PROCESS CAPABILITY

Muhammad Asif
At 3-sigma away from process mean expect 99.73% of observations fall within these limits. At $+2\sigma$ expect 95.46%
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Process Capability

The ability of a process to meet product design/technical specifications

Conducted only when the process is normally distributed
HOW TO MEASURE $C_p$

$$C_p = \frac{(UTL-LTL)}{6\sigma}$$

$$C_p = \frac{6\sigma}{6\sigma} = 1$$
Process Capability Index: 
\( C_p \) -- Measure of *Potential Capability*

\[
C_p = \frac{\text{allowable process variation}}{\text{actual process variation}} = \frac{USL - LSL}{6\sigma}
\]

- \( C_p > 1 \)
- \( C_p = 1 \)
- \( C_p < 1 \)
$C_p = \frac{12\sigma}{6\sigma} = 2$

SIX SIGMA PROCESS
What value of \( cp \) is acceptable

- \( Cp<1.0 \): poor process
- \( Cp=1.0 \): so ok
- \( Cp=1.3 - 1.5 \): good
- \( Cp=2 \): Excellent, that is 6 sigma
A manufacturing process produces a certain part with a mean diameter of 2 inches and a standard deviation of 0.03 inches. The lower and upper engineering specification limits are 1.90 inches and 2.05 inches.

\[
C_p = \frac{USL - LSL}{6\sigma} = \frac{2.05 - 1.90}{6(0.03)} = 0.83
\]

*Process is not a capable process since \( C_p < 1.0 \)*
Process Capability Index: 
$C_{pk}$ -- Measure of Actual Capability

$$C_{pk} = \min \left[ \frac{\bar{X} - LSL}{3\sigma}, \frac{USL - \bar{X}}{3\sigma} \right]$$

$\sigma$ is the standard deviation of the process
WHY $C_{pk}$ IS NEEDED?

IS $C_p$ NOT ENOUGH?
Impact of Process Location on Process Capability

\[ \sigma = 2 \]

- \( C_p = 2.0 \)
- \( C_{pk} = 2.0 \)

- \( C_p = 2.0 \)
- \( C_{pk} = 1.5 \)

- \( C_p = 2.0 \)
- \( C_{pk} = 1.0 \)

- \( C_p = 2.0 \)
- \( C_{pk} = 0 \)
WHY Cpk IS NEEDED?

IS Cp NOT ENOUGH?

Cp TELS U ONLY ABOUT THE SMARTNESS OF CURVE

Cpk TELS U ABOUT THE POSITIONING / LOCATION OF THE CURVE
Process Capability Index

Example

A manufacturing process produces a certain part with a mean diameter of 2 inches and a standard deviation of 0.03 inches. The lower and upper engineering specification limits are 1.90 inches and 2.05 inches.

\[
C_{pk} = \min \left[ \frac{\bar{X} - LSL}{3\sigma}, \frac{USL - \bar{X}}{3\sigma} \right] = \min \left[ \frac{2 - 1.90}{3(0.03)}, \frac{2.05 - 2}{3(0.03)} \right]
\]

\[
= \min[1.11, 0.56] = 0.56
\]

Therefore, the process is not capable (the variation is too large and the process mean is not on target)
Six Sigma

- Is the relentless and rigorous pursuit of the reduction of variation in all critical processes to achieve continuous and breakthrough improvements that impact the bottom line of the organization and increase customer satisfaction.

- It is an organizational initiative designed to create manufacturing, service and administrative processes that produce approximately 3.4 defects per million Opportunities (DPMO).
WITH OUT ANY DEVIATION

- 317300 ppm outside (deviation)

Result: 317300 ppm outside (deviation)

- $\pm \sigma$: $68.27\%$
- $\pm 2\sigma$: $95.45\%$
- $\pm 3\sigma$: $99.73\%$
- $\pm 4\sigma$: $99.9937\%$
- $\pm 5\sigma$: $99.999943\%$
- $\pm 6\sigma$: $99.9999998\%$

Between $\pm 1\sigma$: 45500 ppm

Between $\pm 2\sigma$: 2700 ppm

Between $\pm 3\sigma$: 63 ppm

Between $\pm 4\sigma$: 0.57 ppm

Between $\pm 5\sigma$: 0.002 ppm
Distribution shifted $\pm 1.5$

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Sigma Level

Defects per Million Opportunities
DMAIC Model

- The model that is used to improve a process in Six Sigma management is called the DMAIC model. This stands for:
  - Define
  - Measure
  - Analyze
  - Improve
  - Control
LEADERSHIP

- Six Sigma must be implemented from the top-down.
Green Belts are project leaders who receive two weeks of training on the Six Sigma roadmap and essential elements of Statistical methodologies supporting Six Sigma projects.

Successful Green Belts are able to allocate 50% of their time to their four to six month Six Sigma Project.
An executive level business leader who facilitates the leadership, implementation, and deployment of Six Sigma philosophies.
Black Belts are project leaders who receive four weeks of training focusing on the Six Sigma Road map and extensive statistical methodologies. Successful Black Belts normally dedicate at least 75% of their time to 4-6 month Six Sigma Project.
Master Black Belt.

- Master Black Belt provides technical leadership to Six Sigma program.
- The mentor and teacher of the Six Sigma Black Belt and Six Sigma Green Belt.
6 Sigma Support Personnel

- 6σ Master Black Belts
- 6σ Black Belts
- 6σ Green Belts
- Process operators and service delivery personnel
Process capability calculation using Minitab

*Please refer to the examples discussed in class*
### Process Data
- **LSL**: 598
- **Target**: 600
- **USL**: 602
- **Sample Mean**: 599.548
- **Sample N**: 100
- **StDev(Within)**: 0.576429
- **StDev(Overall)**: 0.620865

### Potential (Within) Capability
- **Cp**: 1.16
- **CPL**: 0.90
- **CPU**: 1.42
- **Cpk**: 0.90
- **Cpm**: 0.87

### Overall Capability
- **Pp**: 1.07
- **PPL**: 0.83
- **PPU**: 1.32
- **Ppk**: 0.83
- **Cpm**: 0.87

### Observed Performance
- **PPM < LSL**: 10000.00
- **PPM > USL**: 0.00
- **PPM Total**: 10000.00

### Exp. Within Performance
- **PPM < LSL**: 3621.06
- **PPM > USL**: 10.51
- **PPM Total**: 3631.57

### Exp. Overall Performance
- **PPM < LSL**: 6328.16
- **PPM > USL**: 39.19
- **PPM Total**: 6367.35
Process Capability of Supp2

**Process Data**
- LSL: 598
- Target: 600
- USL: 602
- Sample Mean: 600.23
- Sample N: 100
- StDev(Within): 1.67231
- StDev(Overall): 1.87861

**Potential (Within) Capability**
- Cp: 0.40
- CPL: 0.44
- CPU: 0.35
- Cpk: 0.35

**Overall Capability**
- Pp: 0.35
- PPL: 0.40
- PPU: 0.31
- Ppk: 0.31
- Cpm: 0.35

**Observed Performance**
- PPM < LSL: 110000.00
- PPM > USL: 180000.00
- PPM Total: 290000.00

**Exp. Within Performance**
- PPM < LSL: 91186.50
- PPM > USL: 144933.01
- PPM Total: 236119.51

**Exp. Overall Performance**
- PPM < LSL: 117604.95
- PPM > USL: 173049.32
- PPM Total: 290654.27
Process Capability Sixpack of Supplier 2

Xbar Chart

Sample Mean

\[ \bar{X} = 600.23 \]

\[ \text{UCL} = 602.474 \]

\[ \text{LCL} = 597.986 \]

R Chart

Sample Range

\[ \bar{R} = 3.890 \]

\[ \text{UCL} = 8.225 \]

\[ \text{LCL} = 0 \]

Last 20 Subgroups

Normal Prob Plot

AD: 0.287, P: 0.615

Capability Plot

Within StDev: 1.67231
Cp: 0.4
Cpk: 0.35

Overall StDev: 1.87861
Pp: 0.35
Ppk: 0.31
Cpm: *