Eqs.(2. 10), (2. 13), and (2. 17), we obtained

$$\frac{1}{3}\bigg(\Theta - \sigma\bigg)' - \sigma\frac{R'}{R} = 0. \tag{2. 18}$$

The Misner-Sharp mass function is given by

$$m(r,t) = \frac{R}{2}(\frac{\dot{R}^2}{A^2} - \frac{R'^2}{B^2} + 1). \tag{2. 19}$$

We have following non trivial components of the Bianchi identities, $T^b_{a;b} = 0$,

$$T^{ab}_{;b}V_a = -\frac{1}{A}(\dot{\mu} + 2(\mu + p_\perp)\frac{\dot{R}}{R} + (\mu + p_r)\frac{\dot{B}}{B}) = 0. \tag{2. 20}$$

and

$$T^{ab}_{;b}\chi_a=\frac{1}{B}\big(p_r'+\frac{A'}{A}\big(\mu+p_r\big)+2\big(p_r-p_\perp\big)\frac{R'}{R}\big)=0. \tag{2. 21}$$

From Eqs.(2. 10) and (2. 20), we get

$$\dot{\mu}+2\big(p_\perp-p_r\big)\frac{\dot{R}}{R}+\big(\mu+p_r\big)A\Theta=0, \tag{2. 22}$$

3. Perturbed Equations

Here, the perturbation scheme is used on the field equations and the dynamical equations which is the small change in the physical system. In this work, we want to analyze the stability or instability of the gravitating source after slightly disturbing the source by applying the perturbation. For this purpose, initially, we assume that geometry and fluid components depend only on the radial dependent coordinates, this implies that the system is in static equilibrium. With the passage of time all the matter quantities depend upon the radial and time. We have concern with the non-linear partial differential equations, in order to analyze roll of radial dependent variables on the dynamical instability of the source, we adopt the particular mathematical method to solve the equations. In this context, we considered the following linear perturbation on the metric and matter functions [20, 35]

$$A(r,t)=A_0(r)+\epsilon a(r)T(t), \tag{3. 23}$$

$$B(r,t)=B_0(r)+\epsilon b(r)T(t), \tag{3. 24}$$

$$R(r,t)=R_0(r)+\epsilon c(r)T(t), \tag{3. 25}$$

$$\mu(r,t)=\mu_0(r)+\epsilon\bar{\mu}(r,t), \tag{3. 26}$$

$$p_r(r,t)=p_{r0}(r)+\epsilon\bar{p_r}(r,t), \tag{3. 27}$$

$$p_\perp(r,t)=p_{\perp0}(r)+\epsilon\bar{p_\perp}(r,t), \tag{3. 28}$$

$$m(r,t)=m_0(r)+\epsilon\bar{m}(r,t), \tag{3. 29}$$

$$\Theta(r,t)=\epsilon\bar{\Theta}(r,t), \tag{3. 30}$$

$$\sigma(r,t)=\epsilon\bar{\sigma}(r,t), \tag{3. 31}$$

where $(0<\epsilon\ll 1)$. For the static configuration, we used Eqs.(3. 23 - 3. 28) and Eqs.(2. 14 - 2. 17) with $R_0=r$ (Schwarzschild coordinates), and then obtained the following equations

$$\kappa\mu_0=\frac{1}{B_0^2r^2}\left(\frac{2rB_0'}{B_0}+B_0^2-1\right)-2\lambda\kappa\left[\frac{1}{r^2B_0^2}-\frac{1}{r^2}+\frac{A_0''}{A_0B_0}-\frac{A_0'B_0}{A_0B_0^3}\right], \tag{3. 32}$$

$$\kappa P_{r0}=\frac{1}{B_0^2r^2}\left(\frac{2rA_0'}{A_0}-B_0^2+1\right)+2\lambda\kappa\left[\frac{1}{r^2}-\frac{B_0^2}{r^2}+\frac{A_0''B_0}{A_0}-\frac{A_0'B_0'}{A_0B_0}\right], \tag{3. 33}$$

$$\kappa p_{\perp 0}=\frac{1}{B_0^2}\left(\frac{A_0''}{A_0}-\frac{A_0'B_0'}{A_0B_0}+\frac{1}{r}\left(\frac{A_0'}{A_0}-\frac{B_0'}{B_0}\right)\right)$$
$$+\lambda\kappa\left[\frac{1}{B_0^2}-1+\frac{r^2A_0''}{A_0B_0}-\frac{A_0'B_0'r^2}{A_0B_0^3}\right]. \qquad (3.\ 34)$$

The perturbed form up to the first order in ϵ are given by

$$\kappa\bar{\mu}=\frac{-2}{B_0^2}\left[\left(\frac{c}{r}\right)''-\frac{1}{r}\left(\frac{b}{B_0}\right)'-\left(\frac{B_0'}{B_0}-\frac{3}{r}\right)\left(\frac{c}{r}\right)'-\left(\frac{B_0'}{r}\right)^2\left(\frac{b}{B_0}-\frac{c}{r}\right)\right]T$$
$$-2\lambda\kappa\left[\left(\frac{2c''}{B_0^2r}-\frac{2}{B_0^2r^3}+\frac{c'}{A_0B_o^2r^2}-\frac{b}{B_o^3r^2}+\frac{a}{A_0B_o^2r^2}+\frac{2c}{r^3}-\frac{2a}{A_0r^2}\right.\right.$$
$$-\frac{2B_0'c''}{B_0^3r}+\frac{A_0'c''}{A_0B_0r}-\frac{A_0''b}{A_0B_0^2}+\frac{A_0''a}{A_0^2B_0}+\frac{a''}{A_0^2B_0}+\frac{2A_0'B_0'b}{A_0B_0^4}-\frac{B_0'a'}{A_0B_0^3}+\frac{A_0'B_0'b}{A_0B_0^4}$$
$$\left.\left.-\frac{A_0'B_0'a}{A_0^2B_0^3}-\frac{A_0'b'}{A_0B_0^3}\right)T-\frac{\ddot{T}c}{A_0^2r}-\frac{\ddot{T}b}{A_0^2B_0}\right]-2\kappa\mu_0\frac{a}{A_0}T, \qquad (3.\ 35)$$

$$\frac{2\dot{T}}{A_0B_0}\left[\left(\frac{c}{r}\right)'-\frac{b}{rB_0}-\left(\frac{A_0'}{A_0}-\frac{1}{r}\right)\frac{c}{r}\right]=0, \qquad (3.\ 36)$$

$$\kappa\bar{p}_r=-\frac{2c}{A_0^2r}\ddot{T}+\left[\left(\frac{a}{A_0}\right)'+\left(1+\frac{rA_0'}{A_0}\right)\left(\frac{c}{r}\right)'+\frac{B_0^2}{r}\left(\frac{c}{r}-\frac{b}{B_0}\right)\right]\frac{2T}{rB_0^2}$$
$$+2\lambda\kappa\left[\left(\frac{2c''}{B_0^2r}-\frac{2c}{B_0^2r^3}+\frac{2c'}{B_0^2r^2}+\frac{2c}{r^3}-\frac{2b}{B_0r^2}-\frac{2B_0'c''}{B_0^3r}+\frac{c''}{A_0B_0r}+\frac{a'}{A_0B_0}+\frac{A_0''b}{A_0B_0^2}\right.\right.$$
$$\left.\left.-\frac{A_0''a}{A_0^2B_0}+\frac{A_0'B_0'b}{A_0B_0^4}-\frac{A_0'b'}{A_0B_0^3}+\frac{A_0'B_0'a}{A_0^2B_0^3}-\frac{B_0'a'}{A_0B_0^3}\right)T-\left(\frac{c}{A_0^2r}+\frac{b}{A_0^2B_0^2}\right)\ddot{T}-2\kappa p_{r0}\frac{b}{B_0}T\right. \qquad (3.\ 37)$$

$$\kappa\bar{p}_\perp=-\left[\frac{c}{r}+\frac{b}{B_0}\right]\frac{\ddot{T}}{A^2}+\left[\left(\frac{a}{A_0}\right)''+\left(\frac{c}{r}\right)''+\left(\frac{\frac{1}{r^2}+2A_0'}{A_0}-\frac{B_0'}{B_0}\right)\left(\frac{a}{A_0}\right)'\right.$$
$$\left.+\left(\frac{2}{r}+\frac{A_0'}{A_0}-\frac{B_0'}{B_0}\right)\left(\frac{c}{r}\right)'-\left(\frac{1}{r}+\frac{A_0'}{A_0}\right)\left(\frac{b}{B_0}\right)'\right]\frac{T}{A_0^2}+2\lambda\kappa\left[\left(\frac{2c''}{B_0^2r}-\frac{2b}{B_0^3r^2}+\frac{2c'}{B_0^2r^2}\right.\right.$$
$$-\frac{2B_0'c''}{B_0^3r}+\frac{A_0'c''}{A_0B_0r^2}+\frac{a''}{A_0B_0}+\frac{2A''c}{A_0B_0r}-\frac{A''b}{A_0B_0^2}-\frac{A_0''a}{A_0^2B_0}+\frac{A_0'B_0'b}{A_0B_0^4}-\frac{A_0'b'}{A_0B_0^3}-\frac{A_0'b}{A_0B_0^3}$$
$$\left.\left.+\frac{A_0'B_0'a}{A_0^2B_0^3}-\frac{B_0'a'}{A_0B_0^3}+\frac{2A_0'B_0'b}{A_0B_0^4}-\frac{2A_0'B_0'c}{A_0B_0^3r^2}\right)T-\left(\frac{c}{A_0^2r}+\frac{b}{A_0^2}\right)\ddot{T}-2\kappa p_{\perp 0}\frac{c}{r}T.\right. \qquad (3.\ 38)$$

From Eqs.(2. 10), (3. 30) and (3. 31), we have

$$\bar{\Theta}=\frac{\dot{T}}{A_0}\left(\frac{b}{B_0}+2\frac{c}{r}\right), \qquad (3.\ 39)$$

Eq.(2. 13) with Eqs.(3. 30) and (3. 31), gives us

$$\bar{\sigma}=\frac{\dot{T}}{A_0}\left(\frac{b}{B_0}-\frac{c}{r}\right). \qquad (3.\ 40)$$

From Eqs.(2. 20), (2. 21), and (3. 23 - 3. 28), we get the following static configuration

$$p'_{r0} + \frac{A'_0}{A_0}\left(\mu_0 + p_{r0}\right) + \frac{2}{r}\left(p_{r0} - p_{\perp}\right) = 0, \tag{3. 41}$$

and the perturbed quantities are given below

$$\frac{1}{A_0}\left[\dot{\bar{\mu}} + 2\left(\mu_0 + p_{\perp 0}\right)\frac{c}{r}\dot{T} + \left(\mu_0 + p_{r0}\right)\frac{b}{B_0}\dot{T}\right] = 0, \tag{3. 42}$$

and

$$\frac{1}{B_0}\left[\bar{p}'_r + \left(\bar{\mu} + \bar{p_r}\right)\frac{A'_0}{A_0} + \left(\mu_0 + p_{r0}\right)\left(\frac{a}{A_0}\right)'T + 2\left(p_{r0} - p_{\perp 0}\right)\left(\frac{c}{r}\right)'T \right.$$
$$\left. + \frac{2}{r}\left(\bar{p_r} - \bar{p_\perp}\right)\right] = 0. \tag{3. 43}$$

Integrating Eq.(3. 42), we get

$$\bar{\mu} = -\left[\left(p_{r0} + \mu_0\right)\frac{b}{B_0} + 2\left(p_{\perp 0} + \mu_0\right)\frac{c}{r}\right]T. \tag{3. 44}$$

From Eqs.(2. 19), (3. 23 - 3. 25), and (3. 29), we get

$$m_0 = \frac{1}{2}\left[r - \frac{r}{B_0}\right], \tag{3. 45}$$

and

$$\bar{m} = \left[\frac{r}{B_0^2}\left(\frac{b}{B_0} - c'\right) + \frac{c}{2}\left(1 - \frac{1}{B_0^2}\right)\right]T. \tag{3. 46}$$

Eq.(3. 27) with condition $p_r =^{\Sigma^{(e)}} 0$ takes the following form

$$p_{r0} =^{\Sigma^{(e)}} 0\ ,\ \bar{p_r} =^{\Sigma^{(e)}} 0. \tag{3. 47}$$

We assumed that the value of c is not equal to zero, so from Eqs.(3. 37) and (3. 47), we have

$$\ddot{T} - \beta T =^{\Sigma^{(e)}} 0, \tag{3. 48}$$

where

$$\beta = \frac{A_0^2 r}{(-2B_0^2 c - 2\kappa\lambda B_0^2 c - \kappa\lambda b r)}\left[\frac{A_0^2}{B_0^2 c}\left(\left(\frac{c}{r}\right)'\left(\frac{rA'_0}{A_0} + 1\right) + \left(\frac{a}{A_0}\right)' - \frac{B_0^2}{r}\left(\frac{b}{B_0} - \frac{c}{r}\right)\right)\right.$$
$$+2\kappa\lambda\left(\frac{2c''}{r} - \frac{2c}{r^3} + \frac{4c'}{r^2} + \frac{B_0^2 c}{r^3} - \frac{2B_0 b}{r^2} - \frac{2B'_0 c''}{B_0 r} + \frac{B_0 c''}{A_0 r} + \frac{B_0 a''}{A_0} + \frac{A''_0 b}{A_0} - \frac{A''_0 B_0 a}{A_0^2}\right.$$
$$\left.\left. + \frac{A'_0 B'_0 b}{A_0 B_0^2} - \frac{A'_0 b'}{A_0 B_0} + \frac{A'_0 B'_0 a}{A_0^2 B_0} - \frac{B'_0 a'}{A_0 B_0}\right)\right]. \tag{3. 49}$$

Equation (3. 48) provides solution containing the oscillating and non-oscillating functions. The oscillating functions corresponds to the stable state of gravitating source while the non-oscillating functions corresponds to unstable state of gravitating source. In this work, we would like to discuss the instability of collapsing source, therefore, we have concerned with the non-oscillating functions. The solution of Eq.(3. 48) is given bellow

$$T(t) = -\exp(\sqrt{\beta_{\Sigma^{(e)}}}t). \tag{3. 50}$$

Equation (3. 50) provides us $T(-\infty) = 0$ when $t = -\infty$, a static system starts to collapsing and areal radius of this system is start to decreasing as t increases. We consider the law of thermodynamics which relates $\bar{p}$ and $\bar{\mu}$. Here, we take the following equation of state [17],

$$\bar{p_r} = \Gamma\frac{p_{r0}}{\mu_0 + p_0}\bar{\mu}, \tag{3. 51}$$

where Γ represents the adiabatic index, which used to measure the variation of pressure with respect to the density variation, this implies that it measures the stiffness of the fluid, and it remains constant throughout the fluid distribution.

4. Newtonian and post Newtonian terms

We can distinguish between the various types of terms such as N and pN approximation, which are useful to established the dynamical equations for unstable model of gravitating source. We considered the following inequalities for N approximation [23],

$$\mu_0 \gg p_{r0}\ ,\ \mu_0 \gg p_{\perp 0}. \tag{4. 52}$$

The metric components up to PN approximation are given below

$$A_0 = 1 - \frac{Gm_0}{C^2 r}\ ,\ B_0 = 1 + \frac{Gm_0}{C^2 r}, \tag{4. 53}$$

where G represents gravitational constant. The difference between N and pN approximations can be determined with the help of the order of C, which are given bellow

$$N\ order : terms\ of\ order\ C^0, \tag{4. 54}$$

$$pN\ order : terms\ of\ order\ C^{-2}, \tag{4. 55}$$

5. Dynamical equation

Here, we have formulated the dynamical equation with the help of equations given in previous sections under expansion free condition. On the basis of this dynamical equation will able to determined the stability or instability conditions. Eq.(3. 39) with $\Theta = 0$ provides us

$$\frac{b}{B_0} = -\frac{2c}{r}. \tag{5. 56}$$

From Eq.(3. 36) and (5. 56), we have

$$\frac{2\dot{T}}{r^3 B_0}\left(\frac{r^2 c}{A_0}\right)' = 0. \tag{5. 57}$$

Eq.(5. 57) provides us

$$c = \eta\frac{A_0}{r^3}, \tag{5. 58}$$

where η is constant. From Eqs.(5. 56) and (3. 44), we get

$$\bar{\mu} = 2\big[p_{r0} - p_{\perp 0}\big]T\frac{c}{r}. \tag{5. 59}$$

Above equation represents that the energy density in perturbed form depends on the static anisotropic pressure. Eqs.(3. 51) and (5. 59) provide us the following equation

$$\bar{p_r} = 2\Gamma\frac{p_{r0}}{\mu_0 + p_{r0}}\big[p_{r0} - p_{\perp 0}\big]T\frac{c}{r}. \tag{5. 60}$$

Eq.(3. 41) give us

$$\frac{A_0'}{A_0} = -\frac{1}{\mu_0 + p_{r0}}\Bigg(p_{r0}' + \frac{2}{r}\Bigg(p_{r0} - p_{\perp 0}\Bigg)\Bigg). \tag{5. 61}$$

From Eqs.(3. 32) and (3. 45), we get

$$\frac{B_0'}{B_0} = \frac{\kappa\mu_0 r^3 - 2m_0}{2r^2 - 4rm_0}, \tag{5. 62}$$

Here, we would like to discuss the instability of self-gravitating source with $\Theta = 0$ up to pN approximation. Due to lengthy calculations of dynamical equations for instability, we put them in an **appendix A**. We used $\mu_0 = \delta r^n$ (where $n \in(-\infty, \infty)$ and $\delta > 0$ are constants) in Eq.(9. 72) (see **appendix A**) and obtained the N and pN approximation with $G = C = 1$. The N approximation can be obtained from Eq.(9. 72), which is given below,

$$\begin{aligned}
&2r^{-2(n+6)}(2\kappa^2\lambda r^5(\lambda\eta(r(2\delta m_0' r^n(2|p_{r0}'|(7r^2+3) - \delta r^{n+1}(3ar^2 - 2m_0' - 22))\\
&-2m_0'' r(\delta^2(2m_0'+11)r^{2n+1} + |p_{r0}'|(7r^2+3)r^n + |p_{r0}'|^2 r) + \delta^2 m_0''^2 r^{2n+3}) + 2m_0 p_{r0}|p_{r0}'|(2r^2-1))\\
&+\delta r^{n+2}(2m_0' r^2(|p_{r0}'|(2r-1) - a\delta r^n) + m_0'' r^3(a\delta r^n - 2|p_{r0}'|r + |p_{r0}'|)\\
&+p_{r0}\eta(\delta r^{n+2} + |p_{r0}'|))) + \kappa\lambda\eta(\delta^2 r^{2n}(r^6(r(3ar^2 - 8r\eta - 8\eta - 70) + 2m_0''^2 r^2\\
&+2m_0''(r^3 - 3r^2 - 6r + 3)) + 16m_0'^2 m_0''\eta + 4m_0' r^6((3 - 2m_0'')r + 6)) + 8m_0 p_{r0}|p_{r0}'|(3r^2-1)r^5\\
&+\delta|p_{r0}'|r^{n+6}(r(m_0''(r^2 - 8r + 4) - 12(r-1)r\eta) - 2m_0'(r^2 - 8r + 4)) + 4|p_{r0}'|^2 r^9(\eta - 5))\\
&+r^7(-2\delta|p_{r0}'|r^{n+1}(-ar^2 + r^4\eta - 1) + \delta^2 r^{2n}(-(ar^2(1-3\eta) + r^5 - 14\eta)) + 4|p_{r0}'|^2 r^2\eta)\\
&+2\kappa^3\lambda^2 p_{r0} r^9\eta(2|p_{r0}'|^2 - \delta^2 r^{2n})) = 0
\end{aligned} \tag{5. 63}$$

From above equation, it can be observed that the instability is independent of adiabatic index, while it depends on anisotropic pressure, density, different parameters and Rastall parameter λ. For instability condition, we considered that all the terms in Eq.(5. 63) remain positive. Therefore, we assumed that all the dynamical quantities, arbitrary constants and Rastall parameter λ remain positive and $p_{r0}' < 0$, this implies that the pressure is decreasing during collapse of the gravitating source. For instability condition, all the terms must be positive in Eq.(5. 63), therefore, following constraints must be satisfied for the instability condition at N order

$$\begin{aligned}
&(2\delta m_0' r^n(2|p_{r0}'|(7r^2+3) - \delta r^{n+1}(3ar^2 - 2m_0' - 22)) - 2m_0'' r(\delta^2(2m_0'+11)r^{2n+1}\\
&+|p_{r0}'|(7r^2+3)r^n + |p_{r0}'|^2 r) + \delta^2 m_0''^2 r^{2n+3}) > 0,
\end{aligned} \tag{5. 64}$$

$$2r - 1 > 0,\ p_{r0} > p_{\perp 0}, \tag{5. 65}$$

$$(2m_0'r^2(|p_{r0}'|(2r-1)-a\delta r^n)+m_0''r^3(a\delta r^n-2|p_{r0}'|r+|p_{r0}'|)+p_{r0}\eta(\delta r^{n+2}+|p_{r0}'|))>0, \quad (5.\ 66)$$

$$(\delta^2r^{2n}(r^6(r(3ar^2-8r\eta-8\eta-70)+2m_0''^2r^2+2m_0''(r^3-3r^2-6r+3))+16m_0'^2m_0''\eta+4m_0'r^6((3-2m_0'')r+6))+8m_0p_{r0}|p_{r0}'|(3r^2-1)r^5+\delta|p_{r0}'|r^{n+6}(r(m_0''(r^2-8r+4)-12(r-1)r\eta)-2m_0'(r^2-8r+4))+4|p_{r0}'|^2r^9(\eta-5))>0, \quad (5.\ 67)$$

$$(-2\delta|p_{r0}'|r^{n+1}(-ar^2+r^4\eta-1)+\delta^2r^{2n}(-(ar^2(1-3\eta)+r^5-14\eta))+4|p_{r0}'|^2r^2\eta)>0, \quad (5.\ 68)$$

$$(2|p_{r0}'|^2-\delta^2r^{2n})>0, \quad (5.\ 69)$$

If all the above inequalities hold, then the system will be unstable otherwise it will be in the stable state. From Eq.(10. 73) (see **appendix B**), it can be seen that the instability is independent of adiabatic index, while it depends on the energy density, anisotropic pressure, various parameters and Rastall parameter at pN order. The instability condition satisfied when all the terms in Eq.(10. 73) remains positive, therefore, Eq.(10. 73) must be satisfy the constraints which are given in **appendix C**. If the inequalities given in the **appendix C** satisfied then the gravitating system remain unstable, but the system remains in equilibrium state if these conditions are not satisfied.

From Eq.(5. 63) at N order and Eq.(10. 73) at pN order can be observed that the instability is independent of a parameter Γ, while it depends on the various structural properties of fluid like energy density and anisotropic pressure as well as depends on some parameters. In this regards, it will be sufficient that $p_{r0} > p_{\perp 0}$, Eqs.(5. 63) and (10. 73) must be positive for the various values parameters. Here, we can say that adiabatic index does not play any roll in Eqs.(5. 63) and (10. 73), therefore, the expansion-free collapse would be happened without any compression of the fluid. The collapse rate of gravitating source with $\Theta = 0$ may be increased or decreased due to energy density, anisotropic pressure and Rastall parameter λ at N and pN order.

6. Notations and Preliminaries

The expectations for children's actions in the mathematics class were quite different from their previous experiences in school. However, in this mathematics class it was necessary for children to express their thinking in order to create opportunities for learning and so that their existing constructions could be investigated by both the teacher and researchers

7. Discrete Evolution Semigroup

Using these premises of children's learning as her guideline, the teacher initiated the mutual construction of a different set of norms for mathematics lessons as she acted to help the students reconceptualize their role during mathematics instruction. Her intention was for the children to figure things out for themselves and to express their ideas in the public arena of whole-class discussions. Additionally, during small-group work she expected them to cooperate and work together to solve problems and to agree on an answer. Her expectation that the children would express their thoughts placed the students under the obligation of having to recall their solutions and explain them to others during the whole-class discussion.

8. Conclusion

Here, we discussed instability of a spherically symmetric collapsing object under expansion-free condition at N and pN order in Rastall theory. We derived set of the partial differential equations also called field equations. We used a linear perturbation scheme on the fluid material components and metric components to established dynamical equations for the instability collapsing object. From Eq.(3. 48), we obtained a solution which contains oscillating function and non-oscillating function. The oscillating function corresponds to stable stage of gravitating source while non-oscillating function corresponds to unstable stage of the spherically symmetric source. We concerned with non-oscillating functions to study the instability of collapsing source. A solution given in Eq.(3. 50) provides a result $T(-\infty) = 0$ when $t = -\infty$, this implies that a static system starts collapsing and the areal radius of this system decreases as t increases. The perturbed form of dynamical equations are used to obtained the main collapse equation (9. 72) (see appendix). The energy density profile $\mu_0 = \delta r^n$ (where $n \in(-\infty, \infty)$ and $\delta > 0$ are constants) has been used in (9. 72) to obtained equations upto N and pN order. An Eq.(5. 63) at N order and Eq.(10. 73) at pN order under expansion-free condition show that the adiabatic index does not contribute in the instability of collapsing object, while the instability depends on the energy density, anisotropic pressure, Rastall parameter λ and other parameters involved in Eq.(5. 63) and Eq.(10. 73). Also, for instability some constraints must be satisfy at N and pN approximation. Moreover any possible model is further constrained by physical requirements such as positivity of energy density, energy density greater than pressure, and stability of local oscillations modes.

9. Appendix A

From Eqs.(5. 59), (5. 60), (3. 43), (3. 48), and (3. 38), we get

$$\kappa\left(\mu_0+p_{r0}\right)\left(\frac{a}{A_0}\right)'+2\kappa r\left(p_{r0}-p_{\perp 0}\right)\left(\frac{c}{r}\right)'-\left(\frac{2\alpha c}{A_0^2 r}+\frac{2}{B_0^2}\left(\left(\frac{a}{A_0}\right)''+\left(\frac{c}{r}\right)''\right.\right.$$
$$+\left(\frac{2A_0'}{A_0}-\frac{B_0'}{B_0}+\frac{1}{r}\right)\left(\frac{a}{A_0}\right)'+\left(\frac{A_0'}{A_0}-\frac{B_0'}{B_0}+\frac{2}{r}\right)\left(\frac{c}{r}\right)'\Bigg)+2\kappa\lambda\Bigg(\frac{2c'}{B_0 r}\Bigg(\frac{1}{B_0}-\frac{B_0'}{B_0}$$
$$+\frac{A_0'}{A_0}\Bigg)+\frac{A_0'}{A_0B_0}\left(4\frac{c}{r}-\frac{a}{A_0}\right)+\frac{2}{B_0r^2}\left(\frac{c'}{B_0}+\frac{2c}{r}\right)+\frac{A_0'B_0'}{A_0B_0^3}\left(\frac{a}{A_0}-\frac{2c}{r}\right)+\frac{4A_0'}{A_0B_0}\left(\frac{c}{r}\right)'$$
$$\left.\left.+\frac{1}{A_0B_0}\left(a''-\frac{a'B_0'}{B_0^2}\right)-\alpha\frac{B_0}{A_0^2}\left(\frac{c}{rB_0}-\frac{2c}{r}\right)\right)-2\kappa p_\perp\frac{c}{r}\right)=0. \qquad (9.\ 70)$$

From Eqs.(3. 37),(3. 48),(5. 56), and (5. 58), we get

$$\left(\frac{a}{A_0}\right)' = \left(1+\frac{\kappa\lambda r B_0'}{B_0}\right)^{-1}\left(-\frac{\kappa A_0}{r^2}\left(2\kappa p_{r0}B_0^2+\left(\frac{A_0'}{A_0}\right)^2-\frac{2rA_0^2}{A_0}+\frac{3}{r}\left(B_0^2-1\right)-\alpha\left(\frac{B_0}{A_0}\right)^2\right)\right.$$
$$-\kappa\lambda\eta\left(\left(2+\frac{B_0}{A_0}-\frac{2B_0'}{B_0}\right)\left(\frac{A_0''}{r^2}-\frac{4A_0'}{r^3}+\frac{6A_0}{r^4}\right)+\left(\frac{2A_0'}{A_0 r}-2\right)\left(\frac{A_0'}{r^3}-\frac{3A_0}{r^4}\right)\right.$$
$$\left.\left.-\frac{2A_0B_0^2}{r^4}-\frac{A_0''B_0}{r^2}-\frac{\alpha B_0^2}{A_0 r^2}-\frac{2\alpha}{A_0 r^2}\right)\right). \qquad (9.\ 71)$$

From Eq.(9. 70) with the help of Eqs.(5. 58) and (9. 71), we obtained

$$
\begin{aligned}
&\Big[\kappa\Big(\mu_0+p_{r0}\Big)-\Big(\frac{2A_0'}{A_0}-\frac{B_0'}{B_0}+\frac{1}{r}\Big)\frac{2}{B_0^2}\Big]\Big[\Big(1+\frac{\kappa\lambda rB_0'}{B_0}\Big)^{-1}\Big(-\frac{\kappa A_0}{r^2}\Big(2\kappa p_{r0}B_0^2+\Big(\frac{A_0'}{A_0}\Big)^2\\
&-\frac{2A_0^2}{rA_0}+\frac{3}{r}\Big(B_0^2-1\Big)-\alpha\Big(\frac{B_0}{A_0}\Big)\Big)-\kappa\lambda\eta\Big(\Big(2+\frac{B_0}{A_0}-\frac{2B_0'}{B_0}\Big)\Big(\frac{A_0''}{r^2}-\frac{4A_0'}{r^3}+\frac{6A_0}{r^4}\Big)\\
&+\Big(\frac{2A_0'}{rA_0}-2\Big)\Big(\frac{A_0'}{r^3}-\frac{3A_0}{r^4}\Big)-\frac{2A_0B_0^2}{r^4}-\frac{2A_0''B_0}{r^2}-\frac{\alpha B_0^2}{A_0r^2}-\frac{2\alpha}{A_0r^2}\Big)\Big)\Big]\\
&+2\kappa\eta\Big(p_{r0}-p_{\perp 0}\Big)\Big(\frac{A_0'}{r^2}-\frac{3A_0}{r^3}\Big)\\
&-\frac{2}{B_0}\Big(-\kappa\lambda\Big(1+\frac{\kappa\lambda rB_0'}{B_0}\Big)^{-2}\Big(\frac{rB_0''}{B_0}+\frac{B_0'}{B_0}-\frac{rB'^2}{B_0^2}\Big)\Big(-\frac{\kappa A_0}{r^2}\Big(2\kappa p_{r0}B_0^2+\Big(\frac{A_0'}{A_0}\Big)^2-\frac{2rA_0^2}{A_0}\\
&+\frac{3}{r}\Big(B_0^2-1\Big)-\alpha\Big(\frac{B_0}{A_0}\Big)^2\Big)-\kappa\lambda\eta\Big(\Big(2+\frac{B_0}{A_0}-\frac{2B_0'}{B_0}\Big)\Big(\frac{A_0''}{r^2}-\frac{4A_0'}{r^3}+\frac{6A_0}{r^4}\Big)\\
&+\Big(\frac{2A_0'}{A_0r}-2\Big)\Big(\frac{A_0'}{r^3}-\frac{3A_0}{r^4}\Big)-\frac{2A_0B_0^2}{r^4}-\frac{A_0''B_0}{r^2}-\frac{\alpha B_0^2}{A_0r^2}-\frac{2\alpha}{A_0r^2}\Big)\Big)\\
&+\Big(1+\frac{\kappa\lambda rB_0'}{B_0}\Big)^{-1}\Big(2\kappa^2\Big(\frac{p_{r0}'B^2}{r^2}+\frac{2p_{r0}B_0B_0'}{r^2}-\frac{2p_{r0}B_0^2}{r^3}\Big)+2\kappa\Big(\Big(\frac{2A_0A_0'}{r^3}-\frac{3A_0^2}{r^4}\Big)\\
&+3\Big(\frac{2B_0B_0'}{r}-\frac{B_0^2}{r^2}+\frac{1}{r^2}\Big)-2\alpha\Big(\frac{B_0}{A_0}\Big)\Big(\frac{B_0'}{A_0}-\frac{A_0'B_0}{A_0^2}\Big)\Big)\\
&-\kappa\eta\lambda\Big(\Big(2+\frac{B_0}{A_0}-\frac{2B_0'}{B_0}\Big)\Big(\frac{A_0'''}{r^2}-\frac{6A_0''}{r^3}-\frac{16A_0'}{r^4}-\frac{24A_0}{r^5}\Big)+\Big(\frac{B_0'}{A_0}-\frac{A_0'B_0}{A_0}\\
&-\frac{2B_0''}{B_0}+\frac{2B_0'^2}{B_0^2}\Big)\Big(\frac{A_0''}{r^2}-\frac{4A_0'}{r^3}-\frac{6A_0}{r^4}\Big)+\Big(\frac{2A_0'}{A_0r}-2\Big)\Big(\frac{A_0''}{r^3}-\frac{6A_0'}{r^4}+\frac{12A_0}{r^5}\Big)\\
&+\Big(\frac{2A_0''}{A_0r}-\frac{2A_0'^2}{A_0^2r}-\frac{A_0'}{A_0r^2}\Big)\Big(\frac{A_0'}{r^3}-\frac{3A_0}{r^4}\Big)-\Big(\frac{A_0B_0A_0'}{r^4}+\frac{A_0'B_0^2}{r^4}-\frac{4A_0B_0^2}{r^5}\Big)\\
&-2\Big(\frac{A_0''B_0}{r^2}+\frac{A_0''B_0'}{r^2}-\frac{2A_0''B_0}{r^3}\Big)-\alpha\Big(\frac{2B_0B_0'}{A_0r^2}-\frac{A_0'B_0^2}{A_0^2r^2}-\frac{2B_0^2}{A_0r^3}\Big)+2\alpha\Big(\frac{A_0'}{A_0^2r^2}+\frac{2}{A_0r^3}\Big)\Big)\Big)\Big)\\
&-\eta\Big(\frac{2\alpha}{r^3}+\Big(\frac{A_0''}{r^5}-\frac{6A_0'}{r^4}+\frac{14A_0}{r^5}+\Big(\frac{A_0'}{A_0}-\frac{B_0'}{B_0}+\frac{2}{r}\Big)\Big(\frac{A_0'}{r^3}-\frac{3A_0}{r^4}\Big)\frac{2}{B_0}\Big)\\
&+2\kappa\eta\lambda\Big(\frac{2}{B_0}\Big(\frac{A_0'}{r^3}-\frac{3A_0}{r^4}\Big)\Big(\frac{1}{B_0}-\frac{B_0'}{B_0^2}+\frac{A_0'}{A_0}\Big)+4\frac{A_0'}{r^3B_0}+\frac{2}{B_0r^2}\Big(\Big(\frac{A_0}{r^2}-\frac{2A_0}{r^3}\Big)\frac{1}{B_0}+\frac{2A_0}{r^3}\Big)\\
&+\frac{A_0'B_0'}{B_0^3r^3}+\frac{4A_0'}{A_0B_0}\Big(\frac{A_0'}{r^3}-\frac{3A_0}{r^4}\Big)-\frac{\alpha B_0}{A_0^2}\Big(\frac{A_0}{B_0r^3}-\frac{2A_0}{r^3}\Big)\Big)\Big)-2\kappa\eta p_{\perp 0}\frac{A_0}{r^3}\Big)=0. \qquad (9.\ 72)
\end{aligned}
$$

10. APPENDIX B

Eq.(9. 72) gives us the following equation up to pN order,

$$
\begin{aligned}
& r^{-4(n+2)}(2\delta(m_0((r^6-6\eta r^5+6r^2-28\eta r+m_0''\eta)\delta^3r^{3n}-2|p_{r0}'|^2(6\eta-7)\delta r^{n+3}\\
& +|p_{r0}'|(4\eta r^4-2ar^2-12r-47\eta-6)\delta^2r^{2n+2}+6|p_{r0}'|^3r^4)\\
& -2r^2(p_{r0}(2(-ar^2+(r^4-9)\eta-1)\delta^2r^{2n}-|p_{r0}'|(-ar^2+(r^4-1)\eta-7)\delta r^{n+1}\\
& +4|p_{r0}'|^2\eta r^2)-2p_{\perp0}r^n\delta(r^n(\eta r^4-ar^2-1)\delta-4|p_{r0}'|r\eta)))r^{n+1}\\
& +\kappa^3\lambda\delta(4\lambda^2m_0(2m_0'-m_0''r)\eta\delta(2|p_{r0}'|(7r^2+3)\delta r^n-(3ar^2+m_0''r-2m_0'-22)\delta^2r^{2n+1}\\
& +2|p_{r0}'|^2r)r^n+2\delta^3(r^n(a(r-1)r-3)\delta-|p_{r0}'|r(2ar^3+2r^2-3r+3))r^{3n+5}\\
& -\lambda(4p_{r0}\eta\delta(p_{\perp0}(2(2r^2-1)\delta r^n+8|p_{r0}'|r)+m_0(|p_{r0}'|\delta r^n+\delta^2r^{2n}+2|p_{r0}'|^2r))r^n\\
& -\delta^2(4m_0(2m_0'-m_0''r)(|p_{r0}'|(2r-1)-ar^n\delta)+\eta(-|p_{r0}'|(3ar^4+28r^3-16r^2\\
& +m_0''(r^2+8r+4)r-2m_0'(r^2+8r+4)-33)\delta r^n+2(3ar^3+(m_0''^2+2m_0'-22)r\\
& -4m_0'+57)\delta^2r^{2n+1}+|p_{r0}'|^2(20r^2-2r+29)r)r^{2n+1}+4p_{r0}^2\eta(2|p_{r0}'|(2r^2-5)\delta r^n\\
& -5\delta^2r^{2n+1}+6|p_{r0}'|^2r)r)r^2)r^{n+1}-2\kappa^4\lambda^2\delta^3(\delta m_0''(a\delta r^n+|p_{r0}'|)r^2+2m_0'|p_{r0}'|(2r-1)r\\
& -p_{r0}\eta(\delta r^n+|p_{r0}'|))r^{n+2}+\lambda\eta(-2m_0'\delta(r^{n+1}(3ar^2-22)\delta-2|p_{r0}'|(7r^2+3))r^n\\
& +m_0''^2\delta^2r^{2n+3}-m_0''(2|p_{r0}'|(7r^2+3)\delta r^n-(3ar^2-4m_0'-22)\delta^2r^{2n+1}+2|p_{r0}'|^2r)r))r^{3n+4}\\
& -\kappa(\delta^3(36m_0r^n-36m_0\delta r^{n+2}+8am_0\delta r^{n+3}+2|p_{r0}'|\delta r^{n+3}+|p_{r0}'|\eta\delta r^{n+3}\\
& -a\delta^2r^{2n+3}-a\delta^2r^{2n+4}+2\eta\delta^2r^{2n+6}+8am_0|p_{r0}'|r^4+4p_{r0}^2\eta r^2+24m_0|p_{r0}'|r\\
& +4p_{\perp0}(4ar^3-p_{r0}\eta r+4)r-16p_{r0}(ar^4+r))r^{3n+3}+2\lambda(24m_0^2p_{r0}|p_{r0}'|\eta\delta^2r^{2n+2}\\
& +\eta(-p_{r0}\delta(-|p_{r0}'|((m_0''-12\eta)r^3-4(2m_0''+\eta-23)r^2+(4m_0''+2)r\\
& -2m_0'(r^2-8r+4)+79)\delta r^n+2(-2m_0'(r-8)+m_0''(r^2-8r+4)\\
& +3r(ar^2-4\eta r+4\eta+4))\delta^2r^{2n}+4|p_{r0}'|^2(1-2r(\eta-5))r^2)r^n+2p0t\delta^2((m_0''(r^2+4)\\
& +3r(ar^2-4\eta r+4\eta))\delta r^n+|p_{r0}'|(r^2(8\eta-44)-48))r^{2n}+8p_{r0}^3|p_{r0}'|r^2)r^2\\
& +m_0(16p_{\perp0}p_{r0}\eta\delta((3r^2-1)\delta r^n+6|p_{r0}'|r)r^n+\delta(2|p_{r0}'|(a(2\eta-1)r^4+2\eta(m_0''-10\eta)r^3\\
& +(20\eta^2-(4m_0'+14m_0''+25)\eta-1)r^2+2(14m_0'+m_0''-3)\eta r-(4m_0'+25)\eta)\delta^2r^{2n}\\
& -(a(2\eta-1)r^3+\eta(-16m_0'^2+2(8m_0''r+r-40)m_0'-4m_0''^2r^2-m_0''(r^2-40r+6)\\
& +2r(-4\eta r^2+16\eta r+2r+10\eta+45)))\delta^3r^{3n}+|p_{r0}'|^2((12\eta^2-52\eta+3)r^2\\
& +12\eta r-62\eta)\delta r^{n+1}+2|p_{r0}'|^3r^4)r^{n+1}+8p_{r0}^2\eta(2\delta^2r^{2n}-14|p_{r0}'|\delta r^{n+1}+9|p_{r0}'|^2r^2))))\\
& +\kappa^2(\lambda\delta^2(8p_{\perp0}p_{r0}\eta\delta r^n+4m_0p_{r0}|p_{r0}'|\eta\delta r^n+12m_0\delta^2r^{2n}+4m_0p_{r0}\eta\delta^2r^{2n}+4m_0|p_{r0}'|\delta r^{n+1}\\
& +4m_0|p_{r0}'|\delta r^{n+2}+4p_{r0}^2\eta\delta r^{n+2}+8m_0|p_{r0}'|\delta r^{n+3}+16|p_{r0}'|^2\eta\delta r^{n+3}+8am_0|p_{r0}'|\delta r^{n+4}\\
& -4am_0\delta^2r^{2n+1}-6|p_{r0}'|\eta\delta^2r^{2n+1}-4am_0\delta^2r^{2n+2}+4|p_{r0}'|\eta^2\delta^2r^{2n+3}-2|p_{r0}'|\delta^2r^{2n+3}\\
& -8|p_{r0}'|\eta\delta^2r^{2n+3}-2a|p_{r0}'|\delta^2r^{2n+5}+12\eta^2\delta^3r^{3n+2}-16\eta\delta^3r^{3n+2}+3a\eta\delta^3r^{3n+3}+a\delta^3r^{3n+4}\\
& +2|p_{r0}'|^3r^5+8p_{r0}^2|p_{r0}'|\eta r-2m_0''(2p_{r0}r(2r-1)(|p_{r0}'|r-2r^n\delta)-r^n\delta(\eta\delta(r^n\delta-2|p_{r0}'|)r^{n+2}\\
& +4p_{\perp0}(1-2r)r+2m_0((ar+6)\delta r^n+|p_{r0}'|(3-8r)r)))r+4p_{r0}^2|p_{r0}'|\eta\\
& +4m_0'(2p_{r0}r^2(|p_{r0}'|(2r-1)-4r^n\delta)-r^n\delta(\eta\delta(r^n\delta-2|p_{r0}'|)r^{n+2}+4p_{\perp0}(1-2r)r\\
& +2m_0((ar+6)\delta r^n+|p_{r0}'|(3-8r)r))))r^{2n+3}+4\delta^4(r^{n+2}(ar-3)\delta-|p_{r0}'|)r^{4n+6}\\
& +2\lambda^2\eta(4\delta(2m_0'(4p_{\perp0}|p_{r0}'|\delta r^n+p_{r0}(|p_{r0}'|(7r^2-1)\delta r^n-14\delta^2r^{2n+1}+2|p_{r0}'|^2r))\\
& -m_0''(2p_{\perp0}\delta r^n-2p_{r0}\delta r^n+p_{r0}|p_{r0}'|r)((7r^2+3)\delta r^n+2|p_{r0}'|r))r^{n+2}+8m_0^2p_{r0}|p_{r0}'|\delta^2r^{2n+2}\\
& +m_0(8p_{\perp0}p_{r0}\delta((2r^2-1)\delta r^n+4|p_{r0}'|r)r^n+\delta^2(|p_{r0}'|(3ar^4-67m_0''r^3+8(m_0''+5)r^2\\
& +(6-32m_0'')r+2m_0'(67r^2-8r+32)-9)\delta r^n+(8(2r-1)m_0'^2-4(5ar^3+4m_0''r^2\\
& -2(m_0''+32)r+6)m_0'+2m_0''^2r^2(2r-1)-r(6ar^3+3ar^2-44r+26)\\
& +2m_0''(5ar^4-64r^2+6r+3))\delta^2r^{2n}+|p_{r0}'|^2(-20r^2+(2-12m_0'')r+24m_0'-21)r)r^{2n+1}\\
& +8p_{r0}^2(\delta^2r^{2n}+|p_{r0}'|(2r^2-5)\delta r^{n+1}+3|p_{r0}'|^2r^2))))=0. \qquad (10.\ 73)
\end{aligned}
$$

11. APPENDIX C

$$p_{r0} > p_{\perp 0}, \tag{11. 74}$$

$$\begin{aligned}
&(m_0((r^6 - 6\eta r^5 + 6r^2 - 28\eta r + m_0''\eta)\delta^3 r^{3n} - 2|p_{r0}'|^2(6\eta - 7)\delta r^{n+3} \\
&+|p_{r0}'|(4\eta r^4 - 2ar^2 - 12r - 47\eta - 6)\delta^2 r^{2n+2} + 6|p_{r0}'|^3 r^4) \\
&-2r^2(p_{r0}(2(-ar^2 + (r^4 - 9)\eta - 1)\delta^2 r^{2n} - |p_{r0}'|(-ar^2 + (r^4 - 1)\eta - 7)\delta r^{n+1} \\
&+4|p_{r0}'|^2\eta r^2) - 2p_{\perp 0} r^n \delta(r^n(\eta r^4 - ar^2 - 1)\delta - 4|p_{r0}'|r\eta))) > 0,
\end{aligned} \tag{11. 75}$$

$$\begin{aligned}
&(4\lambda^2 m_0(2m_0' - m_0'' r)\eta\delta(2|p_{r0}'|(7r^2 + 3)\delta r^n - (3ar^2 + m_0'' r - 2m_0' - 22)\delta^2 r^{2n+1} \\
&+2|p_{r0}'|^2 r)r^n + 2\delta^3(r^n(a(r-1)r - 3)\delta - |p_{r0}'|r(2ar^3 + 2r^2 - 3r + 3))r^{3n+5} \\
&-\lambda(4p_{r0}\eta\delta(p_{\perp 0}(2(2r^2 - 1)\delta r^n + 8|p_{r0}'|r) + m_0(|p_{r0}'|\delta r^n + \delta^2 r^{2n} + 2|p_{r0}'|^2 r))r^n \\
&-\delta^2(4m_0(2m_0' - m_0'' r)(|p_{r0}'|(2r - 1) - ar^n\delta) + \eta(-|p_{r0}'|(3ar^4 + 28r^3 - 16r^2 \\
&+m_0''(r^2 + 8r + 4)r - 2m_0'(r^2 + 8r + 4) - 33)\delta r^n + 2(3ar^3 + (m_0''^2 + 2m_0' - 22)r \\
&-4m_0' + 57)\delta^2 r^{2n+1} + |p_{r0}'|^2(20r^2 - 2r + 29)r))r^{2n+1} + 4p_{r0}^2\eta(2|p_{r0}'|(2r^2 - 5)\delta r^n \\
&-5\delta^2 r^{2n+1} + 6|p_{r0}'|^2 r)r)r^2) > 0,
\end{aligned} \tag{11. 76}$$

$$(\delta m_0''(a\delta r^n + |p_{r0}'|)r^2 + 2m_0'|p_{r0}'|(2r - 1)r - p_{r0}\eta(\delta r^n + |p_{r0}'|)) < 0, \tag{11. 77}$$

$$\begin{aligned}
&-2m_0'\delta(r^{n+1}(3ar^2 - 22)\delta - 2|p_{r0}'|(7r^2 + 3))r^n + m_0''^2\delta^2 r^{2n+3} \\
&-m_0''(2|p_{r0}'|(7r^2 + 3)\delta r^n - (3ar^2 - 4m_0' - 22)\delta^2 r^{2n+1} + 2|p_{r0}'|^2 r)r > 0,
\end{aligned} \tag{11. 78}$$

$$\begin{aligned}
&(36m_0 r^n - 36m_0\delta r^{n+2} + 8am_0\delta r^{n+3} + 2|p_{r0}'|\delta r^{n+3} + |p_{r0}'|\eta\delta r^{n+3} \\
&-a\delta^2 r^{2n+3} - a\delta^2 r^{2n+4} + 2\eta\delta^2 r^{2n+6} + 8am_0|p_{r0}'|r^4 + 4p_{r0}^2\eta r^2 + 24m_0|p_{r0}'|r \\
&+4p_{\perp 0}(4ar^3 - p_{r0}\eta r + 4)r - 16p_{r0}(ar^4 + r)) < 0,
\end{aligned} \tag{11. 79}$$

$$\begin{aligned}
&(24m_0^2 p_{r0}|p_{r0}'|\eta\delta^2 r^{2n+2} + \eta(-p_{r0}\delta(-|p_{r0}'|((m_0'' - 12\eta)r^3 - 4(2m_0'' + \eta - 23)r^2 + (4m_0'' + 2)r \\
&-2m_0'(r^2 - 8r + 4) + 79)\delta r^n + 2(-2m_0'(r - 8) + m_0''(r^2 - 8r + 4) \\
&+3r(ar^2 - 4\eta r + 4\eta + 4))\delta^2 r^{2n} + 4|p_{r0}'|^2(1 - 2r(\eta - 5))r^2)r^n + 2p0t\delta^2((m_0''(r^2 + 4) \\
&+3r(ar^2 - 4\eta r + 4\eta))\delta r^n + |p_{r0}'|(r^2(8\eta - 44) - 48))r^{2n} + 8p_{r0}^3|p_{r0}'|r^2)r^2 \\
&+m_0(16p_{\perp 0}p_{r0}\eta\delta((3r^2 - 1)\delta r^n + 6|p_{r0}'|r)r^n + \delta(2|p_{r0}'|(a(2\eta - 1)r^4 + 2\eta(m_0'' - 10\eta)r^3 \\
&+(20\eta^2 - (4m_0' + 14m_0'' + 25)\eta - 1)r^2 + 2(14m_0' + m_0'' - 3)\eta r - (4m_0' + 25)\eta)\delta^2 r^{2n} \\
&-(a(2\eta - 1)r^3 + \eta(-16m_0'^2 + 2(8m_0'' r + r - 40)m_0' - 4m_0''^2 r^2 - m_0''(r^2 - 40r + 6) \\
&+2r(-4\eta r^2 + 16\eta r + 2r + 10\eta + 45)))\delta^3 r^{3n} + |p_{r0}'|^2((12\eta^2 - 52\eta + 3)r^2 \\
&+12\eta r - 62\eta)\delta r^{n+1} + 2|p_{r0}'|^3 r^4)r^{n+1} + 8p_{r0}^2\eta(2\delta^2 r^{2n} - 14|p_{r0}'|\delta r^{n+1} + 9|p_{r0}'|^2 r^2))) < 0,
\end{aligned} \tag{11. 80}$$

$$+\kappa^2(\lambda\delta^2(8p_{\perp0}p_{r0}\eta\delta r^n+4m_0p_{r0}|p'_{r0}|\eta\delta r^n+12m_0\delta^2r^{2n}+4m_0p_{r0}\eta\delta^2r^{2n}+4m_0|p'_{r0}|\delta r^{n+1}$$
$$+4m_0|p'_{r0}|\delta r^{n+2}+4p_{r0}^2\eta\delta r^{n+2}+8m_0|p'_{r0}|\delta r^{n+3}+16|p'_{r0}|^2\eta\delta r^{n+3}+8am_0|p'_{r0}|\delta r^{n+4}$$
$$-4am_0\delta^2r^{2n+1}-6|p'_{r0}|\eta\delta^2r^{2n+1}-4am_0\delta^2r^{2n+2}+4|p'_{r0}|\eta^2\delta^2r^{2n+3}-2|p'_{r0}|\delta^2r^{2n+3}$$
$$-8|p'_{r0}|\eta\delta^2r^{2n+3}-2a|p'_{r0}|\delta^2r^{2n+5}+12\eta^2\delta^3r^{3n+2}-16\eta\delta^3r^{3n+2}+3a\eta\delta^3r^{3n+3}+a\delta^3r^{3n+4}$$
$$+2|p'_{r0}|^3r^5+8p_{r0}^2|p'_{r0}|\eta r-2m''_0(2p_{r0}r(2r-1)(|p'_{r0}|r-2r^n\delta)-r^n\delta(\eta\delta(r^n\delta-2|p'_{r0}|)r^{n+2}$$
$$+4p_{\perp0}(1-2r)r+2m_0((ar+6)\delta r^n+|p'_{r0}|(3-8r)r)))r+4p_{r0}^2|p'_{r0}|\eta$$
$$+4m'_0(2p_{r0}r^2(|p'_{r0}|(2r-1)-4r^n\delta)-r^n\delta(\eta\delta(r^n\delta-2|p'_{r0}|)r^{n+2}+4p_{\perp0}(1-2r)r$$
$$+2m_0((ar+6)\delta r^n+|p'_{r0}|(3-8r)r))))>0, \quad (11.\ 81)$$

$$(r^{n+2}(ar-3)\delta-|p'_{r0}|)>0. \quad (11.\ 82)$$

$$4\delta(2m'_0(4p_{\perp0}|p'_{r0}|\delta r^n+p_{r0}(|p'_{r0}|(7r^2-1)\delta r^n-14\delta^2r^{2n+1}+2|p'_{r0}|^2r))$$
$$-m''_0(2p_{\perp0}\delta r^n-2p_{r0}\delta r^n+p_{r0}|p'_{r0}|r)((7r^2+3)\delta r^n+2|p'_{r0}|r))r^{n+2}+8m_0^2p_{r0}|p'_{r0}|\delta^2r^{2n+2}$$
$$+m_0(8p_{\perp0}p_{r0}\delta((2r^2-1)\delta r^n+4|p'_{r0}|r)r^n+\delta^2(|p'_{r0}|(3ar^4-67m''_0r^3+8(m''_0+5)r^2$$
$$+(6-32m''_0)r+2m'_0(67r^2-8r+32)-9)\delta r^n+(8(2r-1)m'^2_0-4(5ar^3+4m''_0r^2$$
$$-2(m''_0+32)r+6)m'_0+2m''^2_0r^2(2r-1)-r(6ar^3+3ar^2-44r+26)$$
$$+2m''_0(5ar^4-64r^2+6r+3))\delta^2r^{2n}+|p'_{r0}|^2(-20r^2+(2-12m''_0)r+24m'_0-21)r)r^{2n+1}$$
$$+8p_{r0}^2(\delta^2r^{2n}+|p'_{r0}|(2r^2-5)\delta r^{n+1}+3|p'_{r0}|^2r^2))>0. \quad (11.\ 83)$$

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