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On Generalization of Quasi s-topological IP-loops

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Abstract.: This paper is based on the investigation and characterization of the properties of quasi $\mathcal{G}$-s-topological loops. Moreover, we have also introduced the notion of quasi $\mathcal{G}$-s-topological loop having inverse property respecting semi continuity.

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1. Introduction

It is always captivating to dig into the relationship between topological and algebraic structures. To bring out the new ideas and results, we primarily make a relationship among them by debilitating the conditions of openness and continuity. With the instauration of semi open set by N. Levine, many of the mathematicians examine and explore several concepts by using semi continuity and semi open sets [10, 11, 6, 7, 12]. A number of new results are obtained when open set is replaced by semi open set and continuity is replaced by semi continuity [9, 8, 4, 15].

A. Csaszar defined generalized topology as: Let X be a set and $\mathcal{G} \subseteq P(X)$. Then $\mathcal{G}$ is said to be generalized topology(denoted by $\mathcal{G}$-topology), if $\phi \in \mathcal{G}$ and $\mathcal{G}$ is closed under arbitrary union [1]. Members of $\mathcal{G}$ are called generalized open sets denoted by $\mathcal{G}$-open sets and their compliments are generalized closed sets($\mathcal{G}$-closed sets). A subset M of X is generalized semi open($\mathcal{G}$-semi open), if there is a $\mathcal{G}$-open set O in X such that

$$O \subseteq M \subseteq \mathcal{G}Cl(O).$$

The collection of all $\mathcal{G}$-semi open sets in X is denoted by $\mathcal{G}SO(X)$. $\mathcal{G}SO(X,t)$ is the collection of all generalized semi open sets containing t. Any union of $\mathcal{G}$-semi open sets remain $\mathcal{G}$-semi open, but their finite intersection may not be $\mathcal{G}$-semi open. Compliment of $\mathcal{G}$-semi open set is $\mathcal{G}$-semi closed. It is to be noted that, every $\mathcal{G}$-open($\mathcal{G}$-closed) set is $\mathcal{G}$-semi open($\mathcal{G}$-semi closed). $M_2 \subseteq X$ is $\mathcal{G}$-semi open neighborhood of $t \in X$, if there exists $M_1 \in \mathcal{G}SO(X)$ such that

$$t \in M_1 \subseteq M_2.$$

If there is $\mathcal{G}$-semi open M' satisfying $t \in M' \subseteq M$, then $t \in X$ is $\mathcal{G}$-semi interior point of M. $\mathcal{G}sInt(M)$ is the set consisting of all $\mathcal{G}$-semi interior points of M. For any $\mathcal{G}$-semi open set M_t, $t \in \mathcal{G}sCl(M')$ if and only if $M_t \cap M' \neq \phi$ [2].
A mapping $f : X \to Y$ is said to be

- $\mathcal{G}$**-semi continuous**, if the set $f^{-1}(M_2)$ is $\mathcal{G}$-semi open in X for each $\mathcal{G}$-open set $M_2 \subseteq Y$. Equivalently, if for all $t \in X$, and every $\mathcal{G}$-open neighborhood M_2 of $f(t)$, there is a $\mathcal{G}$-semi open neighborhood M_1 of t satisfying $f(M_1) \subseteq M_2$ or $t \in M_1 \subseteq f^{-1}(M_2)$ then f is $\mathcal{G}$-semi continuous [1];
- $\mathcal{G}$**-semi open**, if for each $\mathcal{G}$-open set M in X, $f(M)$ is $\mathcal{G}$-semi open in Y [1];
- **Quasi $\mathcal{G}$-s homeomorphism**, if f is bijective, $\mathcal{G}$-semi open, and $\mathcal{G}$-semi continuous.

If there exists a finite(countable) subcover for any $\mathcal{G}$-semi open cover of M in X then a subset M of a $\mathcal{G}$-topological space X is called $\mathcal{G}$-semi compact ($\mathcal{G}$-semi-Lindelof). A $\mathcal{G}$-topological space $(L, \mathcal{G})$ is said to be $\mathcal{G}$-s regular, if for every $t \in L$ and every $\mathcal{G}$-closed set $Q \subseteq L$, there exists $\mathcal{G}$-semi open neighbourhoods M, N containing t and Q respectively with $t \notin Q$ such that $M \cap N = \phi$. A collection $\mathcal{G}_w$ of subsets of a $\mathcal{G}$-topological space $(L, \mathcal{G})$ is $\mathcal{G}$-semi discrete, if every $t \in L$ has $\mathcal{G}$-semi open neighbourhood intersecting maximum one element of $\mathcal{G}_w$. A $\mathcal{G}$-topological space $(L, \mathcal{G})$ is called quasi $\mathcal{G}$-s-homogeneous, if there is a Quasi $\mathcal{G}$-s homeomorphism $f : L \to L$ such that for every $r, s \in L$ implies $f(r) = s$. If a $\mathcal{G}$-topological space $\mathcal{G}'$ cannot be expressed as a union of two nonempty disjoint $\mathcal{G}$-semi open sets then it is said to be $\mathcal{G}$-semi connected. $\mathcal{G}.S.C.(x)$ denotes the $\mathcal{G}$-semi component containing $x \in X$. $\mathcal{G}$-semi components are the maximal $\mathcal{G}$-semi connected subsets of a $\mathcal{G}$-topological space.

A groupoid $(L, *)$ is a loop if the conditions given below are satisfied:

- L contain an identity element.
- for every $t_1 \in L$, the mappings $l_{t_1} : L \to L$ and $r_{t_1} : L \to L$ are bijective, where $l_{t_1}(t_2) = t_1 * t_2$ and $r_{t_1}(t_2) = t_2 * t_1$ for all $t_2 \in L$.

An inverse property loop (IP-loop) L is a loop having two sided inverse t^{-1} such that $(r * t) * t^{-1} = r = t^{-1} * (t * r)$ for all $r, t \in L$.

Left(right) translations and left(right) inverse mappings are defined on a loop $(L, *)$ as follows:

- **Left translation** $l_{t_1} : L \to L$ is given by $l_{t_1}(t_2) = t_1 * t_2$;
- **Right translation** $r_{t_1} : L \to L$ is given as $r_{t_1}(t_2) = t_2 * t_1$;

- **Left inverse mapping** $i_L : L \to L$ is defined by $i_L(t) = (t)_L^{-1}$;
- **Right inverse mapping** $i_R : L \to L$ is defined as $i_R(t) = (t)_R^{-1}$;

where $t, t_1, t_2 \in L$.
We used the standard notions and terminologies as in [5].

2. Quasi Generalized s-topological IP-loop

Definition 2.1. *A triplet $(L, *, \mathcal{G})$ is said to be a quasi $\mathcal{G}$-s-topological IP-loop, if the conditions given below are satisfied:*

- *$(L, *)$ is an IP-loop.*
- *$(L, \mathcal{G})$ is a $\mathcal{G}$-topological space.*
- *Multiplication mapping is separately $\mathcal{G}$-semi continuous in $(L, *, \mathcal{G})$.*
- *Inverse mapping is $\mathcal{G}$-semi continuous in $(L, *, \mathcal{G})$.*

To illustrate quasi $\mathcal{G}$-s-topological IP-loop we have provided an interesting example of an IP-loop L having order 10.

Example 2.2. *In a commutative IP-loop L of order 10 with $\mathcal{G}$-topology $\mathcal{G} = \{\phi, \{9, 10\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{1, 2\}, \{1, 2, 3, 4\}, \{1, 2, 5, 6\}, \{1, 2, 9, 10\}, \{1, 2, 7, 8\}, \{3, 4, 5, 6\}, \{3, 4, 7, 8\}, \{3, 4, 9, 10\}, \{5, 6, 7, 8\}, \{1, 2, 3, 4, 5, 6, 7, 8\}, \{7, 8, 9, 10\}, \{1, 2, 3, 4, 5, 6\}, \{1, 2, 7, 8, 9, 10\}, \{1, 2, 3, 4, 9, 10\}, \{1, 2, 5, 6, 7, 8\}, \{1, 2, 5, 6, 9, 10\}, \{1, 2, 3, 4, 7, 8\}, \{3, 4, 5, 6, 7, 8\}, \{5, 6, 9, 10\}, \{3, 4, 7, 8, 9, 10\}, \{3, 4, 5, 6, 9, 10\}, \{1, 2, 3, 4, 5, 6, 9, 10\}, \{1, 2, 3, 4, 7, 8, 9, 10\}, \{5, 6, 7, 8, 9, 10\}, \{1, 2, 5, 6, 7, 8, 9, 10\}, \{3, 4, 5, 6, 7, 8, 9, 10\}, L\}$ inverse mapping is $\mathcal{G}$-semi continuous, because inverse image of each $\mathcal{G}$-open set is $\mathcal{G}$-semi open. But multiplication mapping is not separately $\mathcal{G}$-semi continuous as inverse image of $\mathcal{G}$-open set $\{9, 10\}$ under left translation l_6 is $\{4, 8\}$ which is not $\mathcal{G}$-semi open (see Table 1). Hence, it is not a quasi $\mathcal{G}$-s-topological IP-loop.*

TABLE 1. Commutative IP-loop L of order 10

*	1	2	3	4	5	6	7	8	9	10
e=1	1	2	3	4	5	6	7	8	9	10
2	2	1	4	3	6	5	9	10	7	8
3	3	4	1	2	7	8	5	6	10	9
4	4	3	2	1	9	10	8	7	5	6
5	5	6	7	9	2	1	10	3	8	4
6	6	5	8	10	1	2	3	9	4	7
7	7	9	5	8	10	3	4	1	6	2
8	8	10	6	7	3	9	1	4	2	5
9	9	7	10	5	8	4	6	2	3	1
10	10	8	9	6	4	7	2	5	1	3

Similarly, with the $\mathcal{G}$-topology $\mathcal{G}' = \{\phi, \{1, 2\}, \{3, 4\}, \{1, 2, 3, 4\}\}$, $(L, *, \mathcal{G}')$ is not a quasi $\mathcal{G}$-s-topological IP-loop. But with the $\mathcal{G}$-topologies $\mathcal{G}_1 = \{\phi\}$ and $\mathcal{G}_2 = \{P(L)\}$, it form quasi $\mathcal{G}$-s-topological IP-loops.

Note: Symmetric group of degree 3 with the topology $\tau = \{\phi, \{e, a, a^2\}, \{b, ab, a^2b\}, S_3\}$ form a quasi $\mathcal{G}$-s-topological IP-loop.

Lemma 2.3. *Let $(L, *, \mathcal{G})$ be a quasi $\mathcal{G}$-s-topological loop with inverse property and $M \subseteq L$. Then $(\mathcal{G}sCl(M))^{-1} \subseteq \mathcal{G}Cl(M^{-1})$.*

Proof. Let $t \in (\mathcal{G}sCl(M))^{-1}$ and N is an $\mathcal{G}$-open neighbourhood of t. So N^{-1} is $\mathcal{G}$-semi open neighbourhood of t^{-1}. As $t^{-1} \in \mathcal{G}sCl(M)$, which implies $N^{-1} \cap M \neq \phi$. Therefore, $N \cap M^{-1} \neq \phi$. Hence, $t \in \mathcal{G}Cl(M^{-1})$. □

Theorem 2.4. *Inverse of a $\mathcal{G}$-semi open set in a quasi $\mathcal{G}$-s-topological IP-loop is $\mathcal{G}$-semi open in conjugate topology.*

Proof. Let $(L, *, \mathcal{G})$ be a quasi $\mathcal{G}$-s-topological IP-loop and $M \in \mathcal{G}SO(L, \mathcal{G})$, then there exists a $\mathcal{G}$-open set N such that $N \subseteq M \subseteq \mathcal{G}Cl(N)$ equivalently $N^{-1} \subseteq M^{-1} \subseteq (\mathcal{G}Cl(N))^{-1}$. Hence, $N^{-1} \subseteq M^{-1} \subseteq (\mathcal{G}Cl(N)^{-1})$. Now $N^{-1} \in \mathcal{G}^{-1}$ implies $M^{-1} \in \mathcal{G}SO(L, \mathcal{G}^{-1})$. □

Theorem 2.5. *The property of being quasi $\mathcal{G}$-s-topological IP-loop is preserved under topological conjugation.*

Proof. Let $(L, *, \mathcal{G})$ be a quasi $\mathcal{G}$-s-topological IP-loop and N be a $\mathcal{G}$-open set in $(L, *, \mathcal{G}^{-1})$, then $M = N^{-1}$ is $\mathcal{G}$-open in $(L, *, \mathcal{G})$. Therefore, $r_t^{-1}(M) \in \mathcal{G}SO(L, \mathcal{G})$ implies $(M * t^{-1})^{-1} \in \mathcal{G}SO(L, \mathcal{G}^{-1})$, $t * M^{-1} \in \mathcal{G}SO(L, \mathcal{G}^{-1})$, $t * N \in \mathcal{G}SO(L, \mathcal{G}^{-1})$, $l_{t^{-1}}^{-1}(N) \in \mathcal{G}SO(L, \mathcal{G}^{-1})$. Hence, l_t is $\mathcal{G}$-semi continuous in $(L, *, \mathcal{G}^{-1})$. Similarly, r_t is $\mathcal{G}$-semi continuous in $(L, *, \mathcal{G}^{-1})$. Let M' is $\mathcal{G}$-open in $(L, *, \mathcal{G}^{-1})$, then $N' = M'^{-1}$ is $\mathcal{G}$-open in $(L, *, \mathcal{G})$. So $i^{-1}(N') = M' \in \mathcal{G}SO(L, \mathcal{G})$. Therefore, $M'^{-1} = i^{-1}(M') \in \mathcal{G}SO(L, \mathcal{G}^{-1})$. Hence, inverse mapping is $\mathcal{G}$-semi continuous in $(L, *, \mathcal{G}^{-1})$. □

Theorem 2.6. *Quasi $\mathcal{G}$-s-topological IP-loop is a quasi $\mathcal{G}$-s-homogeneous space.*

Proof. Suppose that $(L, *, \mathcal{G})$ is a quasi $\mathcal{G}$-s-topological loop and $n = t^{-1} * m$, for $m, n, t \in L$. So $r_n(t) = t * n = t * (t^{-1} * m) = m$. □

Theorem 2.7. *Free product of a $\mathcal{G}$-open subset with any subset in a quasi $\mathcal{G}$-s-topological topological IP-loop is $\mathcal{G}$-semi open.*

Proof. Let $(L, *, \mathcal{G})$ be a quasi $\mathcal{G}$-s-topological topological IP-loop, $M \in O(L)$ and $N \subseteq L$. If $n \in N$ and $t \in M * n$ then $t = m * n$ for some $m \in M$. This implies that $m = t * n^{-1} = r_{n^{-1}}(t)$ for fixed $n^{-1} \in L$, so, there is a $\mathcal{G}$-semi open set S_t satisfying $r_{n^{-1}}(S_t) \subseteq M$, $S_t * n^{-1} \subseteq M$, $S_t \subseteq M * n$. This implies that t is $\mathcal{G}$-semi interior point of $M * n$. Hence, $M * n \in \mathcal{G}SO(L)$. Therefore, $M * N = \cup_{n \in N} M * n$ is $\mathcal{G}$-semi open. □

Corollary 2.8. *Left and right translation are quasi $\mathcal{G}$-s homeomorphism in a quasi $\mathcal{G}$-s-topological IP-loop.*

Theorem 2.9. *Multiplication mapping is jointly $\mathcal{G}$-semi open in quasi $\mathcal{G}$-s-topological IP-loop.*

Proof. Suppose that $(L, *, \tau)$ is a quasi $\mathcal{G}$-s-topological IP-loop and $f : L \times L \to L$ be a multiplication mapping defined by $f(M \times N) = MN$. If $M, N \in \mathcal{G}O(L)$, then $M \times N$ is $\mathcal{G}$-open in $L \times L$. So $f(M \times N) = MN$ is $\mathcal{G}$-semi open in L by Theorem 2.7. Hence, f is $\mathcal{G}$-semi open. □

Theorem 2.10. *Every sub-loop of a quasi $\mathcal{G}$-s-topological IP-loop containing a non-void $\mathcal{G}$-open subset is $\mathcal{G}$-semi open.*

Proof. Suppose that $(L, *, \mathcal{G})$ is a quasi $\mathcal{G}$-s-topological IP-loop and N is its sub-loop. M is a non-empty $\mathcal{G}$-open set in L with $M \subseteq N$. Then by Corollary 2.8, for every $n \in N$ the set $r_n(M) = M * n$ is $\mathcal{G}$-semi open in $(L, *, \mathcal{G})$. Therefore, $N = \cup_{n \in N}(M * n)$ is $\mathcal{G}$-semi open in L. □

Corollary 2.11. *Every $\mathcal{G}$-open sub-loop in a quasi $\mathcal{G}$-s-topological IP-loop is $\mathcal{G}$-semi closed.*

Proof. Let S be a $\mathcal{G}$-open sub-loop of a quasi $\mathcal{G}$-s-topological IP-loop L. Since, l_t is $\mathcal{G}$-semi open, so $t * S$ is $\mathcal{G}$-semi open $\forall t \in L - S$. Therefore, $F = \cup_{t \in L-S}(t * S)$ is $\mathcal{G}$-semi open. Thus, $S = L - F$ is $\mathcal{G}$-semi closed. □

Theorem 2.12. *If a homomorphism f from L to L' (quasi $\mathcal{G}$-s-topological IP-loops) is $\mathcal{G}$-irresolute at e_L, then is $\mathcal{G}$-semi continuous on L.*

Proof. Let for all $t_1 \in L$, suppose $M_2' \in \mathcal{G}O(L', t_2)$ where $t_2 = f(t_1)$. Then, there is $M_1' \in \mathcal{G}SO(L', e_{L'})$ such that $l'_{t_2}(M_1') = t_2 * M_1' \subseteq M_2'$. This implies $f(M_1) \subseteq M_1'$ for some $M_1 \in \mathcal{G}SO(L, e_L)$. So, $t_1 * M_1 \in \mathcal{G}SO(L, t_1)$ and $f(t_1 * M_1) = f(t_1) * f(M_1) = t_2 * f(M_1) \subseteq t_2 * M_1' \subseteq M_2'$. Hence, f is $\mathcal{G}$-semi continuous on L. □

Remark 2.13. *Above result also holds for a homomorphism from a quasi $\mathcal{G}$-irresolute topological IP-loop to a quasi $\mathcal{G}$-s-topological IP-loop.*

Theorem 2.14. *Let $(L, *, \mathcal{G})$ be a quasi $\mathcal{G}$-s-topological IP-loop. Then for every $\mathcal{G}$-open neighbourhod M of e and for each $S \subseteq L$, $\mathcal{G}sCl(S) \subseteq S * M$.*

Proof. Consider $t \in \mathcal{G}sCl(S)$ and for a $\mathcal{G}$-open neighbourhood N of e, $t * N$ is $\mathcal{G}$-semi open neighbourhood of t. So, there exists $s \in S \cap t * N$, that is $s \in t * N$. It gives $s = t * n$ for some $n \in N$. Therefore, $t = s * n^{-1} \in s * N^{-1} \subseteq S * M$ for $M \in \mathcal{G}O(L, e)$. □

Theorem 2.15. *Let μ_e is family of $\mathcal{G}$-open sets containing e in quasi $\mathcal{G}$-s-topological IP-loop $(L, *, \mathcal{G})$. Then for each $S \subseteq L$, $\mathcal{G}sCl(S) = \cap\{S * M : M \in \mu_e\}$.*

Proof. For $t \notin \mathcal{G}sCl(S)$ implies $t \notin S$, and there exists $N \in \mu_e$ such that $t * N \cap S = \phi$. As $M \in \mu_e$ satisfies $M^{-1} \subseteq N$. Therefore, $t * M^{-1} \cap S = \phi$, $\{t\} \cap S * M = \phi$. So $t \notin S * M$. Moreover, by Theorem 2.14 $\mathcal{G}sCl(S) \subseteq S * M$. Thereupon, $\mathcal{G}sCl(S) = \cap\{S * M : M \in \mu_e\}$. □

Theorem 2.16. *$\mathcal{G}$-semi closure of inverse of every $\mathcal{G}$-open neighbourhood of identity element in a quasi $\mathcal{G}$-s-topological IP-loop contain in free product of its inverse with itself.*

Proof. Let M^{-1} is a $\mathcal{G}$-open neighbourhood of e and $t \in \mathcal{G}sCl(M)$. As $t * M^{-1}$ is $\mathcal{G}$-semi open neighbourhood of t so it meets M. Furthermore, there exists $r \in M$ such that $r = t * s^{-1}$ for $s \in M$. Then $t = r * s \in M * M$ □

Theorem 2.17. *For $N \in \mathcal{G}O(L,e)$ and $M \subseteq L$ in quasi $\mathcal{G}$-s-topological IP-loop $(L,*,\mathcal{G})$ such that $N^4 \subseteq M$ and N is symmetric. If $S \subseteq L$ is M-semi disjoint, then $\{s*N : s \in S\}$ being the family of $\mathcal{G}$-semi open sets is $\mathcal{G}$-semi discrete in L.*

Proof. To prove $\mathcal{G}$-semi discreteness of $\{s*N : s \in S\}$, it is sufficient to prove that for every $t \in L$, $t*N$ intersects at most one of $\{s*N : s \in S\}$. Assume contrarily that, for some $t \in L$, there exists $r, s \in S$ such that $t*N \cap s*N \neq \phi$ and $t*N \cap r*N \neq \phi$, then $t^{-1}*s \in N^2$ and $t \in r*N^2$. Here $s = t*(t^{-1}*s) \in r*N^4 \subseteq r*M$. Hence, $s \in r*M$. A contradiction to the fact that S is M-semi disjoint. So, $\{s*N : s \in S\}$ being the family of $\mathcal{G}$-semi open sets is $\mathcal{G}$-semi discrete in L. □

3. Quasi Generalized s-topological loop

Definition 3.1. *A triplet $(L,*,\mathcal{G})$ is said to be a quasi $\mathcal{G}$-s-topological loop, if the conditions given below are satisfied:*

- *$(L,\mathcal{G})$ is a $\mathcal{G}$-topological space.*
- *$(L,*)$ is a loop.*
- *Multiplication mapping is separately $\mathcal{G}$-semi continuous in $(L,*,\mathcal{G})$.*
- *Right and left inverse mappings are $\mathcal{G}$-semi continuous in $(L,*,\mathcal{G})$.*

In the following example, different type of $\mathcal{G}$-topologies are defined and discussed to form quasi $\mathcal{G}$-s-topological loop.

Example 3.2. *Loop $L_5(4)$ of order 6 with the $\mathcal{G}$-topologies $\mathcal{G}_1 = \{\phi\}$ and $\mathcal{G}_2 = \{P(L_5(4))\}$ is a quasi $\mathcal{G}$-s-topological loop. However, with the $\mathcal{G}$-topology $\mathcal{G}_3 = \{\phi, \{e\}, \{1,2\}, \{3,4\}, \{e,1,2\}, \{e,3,4\}, \{1,2,3,4\}, \{e,1,2,3,4\}\}$, $(L_5(4),*,\mathcal{G}_3)$ is not a quasi $\mathcal{G}$-s-topological loop (see Table 2).*

TABLE 2. Loop $L_5(4)$ of order 6

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	5	4	3	2
2	2	3	e	1	5	4
3	3	5	4	e	2	1
4	4	2	1	5	e	3
5	5	4	3	2	1	e

Note: Klein four-group with the topology $\tau = \{\phi, \{e,a\}, \{b,ab\}, K_4\}$ form a quasi $\mathcal{G}$-s-topological loop.

Theorem 3.3. *In a quasi $\mathcal{G}$-s-topological loop, $\mathcal{G}$-openness is sufficient condition for a sub-loop to be a quasi $\mathcal{G}$-s-topological loop.*

Proof. Suppose that $(L,*,\mathcal{G})$ is a quasi $\mathcal{G}$-s-topological loop and L' is a $\mathcal{G}$-open sub-loop of L. So, for every $\mathcal{G}$-open neighbourhood M_1 in L with $t \in L'$ and $M_1 \subseteq L'$, $l_t^{-1}(M_1) \in \mathcal{G}SO(L)$ and $l_t^{-1}(M_1) \subseteq L'$, gives $l_t^{-1}(M_1) \in \mathcal{G}SO(L')$. Clearly, l_t is $\mathcal{G}$-semi continuous in L'. In the same way, $r_t, i_L and i_R$ are $\mathcal{G}$-semi continuous in L'. □

Lemma 3.4. *Left and right inverse mappings are quasi $\mathcal{G}$-s homeomorphisms in a quasi $\mathcal{G}$-s-topological loop.*

Remark 3.5. *In a loop with $\mathcal{G}$-topology, if left(right) inverse mapping is $\mathcal{G}$-semi open, then right(left) inverse mapping is $\mathcal{G}$-semi continuous.*

Theorem 3.6. *For a $\mathcal{G}$-semi discrete sub-loop $L^{'}$, $\mathcal{G}sCl(L^{'}) = L$, where L is a sub-loop and $(L, *, \mathcal{G})$ is a quasi $\mathcal{G}$-s-topological loop having $\mathcal{G}$-semi open left translation.*

Proof. Let L' be a $\mathcal{G}$-discrete sub-loop of a quasi $\mathcal{G}$-s-topological loop $(L, *, \mathcal{G})$. For $t_1, t_2 \in \mathcal{G}sCl(L')$, if M_1 and M_2 are $\mathcal{G}$-open neighborhoods of t_1 and t_2, then for $(t_1)_L^{-1}$, $(t_2)_L^{-1} \in L$, $l_{(t_1)_L^{-1}}(M_1) = (t_1)_L^{-1} * M_1$, and $l_{(t_2)_L^{-1}}(M_2) = (t_2)_L^{-1} * M_2$ are $\mathcal{G}$-semi open neighborhoods of e. So $((t_1)_L^{-1} * M_1) \cap L' \neq \phi$ and $((t_2)_L^{-1} * M_2) \cap L' \neq \phi$. Therefore, $((t_1 * (t_2)_L^{-1}) * ((t_1)_L^{-1} * M_1) \cap (t_1 * (t_2)_L^{-1}) * L') \cup ((t_1 * (t_2)_L^{-1}) * ((t_2)_L^{-1} * M_2) \cap (t_1 * (t_2)_L^{-1}) * L') \neq \phi$. By using distributive law, $((t_1 * (t_2)_L^{-1}) * ((t_1)_L^{-1} * M_1) \cup ((t_1 * (t_2)_L^{-1}) * ((t_2)_L^{-1} * M_2))) \cap ((t_1 * (t_2)_L^{-1}) * L') \neq \phi$. That is, $H \cap ((t_1 * (t_2)_L^{-1}) * L') \neq \phi$, where $H = (t_1 * (t_2)_L^{-1}) * ((t_1)_L^{-1} * M_1) \cup ((t_1 * (t_2)_L^{-1}) * ((t_2)_L^{-1} * M_2))$ is a $\mathcal{G}$-semi open neighborhood of $t_1 * (t_2)_L^{-1}$. Hence, $\forall t_1, t_2 \in \mathcal{G}sCl(L')$ implies $t_1 * (t_2)_L^{-1} \in \mathcal{G}sCl(L')$. Accordingly, $t_1 * (t_2)_R^{-1} \in \mathcal{G}sCl(L')$. □

Remark 3.7. *The sufficient condition for a sub-loop of a quasi $\mathcal{G}$-s-topological loop with $\mathcal{G}$-semi open left translation to be $\mathcal{G}$-semi clopen is its $\mathcal{G}$-openness.*

Theorem 3.8. *In a quasi $\mathcal{G}$-s-topological loop, inverses and free product with finite (countable) set of a $\mathcal{G}$-semi compact ($\mathcal{G}$-semi-Lindelof) set is $\mathcal{G}$-compact ($\mathcal{G}$-Lindelof).*

Proof. Let M is $\mathcal{G}$-semi compact ($\mathcal{G}$-semi-Lindelof), this implies $i_L(M) = M_L^{-1}$, $i_R(M) = M_R^{-1}$ are $\mathcal{G}$-compact ($\mathcal{G}$-Lindelof) [3, 14]. Moreover, let N is finite (countable) and for all $n \in N$, $N.M$ as a finite union of the sets $nM = l_n(M)$ is $\mathcal{G}$-compact ($\mathcal{G}$-Lindelof) [3, 14]. □

Theorem 3.9. *Let $(L, *, \mathcal{G})$ be a quasi $\mathcal{G}$-s-topological loop, and $\mathcal{G}.S.C.(e)$ is $\mathcal{G}$-open. Then $\mathcal{G}.S.C.(e)$ is sub-loop, if for all $t \in \mathcal{G}.S.C.(e)$, $r_{(t)_L^{-1}}$ and $r_{(t)_R^{-1}}$ are $\mathcal{G}$-open.*

Proof. Let $t \in \mathcal{G}.S.C.(e)$, then $(\mathcal{G}.S.C.(e)).(t)_L^{-1} = r_{(t)_L^{-1}}(\mathcal{G}.S.C.(e))$ is $\mathcal{G}$-semi connected and contain e [13]. Hence, $(\mathcal{G}.S.C.(e)).(t)_L^{-1} \subseteq \mathcal{G}.S.C.(e)$. As $(t)_L^{-1} \in (\mathcal{G}.S.C.(e))_L^{-1}$, therefore, $(\mathcal{G}.S.C.(e)).(\mathcal{G}.S.C.(e))_L^{-1} \subseteq \mathcal{G}.S.C.(e)$. Similarly, $(\mathcal{G}S.C.(e)).(\mathcal{G}.S.C.(e))_R^{-1} \subseteq \mathcal{G}.S.C.(e)$. □

4. Conclusion

We have investigated and characterized some new properties of quasi $\mathcal{G}$-s-topological loops by enfeebling the condition of continuity and openness. We have introduced an algebraic structure with above mentioned conditions which has not been discussed yet. We have discussed that the property of being quasi $\mathcal{G}$-s-topological IP-loop is preserved under topological conjugation, and proved that multiplication mapping is jointly $\mathcal{G}$-semi open in a quasi $\mathcal{G}$-s-topological IP-loop. Moreover, some properties of $\mathcal{G}$-semi compact ($\mathcal{G}$-semi-Lindelof), and $\mathcal{G}$-open neighbourhood of identity has also been discussed. Factually, these results have a useful contribution in the field of topological algebra.

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