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Anti Fuzzy Implicative Ideals in BCK-Algebras

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Abstract. The aim of this paper is to introduce the notion of anti fuzzy implicative ideals of *BCK*-algebras and to investigate their properties.We give several characterizations of anti fuzzy implicative ideals. We also introduce the notion of anti Cartesian product of anti fuzzy implicative ideals, and then we study related properties.

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1. Introduction

The main problem in fuzzy mathematics is how to carry out the ordinary concepts to the fuzzy case. The difficultylies in how to pick out the rational generalization from the large number of available approaches. It is worth noting that fuzzy ideals are different from ordinary ideals in the sense that one cannot say which *BCK*-algebra element belongs to the fuzzy ideal under consideration and which one does not. The concept of fuzzy sets was introduced by Zadeh [13]. Since then these ideas have been applied to other algebraic structures such as semigroups, groups, rings, modules, vector spaces and topologies. In 1991, Xi [12] applied the concept of fuzzy sets to *BCK* -algebras which are introduced by Imai and Iséki [5] . In [1], Biswas introduced the concept of anti fuzzy subgroups of groups. Modifying his idea, in [4], S. M. Hong and Y. B. Jun applied the idea to *BCK* -algebras. They introduced the notion of anti fuzzy ideals of *BCK* -algebras. In this paper, we introduce the notion of anti fuzzy implicative ideal of *BCK* -algebras, and investigate some related properties. We show that in an implicative *BCK*-algebra, a fuzzy subset is an anti fuzzy ideal if and only if it is an anti fuzzy implicative ideal. We show that a fuzzy subset of a *BCK*-algebra is a fuzzy implicative ideal if and only if the complement of this fuzzy subset is an anti fuzzy implicative ideal. Moreover, we discuss the pre-image of anti fuzzy implicative ideals. Finally, we introduce the notion of anti Cartesian product of anti fuzzy implicative ideals, and then we characterize anti fuzzy implicative ideals by it .

2. Preliminaries

In this section we cite the fundamental definitions that will be used in the sequel:

Definition 1. [6] An algebra $(X, *, 0)$ of type $(2, 0)$ is called a *BCK* - algebra if it satisfies the following axioms for all $x, y, z \in X$:

$(i)\, ((x * y) * (x * z)) * (z * y) = 0,$

$(ii)\, (x * (x * y)) * y = 0,$

$(iii)\, x * x = 0,$

$(iv)\, 0 * x = 0,$

$(v)\, x * y = 0\, and \quad y * x = 0\, imply\, x = y$

We can define a partial ordering $\leq$ on X by $x \leq y$ if and only if $x * y = 0$.

Proposition 2. [6] *In any* BCK-*algebra* X *, the following hold for all* $x, y, z \in X$ *:*

$(i)\, (x * y) * z = (x * z) * y,$

$(ii)\, x * y \leq x,$

$(iii)\, x * 0 = x,$

$(iv)\, (x * z) * (y * z) \leq x * y,$

$(v)\, x * (x * (x * y)) = x * y,$

$(vi)\, x \leq y \;\; implies\, x * z \leq y * z \quad and \;\; z * y \leq z * x.$

A *BCK*-algebra is said to be implicative if $x*(y * x) = x \quad forall \;\; x, y \in X$ (see[6,9]).

Definition 3. [8] A non-empty subset I of a *BCK*-algebra X is called an ideal of$X$ if it satisfies

$(I_1)\, 0 \in I$

$(I_2)\, x * y \in I and \quad y \in I\; imply \;\; x \in I$.

Definition 4. [8] A non-empty subset I of a *BCK*-algebra X is called an implicative ideal of X if it satisfies $(I_1)$and $(I_3)\, x \in I \;\; whenever\, (x * (y * x)) * z \in I\, and\, z \in I \;\; forall$ $x, y, z \in X.$

Definition 5. [13] Let S be a non-empty set. A fuzzy subset A of S is a function A :S $\longrightarrow$ [0,1]. Let A be a fuzzy subset of S . Then for t $\in$[0,1], the t-level cut of A is the set $A_t = \{x \in S \; | A\,(x) \geq t\}$ and the complement of A, denoted by A^c is the fuzzy subset of$S$ given by $A^c\,(x) = 1 - A\,(x)$ for all $x \in S$.(see [2,3,7]).

Definition 6. [12] A fuzzy subset A of a *BCK*-algebra is called a fuzzy subalgebra of X if

$$A\,(x * y) \geq min\,\{A\,(x)\,, A\,(y)\}\; forall\; x, y \in X.$$

Definition 7. [12] Let X be a *BCK*-algebra. A fuzzy subset A of X is called a fuzzy ideal of X if

$(F_1)\;A\,(0) \geq A\,(x)\,,$

$(F_2)\;A\,(x) \geq min\;\{A\,(x * y)\,, A\,(y)\}\,, for\; all\; x, y \in X.$

Definition 8. [10] A fuzzy subset A of a *BCK*-algebra X is called a fuzzy implicative ideal of X if it satisfies

$(F_1)\; and\; (F_3) \quad A\,(x) \geq min\;\{A\,((x * (y * x)) * z)\,, A\,(z)\}\, forall\; x, y, z \in X.$

Definition 9. [4] A fuzzy subset A of a *BCK*-algebra X is called an anti fuzzy subalgebra of X if

$$A\,(x * y) \leq \;\; max \;\; \{A\,(x)\,, A\,(y)\}\; forall \;\; x, y \in X.$$

Definition 10. [4] A fuzzy subset A of a*BCK*-algebra X is called an anti fuzzy ideal of X if

$(A_1)\, A\,(0) \leq A\,(x)\,,$

$(A_2)\, A\,(x) \leq \;\; max\,\{A\,(x * y)\,, A\,(y)\}\,, forallx, y \in X.$

Proposition 11. [4] *Every anti fuzzy ideal of a* BCK-*algebra X is an anti fuzzy subalgebra of* X.

Definition 12. [4] Let A be a fuzzy subset of a *BCK*-algebra. Then for $t \in [0,1]$ the lower t-level cut of A is the set

$$A^t = \{x \in X \,|A(x) \leq t\}$$

Definition 13. [4] Let A be a fuzzy subset of a *BCK*-algebra. The fuzzification of $A^t, t \in [0,1]$ is the fuzzy subset μ_{A^t} of X defined by

Definition 14. [11] Let $f: X \to Y$ be a mapping of *BCK*-algebras and A be a fuzzy subset of Y . The map A^f is the inverse image of A under f if $A^f(x) = A(f(x)) \, \forall x \in X$.

3. ANTI FUZZY IMPLICATIVE IDEAL

Definition 15. A fuzzy subset A of a *BCK*-algebra X is called an anti fuzzy implicative ideal of X if it satisfies

$(A_1)\ and\ (A_3)\ A(x) \leq max\{A((x*(y*x))*z), A(z)\}\ forall\ x, y, z \in X.$

Example 1. *(1) Every constant function* $A: X \longrightarrow [0,1]$ *is an anti fuzzy implicative ideal of* $.X$ *(2) Let* $X = \{0, a, b, c\}$ *be a* BCK-*algebra with Cayley table as follows:*

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

Let t_0,t_1 be such that $t_0 \prec t_1$. Define A: $X \longrightarrow [0,1]$ by $A(0) = A(a) = A(b) = t_0$ and $A(c) = t_1$ Routine calculations give that A is an anti fuzzy implicative ideal.

Proposition 16. *Every anti fuzzy implicative ideal of a* BCK-*algebra* X *is order preserving.*

Proof. Let A be an anti fuzzy implicative ideal of a BCK-algebra X and let $x, y \in X$be such that $x \leq y$ Then

$$\begin{aligned} A(x) &\leq max\{A((x*(z*x))*y), A(y)\}, \\ &= max\{A((x*y)*(z*x)), A(y)\}, \\ &= max\{A(0*(z*x)), A(y)\}, \\ &= max\{A(0), A(y)\} = A(y). \end{aligned}$$ □

Proposition 17. *Every anti fuzzy implicative ideal of a* BCK-*algebra* X *is an anti fuzzy ideal.*

Proof. Let A be an anti fuzzy implicative ideal of a *BCK* -algebra X, so for all $x, y, z \in X$:

$$A(x) \leq max\{A((x*(y*x))*z), A(z)\},$$

Putting in $y = x$, and $z = y$:

$$\begin{aligned} A(x) &\leq max\{A((x*x)*y), Ay\} \\ &= max\{A(x*y), A(y)\}. \end{aligned}$$ □

Combining Proposition 2.11 and 3.4 yields the following result.

Proposition 18. *Every anti fuzzy implicative ideal of a* BCK-*algebra* X *is an anti fuzzy subalgebra of* X.

Remark 19. An anti fuzzy ideal (subalgebra) of a *BCK*-algebra X may not be an anti fuzzy implicative ideal of X as shown in the following example:

Example 2. *Let X be the* BCK-*algebra in Example 3.2(2) and let* $t_{0,}t_{1,}t_2 \in [0,1]$ *be such that* t_0 $t_1 t$ $_2$ *Define* $A : X \longrightarrow A(0) = t_0, A(a) = A(b) = t_1$ *and*$A(c) = t_2$ *Routine calculations give that A is an anti fuzzy ideal (subalgebra) of X. but not an anti fuzzy implicative ideal of X because*

$$A(a) = t_1 > max\{A((a*(b*a))*0), A(o)\}$$
$$= max\{A(0), A(0)\} = t_0.$$

Proposition 20. *If X is implicative* BCK *-algebra, then every anti fuzzy ideal of X is an anti fuzzy implicative ideal of X.*

Proof. Let A be an anti fuzzy ideal of an implicative *BCK*-al gebra X , so for all $x, z \in X$:

$$A(x) \le max\{A(x*z), A(z)\},$$

Since X is an implicative, then $x*(y*x) = x\ forall\ x, y \in X$. Hence

$$A(x) \le \max\{A((x*(y*x))*z), A(z)\},$$

This shows that A is an anti fuzzy implicative ideal of X. □

By applying Proposition 17 and 20 , we have

Theorem 21. *If X is an implicative* BCK-*algebra, then a fuzzy subset A of X is an anti fuzzy ideal of X if and only if it is an anti fuzzy implicative ideal of X.*

Proposition 22. *A fuzzy subset A of a* BCK-*algebra X is a fuzzy implicative ideal of X if and only if its complement A^c is an anti fuzzy implicative ideal of X.*

Proof. Let A be a fuzzy implicative ideal of a *BCK*-algebra X, and let$x, y, z \in X$. Then

$$A(0) = 1 - A(0) \le 1 - A(x) = A^c(x)\ and$$
$$A^c(x) = 1 - A(x) \le 1 - min\{A((x*(y*x))*z), A(z)\},$$
$$= 1 - min\{1 - A^c((x*(y*x))*z), 1 - A^c(z)\},$$
$$= max\{A^c((x*(y*x))*z), A^c(z)\}.$$

So, A^c is an anti fuzzy implicative ideal of X. Now let A^c is an anti fuzzy implicative ideal of X, and let $x, y, z \in X$. Then

$$A(0) = 1 - A^c(0) \ge 1 = A^c(x) = A(x)\ and$$
$$A(x) - 1A^c(x) \le 1 - max\{A^c((x*(y*x))*z), A^c(z)\},$$
$$= 1 - max\{1 - A((x*(y*x))*z) 1 - A(z)\},$$
$$= min\{A((x*(y*x))*z), A(z)\}.$$

Thus, A is a fuzzy implicative ideal of X. □

Theorem 23. *Let A be an anti fuzzy implicative ideal of a* BCK-*algebra X. Then the set*

$$X_A = \{x \in X\ |A(x) = A(0)\quad\},$$

is an implicative ideal of X.

Proof. Clearly $0 \in X$. Let $x, y, z \in X_A$ be such that $(x*(x*y))*z \in X$ and $z \in X_A$. Then

$$A((x*(y*x))*z) = A(z) = A(0).$$

It follows that

$$A(x) \le max\{A((x*(y*x))*z), A(z)\}$$
$$= max\{A(0), A(0)\} = A(0).$$

Combining Definition 15(A_1), we get $A(x) = A(0)$ and hence$x \in X$. □

Theorem 24. *Let A be a fuzzy subset A of a* BCK-*algebra X. Then A is an anti fuzzy implicative ideal of X if and only if for each $t \in [0,1], t \ge A(0)$, the lowert t-level cut A^t is an implicative ideal of X.*

Proof. Let A be an anti fuzzy implicative ideal of X and let$t \in [0,1]$ with$t \geq A(0)$. Clearly$0 \in A^t$. Let $x, y, z \in X$ be such that $(x*(y*x))*z \in A^t$ and $z \in A^t$ .Then $A((x*(y*x))*z) \leq t$, $A(z) \leq t$,
hence

$$A(x) \leq \max\{A((x*(y*x))*z)\, A(z)\} \leq t\, .$$

And so $x \in A^t$. HenceA^t is an implicative ideal of X.
Conversely, let A^t is an implicative ideal of X, we first show $A(0) \leq A(x)$ for all $x \in X$. If not, then there exists $x_0 \in X$ such that $A(0)¿A(x)$. Putting$t_0 = \frac{1}{2}\{A(0) + A(x_0)\}$ then $0 \leq A(x_0)¡t_0\, ¡A(0) \leq 1$. Hence $x_0 \in A^{t_0}$, so that $A^{t_0} \neq \varnothing$. But A^{t_0} is an implicative ideal of X. Thus $0 \in A^{t_0}$, or $A(0) \leq t_0$, a contradiction. Hence $A(0) \leq A(x)$ for all $x \in X$. Now we prove that$A(x) \leq \max\{A((x*(y*))*z), A(z)\}$ for all $x, y, z \in X$. If not, then there exists $x_0, y_0, z_0 \in X$, such that

$$A(x) > max\{A((x_0*(y_0*x_0))*z_0), A(z_0)\},$$

Taking $s_0 = \frac{1}{2}\{A(x_0) + max\{((x_0*(y_0*x_0))*z_0), A(z_0)\}\}$;
$then\ s_0 \leq A(x_0)\ and$

$$0 \leq max\{A((x_0*(y_0*x_0))*z_0)\}¡\, s_0 \leq 1.$$

Thus we have $s_0\, ¿A((x_0*(y_0*x_0))*z_0)$ and $s_0\, ¿A(z_0)$. Which imply that
$(x_0*(y_0*x_0))*z_{0 \in A^{s}0}$ and $z_0 \in A^{s_0}$ .But $A^{s_0}$is an implicative ideal of X . Thus $x_0 \in A^{s_0}$ or $A(x_0) \leq s_0$ This is a contradiction, ending the proof. □

Theorem 25. *If A is an anti fuzzy implicative ideal of a* BCK*-algebra X. Then μ_{A^t} is also an anti fuzzy implicative ideal of X where $t \in [0,1], t, \geq A(0)$.*

Proof. From Theorem 24, it is sufficient to show that $(\mu_{A^t})^s$ is an implicative ideal of X,where $s \in [0,1]$ and $s \geq \mu_{A^t}(0)$. Clearly, $0 = (\mu_{A^t})$ Let $x, y, z \in X$ be such that $(x*(y*x))*z \in (\mu_{A^t})^s$ and $z \in (\mu_{A^t})^s$,
hence

$$\mu_{A^t}((x*(y*x))*z) \leq s\ ,$$

and

$\mu_{A^t}(z) \leq s$. We claim that $x \in (\mu_{A^t})\ or\ \mu_{A^t}(x) \leq s$. If $(x*(y*x))*z \in A^t$ and $z \in A^t$, then $x \in A^t$ because A^t is an implicative ideal of X. Hence.

$$\begin{aligned} \mu_{A^t}(x) &= A(x) \\ &\leq\ max\{A((x*(y*x))*z), A(z)\}, \\ &=\ max\{\mu_{A^t}((x*(y*x))*z), \mu_{A^t}(z)\} \leq s. \end{aligned}$$

and so $x \in (\mu_{A^t})^s$ If $(x*(y*x))*z \notin A^t$ or $z \in A^t$, then $\mu_{A^t}((x*(y*x))*z) = 0$ or $\mu_{A^t}(z) = 0$ then clearly $\mu_{A^t}(x) \leq s$, and so $x \in (\mu_{A^t})^s$. Therefore $(\mu_{A^t})^s$ is an implicative ideal of X. □

Theorem 26. *Theorem 3.13. Let $f : X \longrightarrow Y$ be a homomorphism of* BCK*-algebras. If A is an anti fuzzy implicative ideal of Y , then A^f is an anti fuzzy implicative ideal of X.*

Proof. Since A is an anti fuzzy implicative ideal of Y, then $A(0') \leq A(f(x))\ forany$ $x \in X$, and so, $A^f(0) = A(f(0)) = A(0') \leq A(f(x)) = A^f(x)$. For any $x, y, z \in X$, we have:

$$\begin{aligned} A^f(x) = A(f(x)) &\leq max\{A((f(x)*(f(y)*f(x)))*f(z)), A(f(z))\}, \\ &= max\{A((f(x)*(f(y*x)))*f(z)), A(f(z))\}, \\ &= max\{A((f(x*(y*x)))*f(z)), Af(z)\}, \\ &= max\{A(f((x*(y*x))*z)), A(f(z))\}, \\ &= max\left\{A^f((x*(y*x))*z), A^f(z)\right\}. \end{aligned}$$

This completes the proof. □

Theorem 27. *Theorem 3.14. Let $f : X \longrightarrow Y$ be an epimorphism of* BCK-*algebras. If A^f is an anti fuzzy implicative ideal of X , then A is an anti fuzzy implicative ideal of Y.*

Proof. Let $y \in Y$, there exists $x \in X$ such that $f(x) = y$. Then
$$A(y) = A(f(x)) = A^f(x) \geq A^f(0) = A(0').$$
Let $x', y', z' \in Y$. Then there exist $x, y, z \in X$. such that $f(x) = x', f(y) = y'$ and $f(z) = z'$. It follows that
$$\begin{aligned} A(x') = A(f(x)) = A^f(x) &\leq max \ \left\{A^f((x*(y*x))*z), A^f(z)\right\}, \\ &= max \left\{A(f((x*(y*x))*z)), A(f(z))\right\}, \\ &= max \left\{A((f(x)*(f(y*x)))*f(z)), A(f(z))\right\}, \\ &= max \left\{A((f(x)*(f(y)*f(x)))*f(z)), A(f(z))\right\}, \\ &= max \left\{A((x'*(y'*x'))*z'), A(z')\right\}. \end{aligned}$$
Hence A is an anti fuzzy implicative ideal of Y. □

4. Anti Cartesian product of anti fuzzy implicative ideals

Definition 28. Let λ and μ be the fuzzy subsets in a set X. The anti Cartesian product $\lambda \times \mu : X \times X \longrightarrow$[0, 1] is defined by $(\lambda \times \mu)(x, y) = max \ \{\lambda(x), \mu(y)\} \ forall$ $x, y \in X$.

Theorem 29. *If λ and μ are anti fuzzy implicative ideals of a* BCK-*algebra X, then $\lambda \times \mu$ is an anti fuzzy implicative ideal of $X \times X$.*

Proof. Let $x, x' \in X$
$$\begin{aligned} &(\lambda \times \mu)(0,0) = max \left\{\lambda(0), \mu(0)\right\} \\ &\leq max \left\{\lambda(x), \mu(x)\right\} = (\lambda \times \mu)(x, x') \end{aligned}$$
For any $(x, x'), (y, y'), (z, z') \in X \times X$ we have
$$\begin{aligned} &(\lambda \times \mu)(x, x') = max \left\{\lambda(x), \mu(x')\right\} \\ &\leq max \left\{max \left\{\lambda(x*(y*x))*z), \lambda(z)\right\}, max \ \left\{\mu(x'*(y'*x'))\, z'), \mu(z')\right\}\right\}, \\ &= max \left\{max \ \left\{\lambda(x*(y*x))*z), \mu(x'(y'*x'))\, z')\right\}, max \left\{\lambda(z), \mu(z')\right\}\right\}, \\ &= max \left\{(\lambda \times \mu)((x, x')*((y, y')*(x, x')))*(z, z')), (\lambda \times \mu)(z, z')\right\}. \end{aligned}$$
Hence $\lambda \times \mu$ is an anti fuzzy implicative ideal of $X \times X$ □

Theorem 30. *Let λ and μ be fuzzy subsets in a* BCK-*algebra X such that $\lambda \times \mu$ is an anti fuzzy implicative ideal of $X \times X$ Then:*
(i) either $\lambda(x) \geq \lambda(0)$ or $\mu(x) \leq \mu(0)$ for all $x \in X$;
(ii) if $\lambda(x) \geq \lambda(0)$ for all $x \in X$,then either $\lambda(x) \geq \mu(0)$ or $\mu(x) \geq \mu(0)$;
(iii) if $\mu(x) \geq \mu(0)$ for all $x \in X$, then either $\lambda(x) \geq \lambda(0)$ or $\mu(x) \geq \lambda(0)$.

Proof. (i) Suppose that $\lambda(x)$¡$\lambda(0)$ and $\mu(y)$ ¡ $\mu(0)$ for some $x, y \in X$. Then
$$(\lambda \times \mu)(x, y) = \text{max} \left\{\lambda(x, \mu(y))\right\} \text{ ¡ max } \left\{\lambda(0), \mu(0)\right\} = (\lambda, \mu)(0, 0).$$
This is a contradiction and we obtain (i).
(ii) Assume that there exist $x, y \in X$ such that $\lambda(x)$¡$\mu(0)$ and $\mu(x)$¡ $\mu(0)$. Then
$$(\lambda \times \mu)(0, 0) = \text{max} \left\{\lambda(0), \mu(0)\right\} = \mu(0).$$
It follows that
$$(\lambda \times \mu)(x, y) = \text{max} \left\{\lambda(x), \mu(y)\right\} \text{ ¡ } \mu(0) = (\lambda \times \mu)(0, 0),$$
which is a contradiction. Hence (ii) holds.
(iii) Similar to (ii). □

Theorem 31. *Let λ and μ be fuzzy subsets in a* BCK-*algebra X such that $\lambda \times \mu$ is an anti fuzzy implicative ideal of $X \times X$. Then either μ or λ is an anti fuzzy implicative ideal of X.*

Proof. By Theorem 30(i), without loss of generality we assume that $\mu(x) \geq \mu(0)$ for all $x \in X$. From (iii) it follows that either $\lambda(x) \geq \lambda(0)$ or $\mu(x) \geq \lambda(0)$. If $\mu(x) \geq \lambda(0)$ for all $x \in X$, then $(\lambda \times \mu)(0, x) = max\{\lambda(0), \mu(x)\} = \mu(x)$. Let $(x, x'), (y, y')$, $(z, z') \in X \times X$, since $\lambda \times \mu$ is an anti fuzzy implicative ideal of $X \times X$.We have

$$(\lambda \times \mu)(x, x') \leq max\{(\lambda \times \mu)(((x, x') * (y, y') * (x, x')) * (z, z')), (\lambda \times \mu)(z, z')\},$$
$$= max(\lambda \times \mu)((x * (y * x)) * z, (x' * (y' * x')) * z'), (\lambda \times \mu)(z, z')\}.$$

Putting$x = y = z = 0$, then

$$\mu(x') = (\lambda \times \mu)(0, x') \leq max\{(\lambda \times \mu)(0, (x' * (y' * x')) * z'), (\lambda \times \mu)(0, z')\},$$
$$= max\{max\{\lambda(0), \mu((x'(y' * x')) * z')\}, max\ \{\lambda(0), \mu(z')\}\},$$
$$= max\{\mu((x' * (y' * x')) * z'), \mu(z')\}.$$

This proves that μ is an anti fuzzy implicative ideal of X. The second part is similar. This completes the proof. □

5. Conclusion

We discussed the notion of anti fuzzy implicative ideals of *BCK*-algebras and gave several characterizations. Also, we introduced the notion of anti cartesian product of anti fuzzy implicative ideals. Our defintions probably can be applied in other kinds of anti ideals of *BCK*-algebras.

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