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Simulation of Rotational Flows in Cylindrical Vessel with Double Rotating Stirrers; Part-A: Analysis of Flow Structure and Pressure Differentials

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Abstract. In this article incompressible rotating mixing flows of Newtonian fluid inside the cylindrical vessel are investigated. The numerical simulations are combination of stationary and rotating double stirrer cases. The influence of rotational speed and rotational direction of stirrers have been observed on the predictions of the flow structure in the dissolution rotating vessel with fixed and rotating stirrers. The problem is relevant to the food industry, of mixing fluid within a cylindrical vessel, where stirrers are located on the lid of the vessel eccentrically configured with fixed and rotating stirrers. Here, the motion is premeditated as driven by the rotation of the outer vessel wall, with various rotational speeds and rotational directions of stirrers. A time-stepping finite element method is employed to foresee the numerical solutions. The numerical technique adopted is based on a semi-implicit Taylor-Galerkin/ pressure-correction scheme, posed in a two-dimensional cylindrical polar coordinate system. Variation with increasing speed of vessel, change in speed of stirrers and change in rotational direction of stirrers in mixer geometry are investigated, with respect to the re-circulating flow pressure isobars. The ultimate objective is to envisage and adjust the design of dough mixers, so that the optimal dough processing may be achieved markedly, with reference to work input on the dough.

AMS (MOS) Subject Classification Codes: 65N30, 65M60, 74S05, 68Q05, 76D05, 35Q30, 76U05

Key Words: Numerical Simulation, Finite Element Method, Newtonian Fluids, Rotational Mixing Flows, Co-rotating Stirrer, Contra-rotating Stirrer and Mix-rotating Stirrer.

1. INTRODUCTION

In mixing industries the analysis of complex mixing of fluids remains a persistent challenge for engineers. In order to improve design of the mixing components and ensure safety and satisfactory operating performance, precise knowledge of their transient response is required. In a first modelling step, design engineers must identify the meaningful physical phenomena in their particular geometry. Generally, industrial problems are demanding to treat with, mainly in the field of chemical process applications, such as mixing of dough in a food processing industry [1−5], granular mixing, powder mixing processes [6], mixing of paper pulp in paper industry and many other industrial processes. The case of a circular cylinder confined in cylindrical fluid domains has already attracted a great attention due to its recurrence in the design of mixing components. Flow between a rotating cylindrical vessel and rotating cylindrical stirrers is perhaps the most popular candidate in experimental and numerical studies of rotational flows. Very few experimental/numerical investigations have been reported in the literature on such type of flows.

Similar type of problems in the literature are as oscillating cylinders in cross or stationary flow frequently observed in offshore structures and power cables with fluidstructure interactions [2]. Building on the understanding of flow over a single stationary cylinder, many researchers have recently focused attention on multiple stationary or oscillating cylinders. Flow around two oscillating cylinders has characteristics of both an oscillating cylinder and multiple cylinders [1, 6 and 7].

In many mixing processes the complicating factors are the use of agitators with stirrer in fact that the agitator may be operated in the transitional regime, the use of the fluids which exhibits very complex rheological behaviour and the rotational speed and rotational direction of stirrers. Previous investigations of identical phenomenon also explored by number of researcher, such complex problem is still persist a challenge [8−11].

The fully three-dimensional incompressible mixing flows had been simulated to obtain the numerical solution for non-Newtonian fluids using generalised Navier-Stokes equation [5] in finite vessel. Whilst, for modelling the dough kneading problem the two-dimensional non-Newtonian mixing flows was investigated with different number and shapes of stirrers [3−6]. Subsequently, the parallel version of this finite element scheme is also developed [1−4].

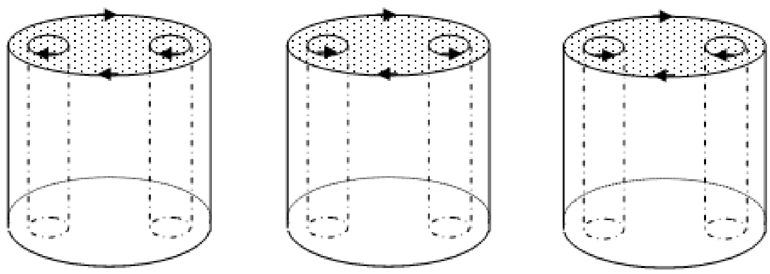
The motivation for this work is to advance fundamental technology in modelling of the dough kneading with the ultimate aim to forecast the optimal design of dough mixers themselves, hence, leading to efficient dough mixing processing. The present work is one of these forms, expressed as the flow between a rotating cylindrical vessel and two stationary as well rotating cylindrical stirrers in co-rotating, contra-rotating and mix-rotating directions against the direction of rotational cylindrical vessel. Stirrers are located on the mixing vessel lid, and placed in an eccentrically. In two-dimension, similar problem was also investigated with different number and shapes of stirrers [4−6].

In the present work a semi-implicit Taylor-Galerkin/Pressure-Correction (TGPC) time-marching scheme adopted, which has been developed and refined over the last couple of decades. The flow is modelled as incompressible via so-called TGPC finite element scheme posed in a two-dimensional cylindrical polar coordinate system which applies a temporal discretisation in a Taylor series prior to a Galerkin spatial discretisation. A semi-implicit treatment for diffusion is employed to address linear stability constraints. An inelastic model with shear rate dependent viscosity is considered [5]. This scheme, firstly conceived in chronological form, is appropriate for the simulation of various types of incompressible Newtonian flows [1].

In Section 2, the complete problem is specified and the governing equations and numerical method are described in Section 3. Numerical results and discussions are presented in Section 4 and conclusions are drawn in Section 5.

2. Problem specification

The problem addressed in this article is a cylindrical vessel with a couple of rotating cylindrical stirrers located eccentrically cinfugered in the side of the vessel. The fluid is driven by the outer vessel wall and two rotating cylindrical stirrers fixed at the top of the vessel. Fixed and rotating stirrers are adopted. initially, the flow analyse between two stationary stirrers and rotating cylindrical vessel, to validate the finite element predications in this cylindrical polar co-ordinate system to compare the present numerical predictions against numerical results obtained in previous investigations [3−6]. Subsequently, three alternative rotational directions (Co-rotating, Contra-rotating and Mix-rotating) and three rotational speeds (half, same and double against the speed of cylindrical vessel) of stirrers are investigated.
Rotational Direction are shown in figure−1 (a), (b) and (c) for Co-rotating, Contra-rotating and Mix-rotating directions of stirrers respectively. Computational domain and finite element mesh for the study involved is shown in figure−2. For the finite element mesh, triangular elements are selected in the current research work. Total numbers of elements are 8960, nodes and degrees-of-freedom, are 18223 and 40057 respectively. Details on mesh convergence, initial and boundary conditions reader is referred to our pervious investigations [1−6]. In this study, solution fields of interest are presented of flow structure and pressure differential through contour plots of streamlines and isobars respectively [1].



(a) (b) (c)

FIGURE 1. Rotational direction of eccentric rotating cylinderical flow, with double stationary and rotating stirrers.

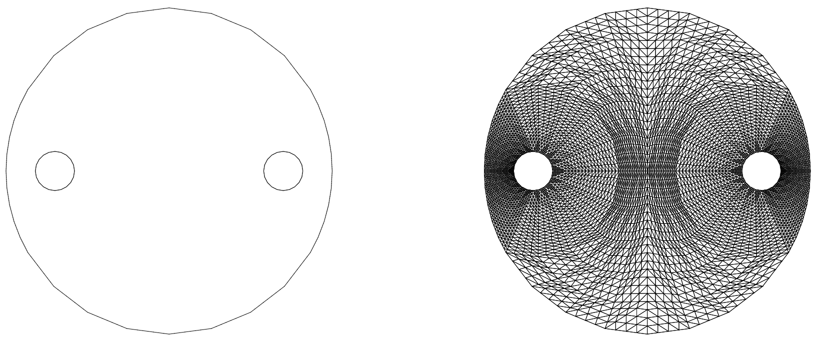


FIGURE 2. Computational domain and finite element mesh of eccentric rotating cylinderical flow, with double stationary and rotating stirrers.

To provide a well-posed specification for each flow problem, it is necessary to prescribe appropriate initial and boundary conditions. Simulations commence from a quiescent initial state. Boundary conditions are taken as follows. For stationary stirrer the fluid may stick to the solid surfaces, so that the components of velocity vanish on the solid inner stirrer sections of the boundary (v_r = 0 and v_θ = 0). For non-stationary stirrer, fixed constant velocity boundary conditions are applied. For co-rotating stirrer, vanishing radial velocity component (v_r = 0) is fixed and for azimuthal velocity component is fixed with three different non-dimensional speeds (v_θ = 0.5, 1 and 2 unit). Similarly, for contra-rotating stirrer only azimuthal velocity component is changed and fixed in reverse direction (i.e., v_θ = -0.5, -1 and -2 unit). On the outer rotating cylinder vessel a fixed constant velocity boundary condition is applied (v_r = 0 and v_θ = 1 unit), and a pressure level is specified as zero for both co-, contra- and mix-rotating stirrers on vessel wall. For stream function, outer cylinder is fixed zero and at inner stirrer is left unconstrained, being solutions on closed streamlines.

3. GOVERNING SYSTEM OF EQUATIONS AND NUMERICAL SCHEME

Incompressible rotational flows of isothermal Newtonian fluid in two-dimensions can be modelled through a system comprising of the generalised momentum transport and conservation of mass equations. The coordinate reference frame is a two-dimensional cylindrical coordinate system taken over domain $\mathbf{\Omega}$ in two-dimensional system. In the absence of body forces, the system of equations can be represented through the conservation of mass equation, as,

$$\nabla \cdot \mathbf{u} = 0, \tag{3. 1}$$

the conservation of momentum transport equation, as,

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{1}{Re}\nabla^2 \mathbf{u} - \mathbf{u}\cdot\nabla\mathbf{u} - \nabla p, \tag{3. 2}$$

where, **u** is the fluid velocity vector field, p is the isotropic fluid pressure (per unit density), t represents time and ∇ is the spatial differential operator.

Relevant non-dimensional Reynolds number is defined as:

$$Re = \frac{\rho V R}{\mu}, \tag{3. 3}$$

The characteristic velocity V is taken to be the rotational speed of the vessel, the characteristic length scale is the radius, R, of a stirrer and ρ is the fluid density and the characteristic viscosity μ is the zero shear-rate viscosity.

Appropriate scaling in each variable takes the form. At a characteristic rotational speed 50 rpm and zero shear viscosity of 105 Pa s, scaling yields dimensional variables p = 2444 p*. To compute numerical solutions through a semi-implicit Taylor-Galerkin/pressure-correction scheme in this study, a time-marching finite element algorithm is employed, based on a fractional-step formulation. Briefly, this involves discretisation, first in the temporal domain, obtain through Taylor series expansion and a pressure-correction operator-split technique, to build a second-order time-stepping scheme. Spatial discretisation is achieved via Galerkin approximation for the momentum equations. The finite element basis functions employed are quadratic (ϕ_j) for velocities, and linear (ψ_k) for pressure. A detail on numerical algorithm, fully discrete system and definition of matrices is described can be found [9−12].

4. NUMERICAL RESULTS AND DISCUSSIONS

Two different directions are employed to analysed the predicted solutions: firstly, change in rotational direction (co-rotating, contra-rotating and mix-rotating) and second, rotational speed ($v_\theta = \frac{1}{2}$, 1, 2; half, same and double respectively) of the stirrer against to the speed of vessel. This leads to testing with respect to increasing viscosity levels (decrease of Reynolds number) for Newtonian fluid and comparison of flow structure and pressure variation across problem instances. The predicted solutions are displayed for Newtonian fluid through contours plots of streamlines, and pressure isobars and these are plotted from minimum value to maximum value, over a range (in all figures Maximum values indicated with SQUARE shape and Minimum Value indicated with OVAL shape).

The Reynolds numbers of Re = 8.0, Re = 0.8 and Re = 0.08, the corresponding zero shear viscosities are μ = 1.05 Pa s, μ = 10.5 Pa s and μ = 105.0 Pa s. Of these levels, a range of material properties is covered from those for model fluids, to model dough, to actual dough, respectively.

4.1. **Flow patterns and pressure differential for stationary stirrers with increasing inertia.** Computations are carried out at Re = 0.08, Re = 0.8 and Re = 8. The effect of increasing Reynolds number upon streamline patterns on left and pressure differential on right isobars are represented in contour plots for stationary stirrers in figure−3. At a low level of inertia, Re = 0.08, an intense recirculating region forms in the centre of the vessel, parallel to the stirrers and symmetrically intersecting the diameter that passes through the centres of the vessel and stirrers. Flow structure remains unaffected as Reynolds number rises to values of O(1); hence the solution field has been suppressed. However, upon increasing Reynolds number up to eight, so O(10), inertia takes hold and the recirculation region twist and shifts towards the upper-half plane, vortex intensity wanes and the vortex eye is pushed towards the vessel wall. The flow becomes asymmetric as a consequence of the shift in vortex core upwards. The diminishing trend in vortex intensity is tabulated in Table 1.

Table–1: Vortex intensity for Newtonian fluids (μ_c = 105, 10.5 and 1.05 Pas)

Problem	Speed of stirrer	Re = 0.08		Re = 0.8		Re = 8.0	
		Min	Max	Min	Max	Min	Max
Stationary stirrer	Zero	0	3.7613	0	3.75964	0	3.61976
Co-rotating stirrer	Half	0	5.68479	0	5.68248	0	5.57629
	Same	0	7.75524	0	7.76814	0	7.99449
	Double	0	11.9254	0	12.0221	0	13.4436
Contra-rotating stirrer	Half	0	2.66273	0	2.66225	0	2.62743
	Same	-0.93386	2.02191	-0.9386	2.01946	-1.49938	1.81904
	Double	-5.019	1.58694	-5.0849	1.58592	-7.41413	1.71834
Mix-rotating stirrer	Half	0	4.25444	0	4.25387	0	4.19542
	Same	0	5.42041	0	5.42056	0	5.42893
	Double	-1.997	7.84833	-1.98886	7.85385	-1.6173	8.11336

Comparable symmetry influence relate across the geometry variants in pressure differential, at Re = 0.08, symmetric pressure isobars appear with equal magnitude in non-dimensional positive and negative extrema on the two sides (upper and lower) of the stirrers in the narrow-gap. As inertia increases from Re = 0.8 to Re = 8, asymmetric isobars are observed, with positive maximum on the top of the stirrer and negative minimum at the outer stirrer tip (near the narrow-gap), numerical results are tabulated in Table 2.

Asymmetrical flow structure is observed in all variables and transversely all occurrence as inertia increase from Re = 0.08 to Re = 8.0, recirculating flow-rate decrease by just five percent. In non-dimensional terms above Re = 0.08 (noting scale differences), there is increase in pressure-differential rise by as much as twenty-two percent, at Re = 8.0, although pressure differential increase on the lower part of the stirrers. For Newtonian fluid, the extrema of recirculating region along with vortex intensity and pressure differential, are tabulated for completeness in tables 1 and 2.

4.2. **Flow patterns and pressure differential for co-rotating stirrers with increasing inertia.** With increasing Reynolds number from Re = 0.08 to Re = 8.0, equivalent field kinematic data for co-rotating stirrers is presented in figure−4 and −5, to make direct comparisons across all instances for Newtonian fluids, with rigorous reference to localised vortex intensity and pressure drops are tabulated in tables 1 and 2.

Stream lines are shown for increasing rotational speed of the stirrer (from left to right), double speed, same speed and half speed (v_θ = 0.5, 1.0 and 2.0 respectively) for co-rotating case, only single vortex is formed, shown in figure−3. At Reynolds number 0.08, half speed of the stirrers the vortex is formed at the centre of the stirrers and is of the elliptical shape in the horizontal direction and smooth in formation, but as the speed of the stirrers is increased to double speed the vortex changes the direction from horizontal to vertical and its size is large as compare to half speed of the stirrers also showing an increase space between the centre of vortex and diameter of secondary streamline. Streamlines tend to increase in density at the edges of the stirrers. At Reynolds number 0.8, half speed of the stirrers the vortex changes the direction from horizontal to vertical and change noted at the same speed is vice-versa, but at the double speed of the stirrers is the vortex twist to the stirrers in rotational direction of the vessel. At Re = 8.0, half speed the diameter of the vortex increases and the shape of the vortex is changed in circular shape similarly at same speed, the diameter of the vortex decreases at the double speed and vortex centre amplifies in the size. Consequently, the fluid recirculates at the centre of the vessel and create vacuum in the centre of the vessel.

In figure−5, illustrates the pressure differential at all comparable parameter values for co-rotating instances, The speed of stirrer is half virtually small change in the pressure differential is observed and remain unaltered for all Reynolds numbers values, the pressure differentials is very low and remain in order of two for all inertial values when increasing the speed of stirrers from same to double speed, consequently, the pressure differentials are noted high and is about seven times in negative extrema compare to stationary stirrer, at Re = 8 and small change is observed in positive maxima. See Table−2.

Table–2: Pressure drop for Newtonian fluids (μ_c = 105, 10.5 and 1.05 Pas)

Problem	Speed of stirrer	Re = 0.08		Re = 0.8		Re = 8.0	
		Min	Max	Min	Max	Min	Max
Stationary stirrer	Zero	-3.35114	3.33577	-3.42489	3.27096	-4.51328	3.29584
Co-rotating stirrer	Half	-1.45201	1.4338	-1.53811	1.356	-3.59659	0.94539
	Same	-1.31591	1.2375	-1.66359	0.87996	-8.18721	0.51811
	Double	-4.51104	4.35319	-5.35531	3.79051	-27.334	2.90016
Contra-rotating stirrer	Half	-5.26616	5.24019	-5.38462	5.14477	-6.71983	4.5179
	Same	-7.18218	7.13253	-7.40446	6.92244	-10.0362	4.98229
	Double	-11.0943	10.9296	-11.8754	10.2863	-24.6213	6.93863
Mix-rotating stirrer	Half	-5.69415	5.67199	-5.81121	5.58275	-8.45116	5.11985
	Same	-8.10922	8.04915	-8.42092	7.76947	-13.7993	5.92129
	Double	-12.9865	12.757	-14.064	11.7682	-29.6726	8.50339

4.3. **Flow patterns and pressure differential for contra-rotating stirrers with increasing inertia.** Corresponding field kinematics data for contra-rotating stirrers situation with increasing Reynolds number at Re = (0.08, 0.8 and 8.0) the streamline contours and pressure differentials are presented in figure−7 respectively. In figure−6, for contra-rotating case, streamlines are demonstrate for increasing speed of the stirrers from half (left) to double (right) against the speed of vessel the six vortices arises, and symmetric behaviour around the stirrer. At the half speed of stirrers and Re = 0.08 there is embro vertices appear in the narrow-gap of both stirrers, and in narrow gap as well in middle of the vessel, two vortices are noted in the upper and lower region of vessel away from stirrer close to vessel wall. In the narrow gap, where stirrers spins in oppose direction of the vessel rotation, a small vortex appear with low vortex intensity, as the speed of stirrers increase these vortices strength up to fifty percent high at low Re = 0.08.

In table 1, minima and maxima of vortex intensity is tabulated when the vortex intensity is observed with increase in inertia. As inertia takes hold in the central region of recirculation, centres of vortices shift towards the upper and lower half plane in the direction of rotation of the both stirrers against to the vessel. For all Reynolds number values, at double speed of stirrers, the central vortices rotate in counter directions against two other vortices. These recirculation regions have different rotational direction which is very important phenomena in homogenisation of the fluid.

For all three instances, comparable equilibrium influence apply across the geometry variants in pressure differential, at Re = 0.08 and double rotational speed of stirrer, symmetric pressure isobars appear with equal magnitude in non-dimensional positive and negative extrema on both sides (upper and lower) of the stirrer in the narrow-gap as shown in figure−7. The associated values of pressure differentials are tabulated in table 2. As inertia increase from Re = 0.8 to Re = 8, asymmetric isobars are observed, with positive maxima on the top of the stirrer and negative minima at the outer stirrer tip (near the narrow-gap), see also Table 2. For the contra-rotating instance, in contrast to co-rotating case, the pressure differentials are somewhat symmetrical in geometry at maxima and minima at twice the speed of stirrer and at half the speed of stirrer for both inertial values Re = 0.08 and Re = 0.8. However, upon increasing Reynolds number up to eight, thus O(10), inertia takes hold the pressure differentials are observed asymmetrical, increasing the speed of the stirrer to double increases the pressure differentials more than twice in negative minima and in contrast it decrease up to thirty five percent in positive maxima. Comparing against co-rotating case at same double speed of stirrer increase in minima is merely eight percent and increase in maxima is about thirty percent.

4.4. **Flow patterns and pressure differential for mix-rotating stirrers with increasing inertia.** For mix-rotating stirrers, the field kinematics data with increasing Reynolds number from 0.08 to 8.0 the streamline contours are shown in figure−8 and in figure−9 pressure differentials. For mix-rotating case, three vortices arise, one in right of the stirrer rotate in same direction of vessel and in right and left of the stirrer rotate in opposite direction of vessel. At the Re = 0.08 and the Re = 0.8 note the same behaviour for half to double speed of stirrers respectively. There is two in the vicinity of both stirrers, and in narrow gap as well in middle of the vessel, two vortices are noted in the upper and lower region of vessel away from stirrer close to vessel wall. In the narrow gap, where stirrers spins in oppose direction of the vessel rotation, a small vortex appear with low vortex intensity, as the speed of stirrers increase these vortices strength up to fifty percent high at low Re = 0.08. The growth in minima of vortex intensity is examine with increase in inertia, however, it dominate in maxima of vortex intensity. As inertia takes hold in the central region of recirculation centres of vortices shift towards the upper and lower half plane in the direction of rotation of the both stirrers against to the vessel. For all Reynolds number values at double speed of stirrer, the central vortices rotate in counter directions against two other vortices. These recirculation regions have different rotational direction which is very important phenomena in homogenisation of the fluid.

5. Conclusions

Computational results of cylindrical rotating vessel with couple of cylindrical rotating stirrers present certain characteristics of symmetry, the rotation of the double stirrer case against stationary stirrer case in co-rotating, contra-rotating, and mix-rotating directions are being investigated with increasing inertia. It is confirmed that, fluid flow structure lose its symmetry and recirculating region move upwards in the direction of vessel motion with increasing inertia and non-dimensional pressure differential increases for stationary stirrer case. For co-rotating stirrer case, single recirculating region develops in the centre of the stirrers and fluid suppressed in direction of rotation of the vessel with increasing inertia and speed of stirrer. Whilst at twice the speed of stirrer pressure differentials are higher and lower at lower speed of stirrer and for the case mix-rotating stirrer different position has been analysed, a re-circulating region develops in the right of the stirrers rotating in direction of vessel and left and right of the stirrer rotating in anti-direction of vessel and fluid suppressed in direction of rotation of the vessel with increasing inertia and speed of stirrer. Whilst at twice the speed of stirrer pressure differentials are higher and lower at lower speed of stirrer.

Contra-rotating case flow structure and pressure differential illustrates completely different picture in contrast to above cases. There is six recirculating regions have been examined with different position of vortex centres. The pressure differentials are generally higher, and similar balance in extrema is noted to those flows. However, the position, in those negative maxima exceeds to positive minima by about four times and in the mix-rotating the narrow gap apears where stirrers spins in oppose direction of the vessel rotation, a small vortex appear with low vortex intensity.

We have productively demonstrated the use of two-dimensional numerical simulations for this complex mixing process using Newtonian fluid flow solver for the industrial flow of dough mixing. Promising future directions of this work are investigation of rotation of two stirrers case in co-rotating, contra-rotating and mixed rotating directions, changing material properties using non-Newtonian fluids and introducing agitator in concentric configured stirrer. Through the calculative capability generated, we shall be able to relate this to mixer design that will ultimately impact upon the processing of dough products.

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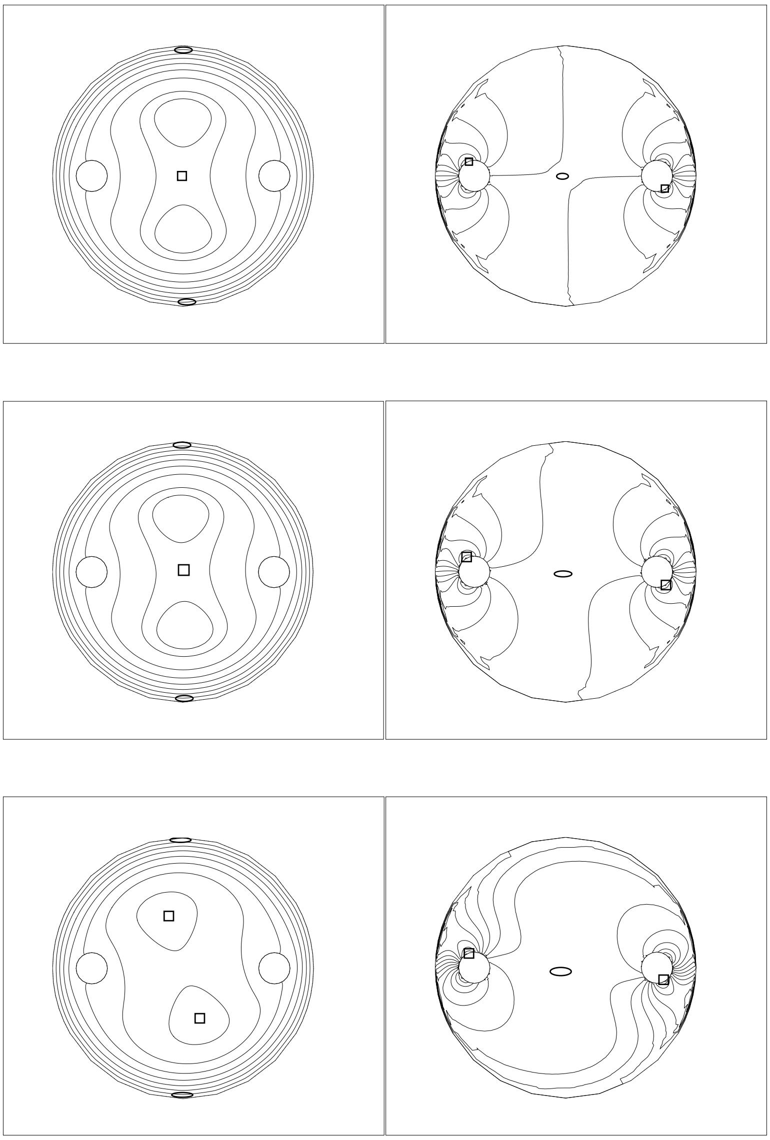
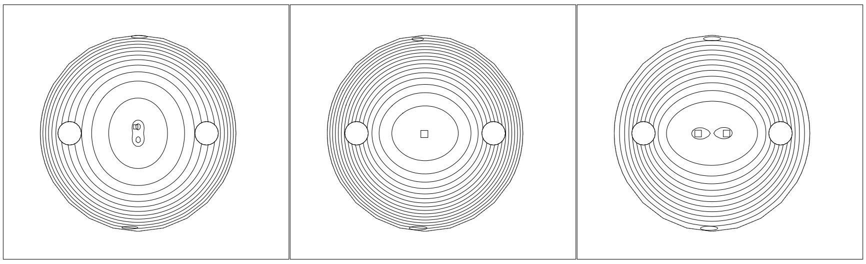
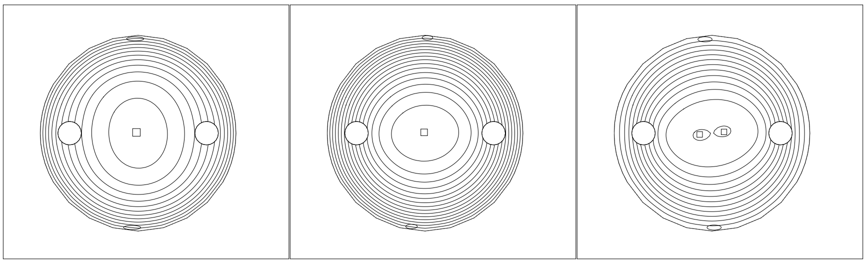


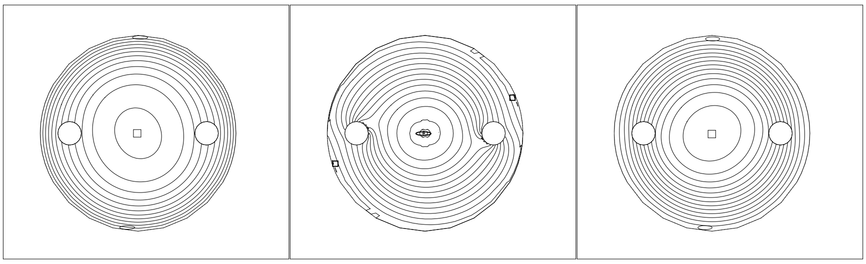
FIGURE 3. Streamline contours and Pressure isobars of stationary stirrers ($v_\theta = 0.0$) with increasing Reynolds number (Re = 0.08, 0.8 and 8.0) from top to bottom



Re= 0.08

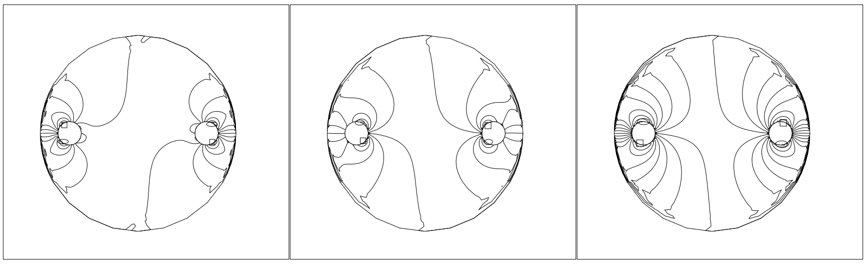


Re = 0.8

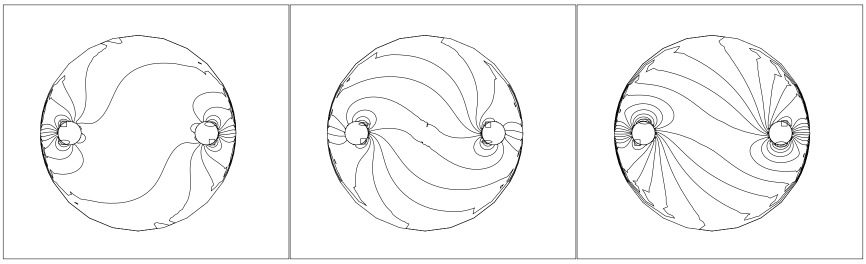


Re = 8.0

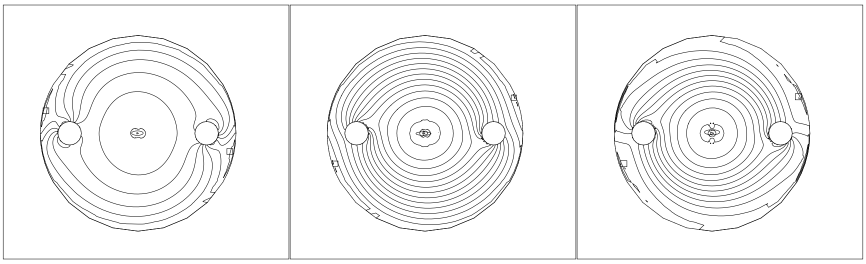
FIGURE 4. Streamline contours for co-rotating stirrers with increasing speed of the stirrers (υ_θ = 0.5, 1.0 and 2.0) from left to right against the speed of the vessel and increasing Reynolds number.



Re = 0.08

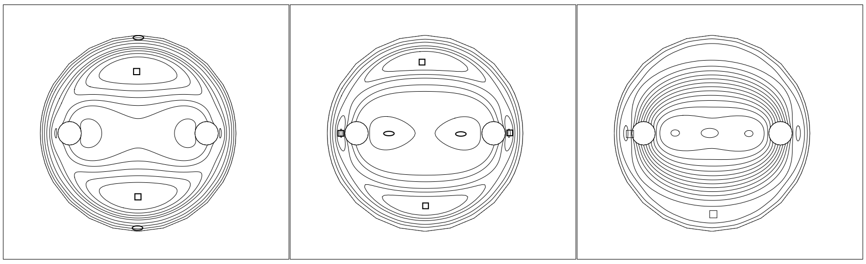


Re = 0.8

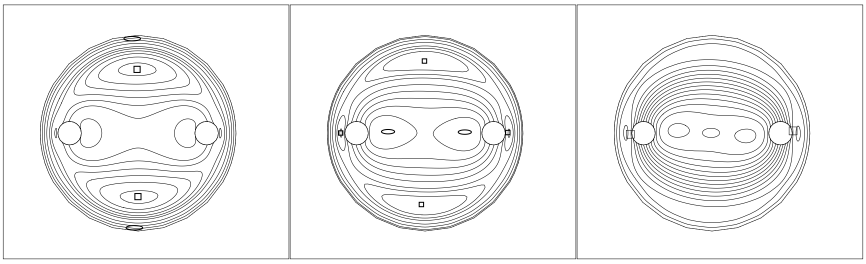


Re = 8.0

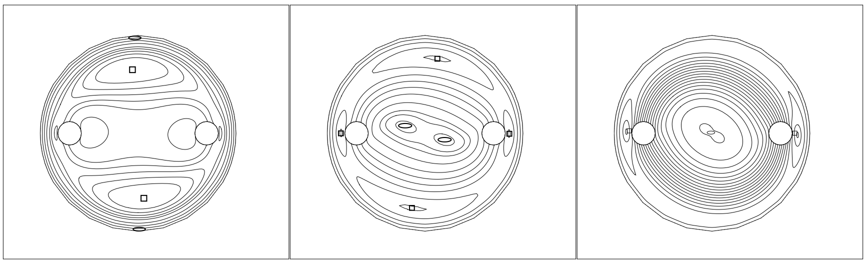
FIGURE 5. Pressure isobars for co-rotating stirrers with increasing speed of the stirrers (v_θ = 0.5, 1.0 and 2.0) from left to right against the speed of the vessel and increasing Reynolds number.



Re = 0.08



Re = 0.8



Re = 8.0

FIGURE 6. Streamline contours for contra-rotating stirrers with increasing speed of the stirrers (v_θ = 0.5, 1.0 and 2.0) from left to right against the speed of the vessel and increasing Reynolds number.

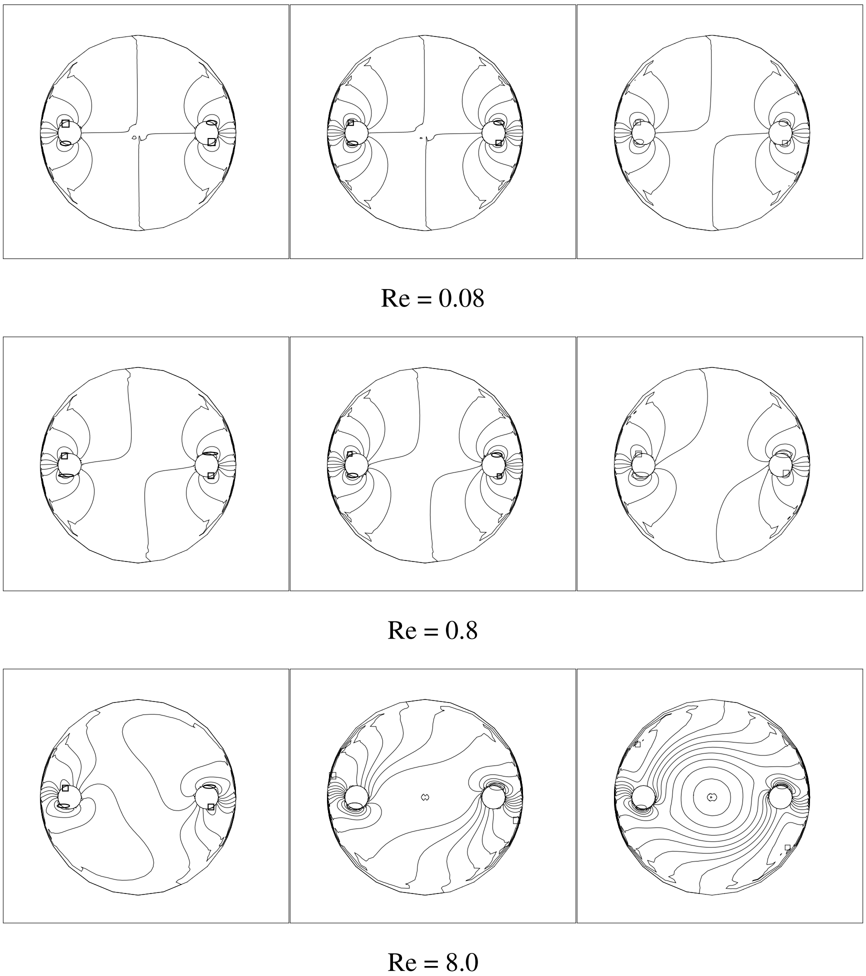
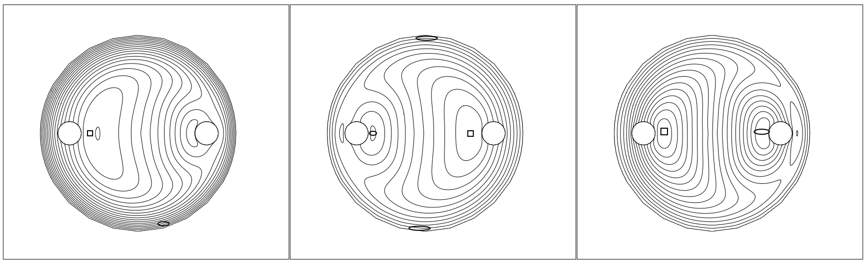
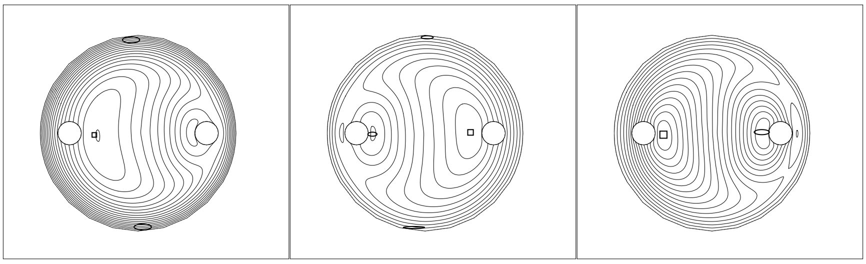


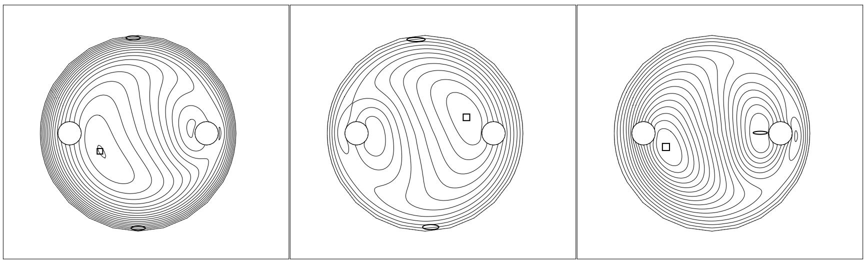
FIGURE 7. Pressure isobars for contra-rotating stirrers with increasing speed of the stirrers (v_θ = 0.5, 1.0 and 2.0) from left to right against the speed of the vessel and increasing Reynolds number.



Re = 0.08



Re = 0.8



Re = 8.0

FIGURE 8. Streamline contours for Mix-rotating stirrers with increasing speed of the stirrers (υ_θ = 0.5, 1.0 and 2.0) from left to right against the speed of the vessel and increasing Reynolds number.

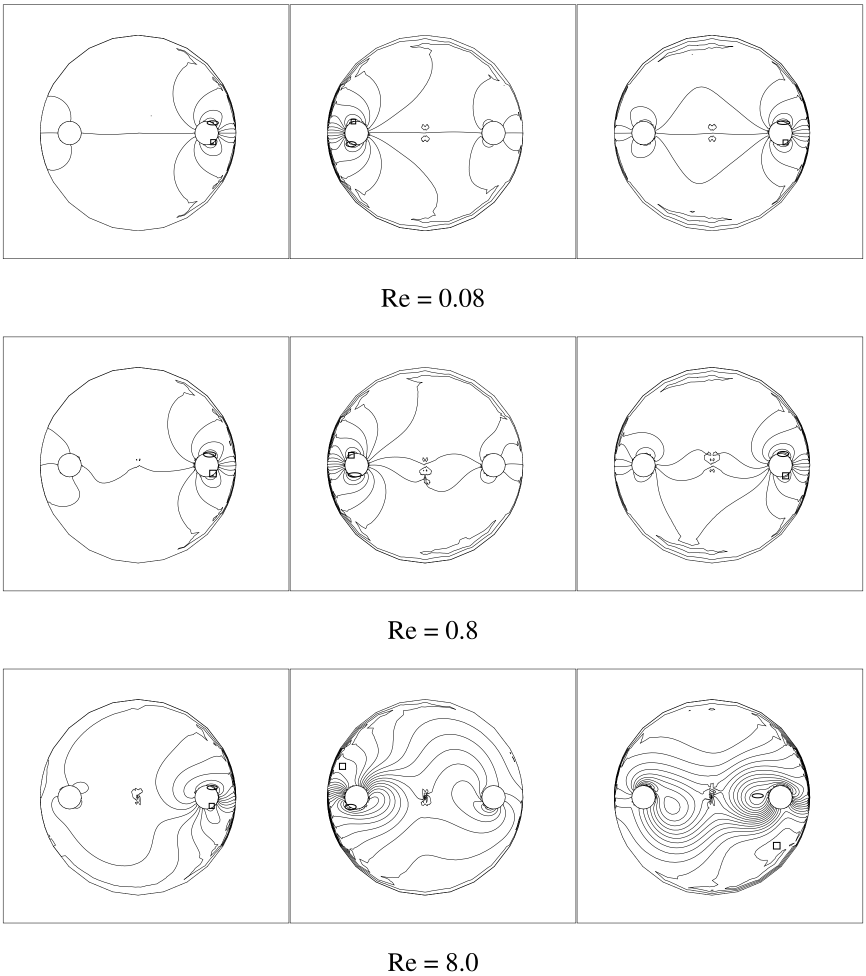


FIGURE 9. Pressure isobars for Mix-rotating stirrers with increasing speed of the stirrers (v_θ = 0.5, 1.0 and 2.0) from left to right against the speed of the vessel and increasing Reynolds number.