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Micropolar Fluid Flow with Heat Generation through a Porous Medium

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Abstract.: The incompressible, steady and laminar micropolar fluid flow through a resistive porous medium between channel walls with mass and heat deportation, by considering the effect of heat generation, is studied numerically. The relevant PDEs governing the flow, heat and concentration are transmuted into nonlinear ordinary ones by employing the powerful tool of similarity transformation and consequently, eight parameters appeared in the final model. Afterward, Quasi-linearization (QL) technique is exploited to solve the relevant nonlinear coupled ODEs. The repercussion of preeminent parameters on flow, heat and mass transfer are deliberated and shown through graphs and tables. The effect of the heat generation is to enhance the rate of heat transfer at both walls of the channel.

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Key Words: Heat Generation, Micropolar Fluid, Porous Medium, Quasi-linearization.

1. Introduction

The rotating micro components of micropolar fluids perturb the hydrodynamics of the fluid flow and this mechanism provides a basis for successful employment of micropolar fluids in modern engineering and bio-technology. Micropolar fluids consist of micro-structured polymeric additives and are exemplified as non- Newtonian fluids. The micropolar fluids can express the flow behaviour of ferro-liquids, paints, exotic lubricants, colloidal fluids, polymeric materials, animal blood, etc. Eringen [1, 2] was the innovator in introducing the micropolar fluids for which the conventional theory of Navier's Stokes was inconsequential. Afterwards, the research community [3, 4, 5, 6] extended this worked towards an inclusive review. In micropolar fluid flow model, an additional transport equation is essentially solved with the usual equations of continuity and momentum. Articles by Ariman et al. [7, 8] epitomize the more theory and applications about micropolar fluids.

Various research scholars have deliberated the different types of fluid flows through channels and parallel plates over different geometries. The impact of radially applied magnetic field on velocity and temperature of a Carreau-Yasuda fluid flowing through a wavy wall was explored by Abbasi et al. [9]. They noticed that the C-Y fluid enhances temperature and reduces velocity with the change in magnetic field. Fusi and Farina [10] scrutinized the impact of magnetic field on temperature and velocity in Bingham Peristaltic fluid and this same fluid was examined in micro channel and permeable tube [11, 12]. The effect of thermal radiation as well as chemical reaction on heat/mass transfer over a vertically moving plate was evaluated by Mohamed and Abo-Dahab [13]. Hayat et al. [14] scrutinized the influence of magnetic force on peristaltic movement of fluid flowing through a curved channel by considering the ratio of wavelength and channel-width so small that can be assumed uniform for pressure of fluid. Khan et al. [15] investigated the viscous flow in porous channel by using Optimal Homotopy Asymptotic Method (OHAM).

During the last few decades researchers have definitely played a pivotal role in micropolar fluid flow, mass and heat transfer through channels. Fakour et al. [16] solved the micropolar fluid, mass and heat transfer problem analytically and numerically. They explained the Least Square Method (LSM) and employed this method to solve the nonlinear ODEs. The results obtained from LSM method were corelated with those achieved from RK fourth order technique. Mirgolbabaee et al. [17] and Sheikholeslami et al. [18] also discussed the same problem by using AGM and HPM (Homotopy Perturbation Method) respectively. The results acquired from both the methods were equated with the results obtained from Runge-kutta fourth order scheme. Ali and Ashraf [19] numerically explored the heat transfer in micropolar fluid flow through a channel by taking one wall of the channel dwindling and other static. Ziabakhsh and Domairry [20] interpreted micropolar fluid flow and mass transport in a porous channel by using DTM (Differential Transformation Method). The micropolar fluid flow through a channel having permeable walls was expounded by Mirzaaghaian and Ganji [21]. They compared the results with the numerical method and came to know that the temperature and concentration are very little bit affected by the Reynolds number. Nwabuzor et al. [22] explained the magneto-hydrodynamic micropolar fluid flow in a porous medium under the effects of heat generation, viscous dissipation, chemical reaction and thermal radiation. Ashraf et al. [23] numerically probed the flow of micropolar fluid through porous medium in a channel. After converting nonlinear PDEs into respective ODEs, Successive over Relaxation (SOR) parameter method along with finite difference discretization was applied. The results were compared with those flourished by Shrestha and Terrill [24]. They reported that the micropolar fluid enhance the couple stress and declines the skin friction coefficientat at both walls of the channel. Singh and Kumar [25] numerically examined the mass and heat transfer in micropolar fluid flow by assuming the viscous effects and thermal radiation through a permeable channel. Ahmad et al. [26] numerically explored the heat and mass transfer flow of an incompressible micropolar fluid with allowance for viscous dissipation through a resistive porous medium between channel walls. They solved fully coupled nonlinear differential equations by means of quasi-linearization. It was found that the effect of viscous dissipation is to increase the heat and mass transfer rates on both walls of the porous channel.

Specifically, the problems related to heat generation within fluid in a porous medium are of extraordinary commonsense. The practical significance of such problems can be observed in geophysical flows, cooling of underground liquid, recovery of petroleum resources, fiber and granular insulations, electric cables, environmental impact of buried heat generating waste and chemical catalytic reactors, solidification of costing, storage of nuclear waste materials and ground water pollution. The flow of micropolar fluid under the influence of heat generation or absorption has been considered by various authors [27, 28, 29]. The present investigation has utilization in industry and biotechnology e.g. air circulation in respiratory system and binary gas diffusion, drying of porous solid surfaces, combustion process in rocket motors, etc [30]. The intent of study this investigation is to analyze the numerical resolution of the flow, heat and mass transfer through a porous medium in channel walls. By employing the suitable non-dimensional coordinates, nonlinear PDEs are transformed into ordinary ones which are then solved by means of quasi-linearization method along with central FD discretization. The impacts of the concerned parameters on concentration, microrotation, flow velocity and temperature are argued and visualized through tables and graphs.

2. Description of Physical Model

The fluid flow is considered in a resistive porous medium between channel walls through which fluid is uniformly injected or removed with a constant speed v_0. $T_1$and $C_1$are temperature and solute concentration at lower channel wall and upper channel wall has temperature $T_2$and solute concentration C_2respectively as appeared schematically in Fig. 1. The channel walls and x-axis are taken parallel whereas walls are placed at $y = \pm h$, where the total width of the channel is $2h$.

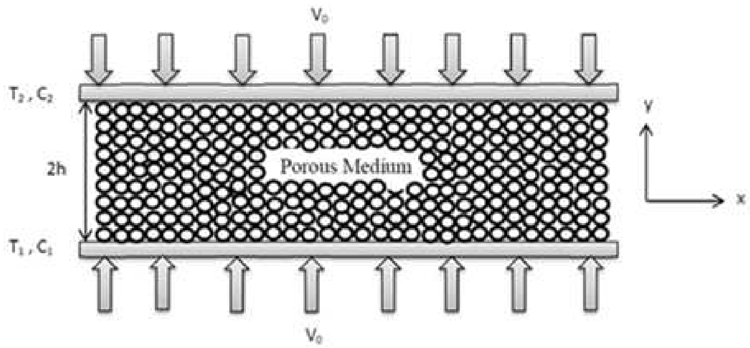

FIGURE 1. Geometry of the problem

The constitutive equations governing the motion of the micropolar fluid as given by Eringen [1] and Ashraf et al. [31] are:

$$\begin{aligned} \frac{\partial p}{\partial t} + \nabla.(\rho V) &= 0 \\ (\lambda + 2\mu + k)\,\nabla(\nabla.V) - (\mu + k)\nabla \times \nabla \times V + k\nabla \times v - \nabla p + \rho f &= \rho V \\ (\alpha + \beta + \gamma)\,\nabla(\nabla.v) - \gamma(\nabla \times \nabla \times v) + k\nabla \times V - 2kv - pl &= \rho j v \end{aligned}$$

where v is the microrotation, V is the fluid velocity vector, ρ is the density, l and f are body couple per unit mass and body force respectively, p is the pressure, j is the micro-inertia, α, β, γ, λ, μ, k are viscosity coefficients (or the material constants), where dot specifies the material derivative. Here the microrotation vector v and the velocity vector V are unknown. Following [16, 25], these equations of flow, heat and concentration in case of porous medium in component form are:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{2. 1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = (\mu + k)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial p}{\partial x} - \frac{\mu + k}{k^*}u + k\frac{\partial N}{\partial y} \tag{2. 2}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = (\mu + k)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial y} - \frac{\mu + k}{k^*}v - k\frac{\partial N}{\partial x} \tag{2. 3}$$

$$\rho\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = -\frac{k}{j}\left(2N + \frac{\partial u}{\partial x} - v\frac{\partial v}{\partial y}\right) + \frac{\mu_s}{j}\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) \tag{2. 4}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_1\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q(x)(T - T_2) \tag{2. 5}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D^*\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \tag{2. 6}$$

where u is the respective velocity component taken along x-axis and v is the respective velocity component taken along y-axis respectively. Moreover, μ, k^*, p, ρ, N, C_p, k, j, $\mu = (\mu + k/2)$, k_1, D^*, T, C and $Q(x)$ are the dynamic viscosity, darcy permeability, pressure, fluid density, angular velocity, specific heat constant, vortex viscosity, microinertia, microrotation viscosity, thermal conductivity, molecular diffusivity, temperature, concentration of the fluid and heat generation coefficient respectively. The expression $''Q(x)''$ for heat generation coefficient is $Q(x) = hA$, here $A$is surface area where heat transfer takes place and h is heat transfer coefficient. Moreover, $h = \frac{q}{\nabla T}$ where q is heat flux and ∇T is the difference in temperatures between the solid surface and surrounding fluid area. The boundary conditions at $y = \pm h$ may be written as:

$$y = -h : u = 0, v = v_0, N = 0, T = T_1, C = C_1 \tag{2. 7}$$

$$y = +h : u = 0, v = -v_0, N \longrightarrow 0, T \longrightarrow T_1, C \longrightarrow C_1 \tag{2. 8}$$

Following similarity variables are defined to alter the governing PDEs in nonlinear ODEs:

$$\eta = \frac{y}{h}, \psi = -v_0 x f(\eta), N = \frac{v_0 x}{h^2} g(\eta), \theta(\eta) = \frac{T - T_2}{T_1 - T_2}, \phi(\eta) = \frac{C - C_2}{C_1 - C_2} \quad (2.\ 9)$$

Here, $T_2 = T_1 - Ax$ and $C_2 = C_1 - Bx$, where A and $B$are fixed. Entreating these similarity variables into equations (2. 2)-(2. 6), we obtain the set of ODEs:

$$-Reff^{'''} + Ref^{'} f^{''} - \varepsilon(1 + C_1) f^{''} + (1 + C_1) f^{iv} - C_1 g^{''} = 0 \quad (2.\ 10)$$

$$C_2 g^{''} + C_1 (f^{''} - 2g) - ReC_3 (fg^{'} - f^{'} g) = 0 \quad (2.\ 11)$$

$$\theta^{''} + Pe_h (f^{'} \theta - f\theta^{'} + H\theta) = 0 \quad (2.\ 12)$$

$$\phi^{''} + Pe_m (f^{'} \phi - f\phi^{'}) = 0 \quad (2.\ 13)$$

with respect to the boundary conditions:

$$\begin{aligned} \eta &= -1 : f = 0, f^{'} = 0, g = 0, \theta = 1, \phi = 1 \quad (2.\ 14) \\ \eta &= 1 : f = -1, f^{'} = 0, g = 0, \theta = 0, \phi = 0 \end{aligned}$$

whereas the parameters involved in the nonlinear system of coupled equations (2. 10)-(2. 13) are defined as:

$$\begin{aligned} \varepsilon &= \frac{h^2}{k^*}, C_1 = \frac{k}{\mu}, Re = \frac{v_0}{\nu} h, C_2 = \frac{\mu_s}{\mu h^2}, \Pr = \frac{\nu \rho C_p}{k_1} \\ C_3 &= \frac{j}{h^2}, SC = \frac{\nu}{D^*}, Pe_m = \frac{v_0 h}{D^*}, Pe_h = \frac{v_0 h \rho C_p}{k_1}, H = \frac{Q(x) h}{v_0 \rho C_p} \end{aligned}$$

where ε, C_1, Re, C_2, $\Pr$, C_3, SC, Pe_m, Pe_h and H are the porosity parameter, vortex viscosity, Reynolds number, spin-gradient viscosity parameter, Prandtl number, microinertia density, the Schmidt number, Peclet numbers for the diffusion of mass and heat and heat generation parameter respectively. Nu_x and Sh_x (Nusselt and Sherwood numbers) are the parameters of primary interest and these may defined as:

$$Nu_x = \frac{q^{''}(x)}{(T_1 - T_2) k_1}|_{y=-h} = -\theta^{'}(-1), Sh_x = \frac{m^{''}(x)}{(T_1 - T_2) k_1}|_{y=-h} = -\phi^{'}(-1) \quad (2.\ 15)$$

where $m^{''}$and $q^{''}$express the mass flux and the local heat flux respectively.

3. Numerical Analysis

Unlike other numerical techniques, quasi-linearization is a well renowned scheme to find the approximate solutions of nonlinear differential equations with very quick convergence. The quasi-linearization method plays a fundamental role to solve the complex nonlinear problems numerically. In addition, one can comment that this technique is a modified form of Newton's method and it can be applied for both boundary and initial value problems. Mostly, the problems comprising nonlinearities (convex or concave) are treated by quasi-linearization method. Due to its numerous usage and implementations, the quasi-linearization technique is quite marvelous providing an ancestry approach to acquire the unique solutions of highly nonlinear boundary value problems. The quasi-linearization method was initially spearheaded by Bellman [32] and Bellman & Kalaba [33]. Lakshmikantham et al. [34, 35, 36] have developed the generalized form of this method and exploited this technique to a wide range of nonlinear problems.

To initiate the numerical computation, quasi-linearization technique is utilized after assembling the sequences $\{f^{(k)}\}, \{g^{(k)}\}, \{\theta^{(k)}\}$and $\{\phi^{(k)}\}$which provide the numerical solution of Eqs. (2. 10)-(2. 13) respectively. In order to detain the terms of first order only, we linearize Eq. (2. 10) that generates $\{f^{(k)}\}$. Initially, we put:

$$-Reff^{'''}+Ref^{'}f^{''}-\varepsilon(1+C_1)f^{''}+(1+C_1)f^{iv}-C_1g^{''}=N(f,f^{'},f^{''},f^{'''},f^{iv}) \quad (3.\ 16)$$

which leads to:

$$\begin{aligned} N(f,f^{'},f^{''},f^{'''},f^{iv}) + \left(f^{(k+1)}-f^{(k)}\right)\frac{\partial N}{\partial f^{(k)}} + \left(f^{(k+1)'}-f^{(k)'}\right)\frac{\partial N}{\partial f^{(k)'}} + \\ \left(f^{(k+1)''}-f^{(k)''}\right)\frac{\partial N}{\partial f^{(k)''}} + \\ \left(f^{(k+1)'''}-f^{(k)'''}\right)\frac{\partial N}{\partial f^{(k)'''}} + \left(f^{(k+1)^{iv}}-f^{(k)^{iv}}\right)\frac{\partial N}{\partial f^{(k)^{iv}}} = 0 \end{aligned} \quad (3.\ 17)$$

After solving (3. 16) and (3. 17), we get:

$$\begin{aligned} (1+C_1)f^{(k+1)^{iv}} - Ref^{(k)}f^{(k+1)'''} + \left[-\varepsilon(1+C_1)+Ref^{(k)'}\right]f^{(k+1)''} + \\ Ref^{(k)''}f^{(k+1)'} = Ref^{(k)'}f^{(k)''} - Ref^{(k)}f^{(k)'''} + C_1g^{(k)''} \end{aligned} \quad (3.\ 18)$$

Now, we might replace the derivatives in Eq. (3. 18) with central differences, generating $\{f^{(k)}\}$sequence. Moreover, to produce $\{g^{(k)}\}, \{\theta^{(k)}\}$and $\{\phi^{(k)}\}$, the linear Eqs. (2. 11)-(2. 13) can be written as:

$$C_2g^{(k+1)''} + C_1(f^{(k)''} - 2g^{(k+1)}) - ReC_3(f^{(k)}g^{(k+1)'} - f^{(k)'}g^{(k+1)}) = 0 \quad (3.\ 19)$$

$$\theta^{(k+1)''} + Pe_h(f^{(k)'}\theta^{(k+1)} - f^{(k)}\theta^{(k+1)'} + H\theta^{(k+1)}) = 0 \quad (3.\ 20)$$

$$\phi^{(k+1)''} + Pe_m(f^{(k)'}\phi^{(k+1)} - f^{(k)}\phi^{(k+1)'}) = 0 \quad (3.\ 21)$$

The following iterative procedure is operated to initiate the numerical process.

- The BCs in Eq. (2. 14) are satisfied by the provided initial guesses $f^{(0)}, g^{(0)}, \theta^{(0)}$and $\phi^{(0)}$.
- Using known $f^{(1)}$, the system of equations (3. 19)-(3. 21) is discretized by finite difference technique and then solved to obtain $g^{(1)}, \theta^{(1)}$and $\phi^{(1)}$.
- The new suggested guesses are $f^{(1)}$, $g^{(1)}, \theta^{(1)}$and $\phi^{(1)}$and then, procedure is repetitive until $\{f^{(k)}\}, \{g^{(k)}\}, \{\theta^{(k)}\}$and $\{\phi^{(k)}\}$converge to $f, g, \theta$and ϕrespectively.
- The four sequences are repeatedly generated as far as

$$\max\left(||f^{(k+1)} - f^{(k)}||, ||g^{(k+1)} - g^{(k)}||, ||\theta^{(k+1)} - \theta^{(k)}||, ||\phi^{(k+1)} - \phi^{(k)}||\right) < 10^{-8}$$

4. RESULTS AND DISCUSSIONS

We obtain the numerical solution of the nonlinear coupled ODEs (2. 10)-(2. 13) subject to the respective BCs (2. 14) by means of quasi-linearization method along with finite-difference discretization for a collection of estimations of the micropolar material parameters C_1, C_2 and C_3, the porosity parameter ε, the Reynolds number Re, the Peclet numbers Pe_m and Pe_h the heat generation parameter H. An effort is made to inspect the influences of the parameters on the flow velocity $F^{'}(\eta)$, microrotation $G(\eta)$, concentration $\phi(\eta)$ and temperature $\theta(\eta)$ as well as on $F^{''}(\pm1)$, $\theta^{'}(\pm1)$and $\phi^{'}(\pm1)$. The step-size η alongwith edge of the boundary layer are accommodated in a best way that the flow, temperature, microrotation and concentration profiles show asymptotic behaviour. A graphical comparison is correlated with the previously accomplished study and examined to be in an exceptional agreement. Our graph may exactly be the same as in [25] if we assume other effects in the flow as were taken in [25].

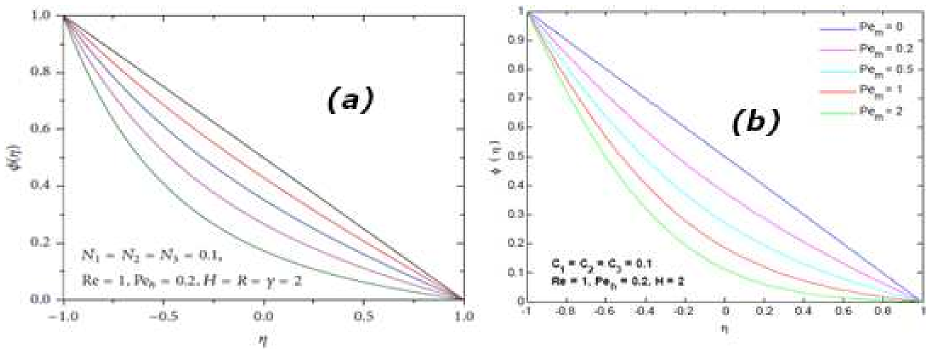

FIGURE 2. Concentration profile $\phi(\eta)$ for various values of Pe_m (a) Ref. [25] and (b) Present.

Table 1 specifies that our numerical results converge in a best way with decreasing values of step-size η and it confirms the accuracy of our numerical procedure. The values of micropolar material parameters C_1, C_2 and C_3 for the five cases are given in Table 2 and these values have been utilized in Table 3 as well as in Figures 3 and 4. The first case $(C_1 = C_2 = C_3 = 0)$ relates with the Newtonian fluid whereas the other ones are taken randomly to find their effects as predicted in the reference articles [37, 38, 31, 39]. From Table 3, it may be decided that the microplar structure of the fluid causes the decrease in the skin friction as predicted in [40] that the micro constituents of the micropolar fluid cause significant reduction in shear stress near a rigid surface.

TABLE 1. The values of temperatures $\theta(\eta)$ on three grid sizes for $C_1 = 3$, $C_2 = 2$, $C_3 = 1$, $Re = 8$, Pe$_h = 4$, Pe$_m = 6$, $H = 0.5$ and $\varepsilon = 2.5$.

$\theta(\eta)$			
η	1^{st} grid (h=0.01)	2^{nd} grid (h=0.005)	3^{rd} grid (h=0.0025)
-0.8	0.939177	0.939170	0.939168
-0.4	0.627332	0.627339	0.627341
0	0.273001	0.273025	0.273030
0.4	0.072824	0.072847	0.072852
0.8	0.009782	0.009789	0.009790

TABLE 2. Set of values of material parameters.

Case No	C_1	C_2	C_3
1(Newtonian)	0	0	0
2	0.5	0.8	0.3
3	1.0	1.2	0.6
4	1.5	1.6	0.9
5	2.0	2.0	1.2

TABLE 3. Shear stress, heat and mass transfer rate for $Re = -8$, Pe$_h = 4$, Pe$_m = 6$, $H = 0.5$, $\varepsilon = 2.5$ and set of values of C_1, C_2 and C_3.

Case N0	$F^{''}(-1)$	$\theta^{'}(-1)$	$\phi^{'}(-1)$	$F^{''}(1)$	$\theta^{'}(1)$	$\phi^{'}(1)$
1(Newtonian)	-12.2429	-0.3832	-0.7030	12.2429	-0.0433	-0.0064
2	-8.3378	-0.2997	-0.6049	8.3378	-0.0398	-0.0058
3	-6.4923	-0.2457	-0.5445	6.4923	-0.0377	-0.0054
4	-5.5265	-0.2124	-0.5083	5.5265	-0.0365	-0.0052
5	-4.9637	-0.1910	-0.4853	4.9637	-0.0357	-0.0050

TABLE 4. Shear stress, heat and mass transfer rate for $C_1 = 3$, $C_2 = 2$, $C_3 = 1$, $Re = 8$, Pe$_h = 4$, Pe$_m = 6$, $H = 0.5$ and various ε.

ε	$F^{''}(-1)$	$\theta^{'}(-1)$	$\phi^{'}(-1)$	$F^{''}(1)$	$\theta^{'}(1)$	$\phi^{'}(1)$
10	-3.5280	-0.1340	-0.4243	3.5280	-0.0338	-0.00473
20	-4.3761	-0.1801	-0.4708	4.3761	-0.0354	-0.00503
30	-5.0924	-0.2134	-0.5056	5.0924	-0.0367	-0.00525
40	-5.7175	-0.2390	-0.5329	5.7175	-0.0376	-0.00543
50	-6.2752	-0.2595	-0.5553	6.2752	-0.0384	-0.00557

TABLE 5. Shear stress, heat and mass transfer rate for $C_1 = 3$, $C_2 = 2$, $C_3 = 1$, $\varepsilon = 2.5$, Pe$_h = 4$, Pe$_m = 6$, $H = 0.5$ and various Re.

Re	$F^{''}(-1)$	$\theta^{'}(-1)$	$\phi^{'}(-1)$	$F^{''}(1)$	$\theta^{'}(1)$	$\phi^{'}(1)$
7	-2.8017	-0.0878	-0.3789	2.8017	-0.0322	-0.00444
14	-2.5960	-0.0731	-0.3649	2.5960	-0.0318	-0.00436
21	-2.4916	-0.0644	-0.3568	2.4916	-0.0315	-0.00430
28	-2.4321	-0.0589	-0.3518	2.4321	-0.0313	-0.00427
35	-2.3947	-0.0551	-0.3485	2.3947	-0.0312	-0.00425

TABLE 6. Heat transfer rate for $C_1 = 3$, $C_2 = 2$, $C_3 = 1$, $\varepsilon = 2.5$, Pe$_h = 2$, Pe$_m = 6$, $Re = 8$ and various H.

H	$\theta^{'}(-1)$	$\theta^{'}(1)$
0.0	-0.6056	-0.0782
0.8	-0.1214	-0.1490
1.2	0.1772	-0.2150
1.6	0.5411	-0.3239
2.0	1.0198	-0.5202

TABLE 7. Heat transfer rate for $C_1 = 3$, $C_2 = 2$, $C_3 = 1$, $\varepsilon = 2.5$, $H = 0.5$, Pe$_m = 6$, $Re = 8$ and various Pe$_h$.

Pe$_h$	$\theta^{'}(-1)$	$\theta^{'}(1)$
0.0	-0.5000	-0.4999
0.3	-0.4872	-0.3960
0.6	-0.4576	-0.3180
0.9	-0.4189	-0.2579
1.2	-0.3754	-0.2107

TABLE 8. Mass transfer rate for $C_1 = 3$, $C_2 = 2$, $C_3 = 1$, $\varepsilon = 2.5$, $H = 0.5$, Pe$_h = 4$, $Re = 8$ and various Pe$_m$.

Pe$_m$	$\phi^{'}(-1)$	$\phi^{'}(1)$
0.0	-0.5000	-0.4999
0.2	-0.5631	-0.3969
0.4	-0.6018	-0.3204
0.6	-0.6244	-0.2620
1.0	-0.6403	-0.1803

It is glaring from Tables 3 and 5 that the repercussions of material parameters and the Reynolds number declines the skin friction as well as heat and mass transport rates on both the walls of channel while porosity parameter acts oppositely as compared with material parameters and Reynolds number that is apparent from Table 4. Both $\theta^{'}(-1)$ and $\theta^{'}(1)$

enhance for heat generation parameter as envisioned in Table 6 but $\theta^{'}(-1)$and $\theta^{'}(1)$ both diminish for the growing values of the parameter Pe$_h$ as predicted in Table 7. The rate of mass transport increases on lower wall and decrease on upper wall with ascending values of the parameter Pe$_m$ as represented in Table 8. Hence, the results reveal that the micropolar material parameters, the Reynolds number and the porosity parameter very slightly affect the mass transfer rate and porous medium strengthens the skin friction coefficient, mass and heat transfer rates on lower and upper walls. It is also inferring here that the effect of the porous medium on the shear stress is more prominent as related to its effect on mass transfer and heat transfer rates on both walls of the channel. This is due to the fact that the porosity parameter does not appear in the heat and concentration equation. The fixed values of parameters (used in numerical calculation) are given in Tables.

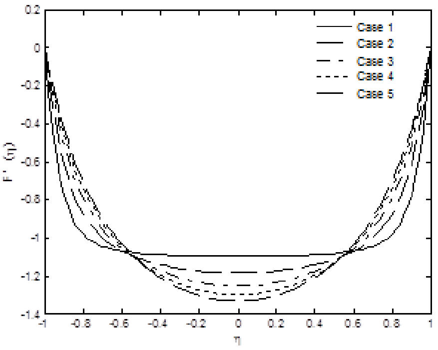

FIGURE 3. $F^{'}(\eta)$ for various values of material parameters.

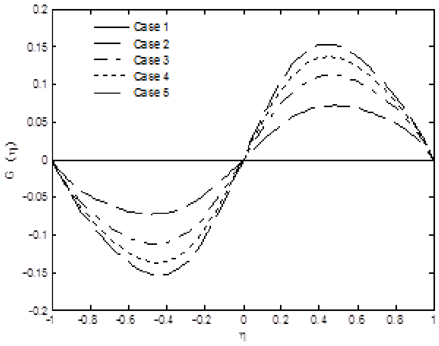

FIGURE 4. $G(\eta)$ for various values of material parameters.

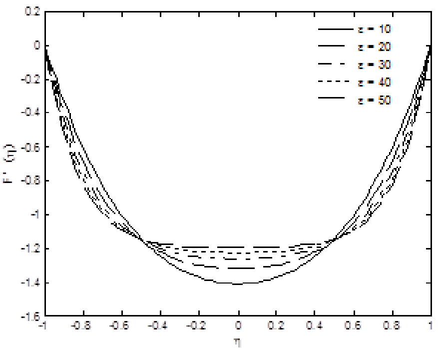

FIGURE 5. $F^{'}(\eta)$ for various values of ε.

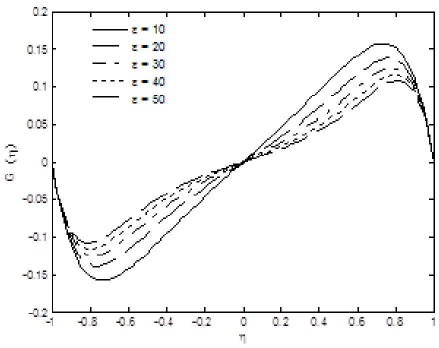

FIGURE 6. $G(\eta)$ for various values of ε.

The streamwise velocity $F^{'}(\eta)$and the angular velocity $G(\eta)$ are represented in Figs. $3-6$ for a variety of micropolar material parameters values and porosity parameter values

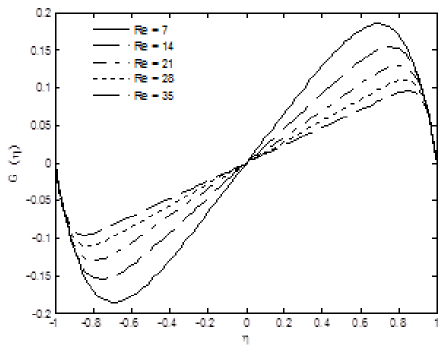

FIGURE 7. $G(\eta)$ for various values of Re.

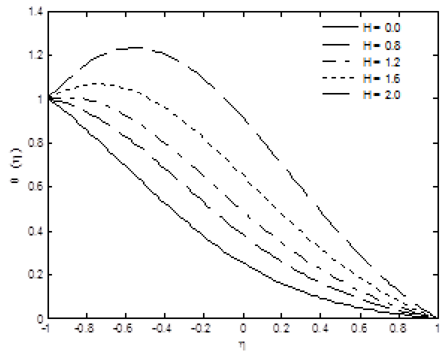

FIGURE 8. $\theta(\eta)$ for various values of H.

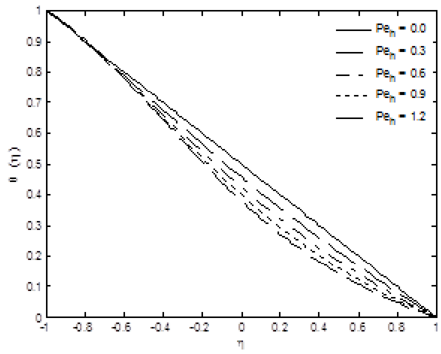

FIGURE 9. $\theta(\eta)$ for various values of Pe_h.

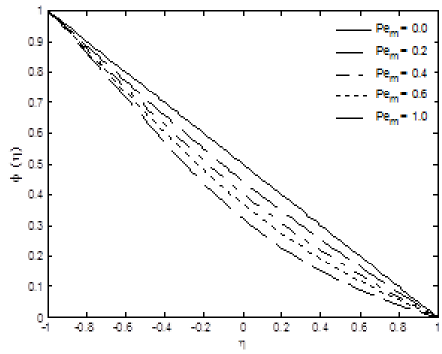

FIGURE 10. $\phi(\eta)$ for various values of Pe_m.

respectively taking the estimations of the stumbling parameters fixed. The results designate that the microrotation and the velocity increase by escalating the micropolar material parameters and an opposite trend as compared with material parameters is noticed in case of porosity parameter. The microrotation profile decreases at lower channel wall and increases at upper channel wall. Fig. 7 indicates that microrotation $G(\eta)$show reduction with ascending Reynolds numbers at both the walls of channel. Fig. 8 exhibits that the temperature profile rise up with increase in the values of the heat generation parameter . An enhancement in the heat generation tends to rise the temperature of the fluid and subsequently temperature on both walls of the channel increases. The effects of the Peclet number for the diffusion of heat and the Peclet number for the diffusion of mass are indicated in Figs. 9 and 10 respectively. Both the temperature and concentration profiles fall with escalating values of the Peclet number for the diffusion of heat $\mathrm{Pe}_h$and the Peclet number for the diffusion of mass Pe_m.

5. CONCLUSIONS

In the recent work, the numerical analysis of micropolar fluid flow through a resistive porous medium between channel walls taking into account the effect of the heat generation is presented. The system of nonlinear PDEs is transmuted into coupled ODEs by using suitable non-dimensional variables and then is solved numerically by using QL method along with finite difference discretization. The main points are mentioned below:

- The Reynolds number and the micropolar material parameters reduce the skin friction coefficient and the rates of mass and heat transport on both walls of the channel.
- It is noticed that the porosity parameter tends to diminish the microrotation and velocity. On the other hand, the micropolar material parameters act in an opposite way to the porosity parameter.
- The heat generation parameter boosts up the heat transfer rate while the Peclet number diminish it.

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