

**Course Code: ECON-322**

**Title: Advanced Mathematical Economics**

**Credit Hours: 03**

**Prerequisite: Calculus -I & Calculus -II**

**Course Objectives:**

In this course, particular focus is on applying the mathematical tools on economic theory and models. The level of sophistication at which the material is to be taught is indicated by the Course Contents of the prescribed textbooks.

### Learning Outcomes:

- On completion of this course, students would be able to
- Apply mathematical techniques for better understanding of economic theory
- Relate mathematical methods most relevant to economics and fully explain each method
- Illustrate how the method is applied in economic analysis
- Use mathematical tools for understanding the relationship between Micro and Macroeconomic variable

### Course Contents:

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| <b>First Order Linear Differential Equations</b>                                     | Meaning and Definition; Homogenous & non-Homogenous Cases. Solution of First Order Linear Differential Equation with Constant Coefficient & Constant Term and its Verification. Economic Applications: Dynamics of Market Price, Dynamics of Consumption using Differential Equations.                                                                                                                                                                                                                                                                                                                               |
| <b>Microeconomic Applications of First Order Linear Differential Equations:</b>      | Dynamics of Investment using Differential Equations. Dynamics of Value of Oil Well, Dynamics of Value of Gold Mine, Dynamics of Future Value of Investment, Dynamics of Fuel Consumption, Limited Growth, Dynamics of Ore Mining, Dynamics of Fish Stock.                                                                                                                                                                                                                                                                                                                                                            |
| <b>Macroeconomic Applications of First Order Linear Differential Equations:</b>      | National Income Determination using Differential Equations, Dynamics of Population, Dynamics of Government Spending, Dynamics of Unemployment.                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <b>First Order Differential Equation with Variable Coefficient and Variable Term</b> | Solution and its Verification of First Order Differential Equation with Variable Coefficient and Variable Term.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <b>Non-linear Differential Equations of the First Order and First Degree.</b>        | Exact Differential Equation, its Solution and Verification. Production Function using Exact Differential Equations, Utility Function using Exact Differential Equations, Average Cost Function using Exact Differential Equations, Point Elasticity Formula using Exact Differential Equations. Separable Variables. Dynamics of Capital using Separable Variables Method. Bernoulli Equation. The Qualitative Graphic Approach. Concept of Phase Diagram, types of Time Paths and their Dynamic Stability. Economic Application: Solow Growth Model: Qualitative Analysis, Phase Diagram and Quantitative Analysis. |

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| <b>Trigonometry and Complex Numbers</b>                   | Fundamentals of trigonometry, Trigonometric identities. Imaginary and Complex Numbers. Complex Roots. Circular Functions. Properties of Sine & Cosine functions. Euler Relations. Alternative Representation of Complex Numbers.                                                                                                                                                                                                                                                                                                                                                                                                        |
| <b>Second Order Differential Equations</b>                | Solution and Verification of Second Order Linear Differential Equations with Constant Coefficient and Constant term-Distinct Real Roots, Repeated Real Roots and Complex Root Cases. Dynamic Stability of Equilibrium. Economic Applications: A Market Model with Price Expectations. The Interaction of Inflation and Unemployment. Phillips Curve: The Phillips Relation, Expectations Augmented Phillips Relation, Feedback from Inflation to Unemployment, Time Path of Expected Rate of Inflation, Time Path of Expected Rate of Inflation – Numerical, Time Path of Inflation – Numerical, Time Path of Unemployment – Numerical. |
| <b>Differential Equations with Variable Term</b>          | Differential Equation with Variable Term, Differential Equation with Variable Term: A Modification.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| <b>First Order Difference Equations</b>                   | Solution and Verification of First Order Difference Equations. The Dynamic Stability of Equilibrium. Economic Applications – The Cobweb Model, A Market Model with Inventory. Harrod Model of Growth, Lagged Income Determination Model, Income Dynamics with Induced Investment, Capital Market, Money Market and Balanced Trade using Difference Equations. Investment Dynamics using Difference Equations. Fish Stock Analysis using Difference Equation, Difference Equations and Adaptive Expectations.                                                                                                                            |
| <b>Second Order Difference Equations</b>                  | Solution and Verification of Second-Order Linear Difference Equations with Constant Coefficients and Constant Term-Distinct Real Roots, Repeated Real Roots and Complex Roots cases. The Convergence of the Time Path. Economic Applications, Samuelson Multiplier-Acceleration Interaction Model. Inflation and Unemployment in Discrete Time. Second-Order Difference Equation with Variable Term, Second-Order Difference Equation with Variable Term ( $ct^n$ )                                                                                                                                                                     |
| <b>Higher Order Differential and Difference Equations</b> | Solution of Higher Order Differential Equations with Constant Coefficient and Constant Term. Convergence and the Routh Theorem. Higher Order Linear Difference Equations and their Solutions. Convergence and Schur Theorem.                                                                                                                                                                                                                                                                                                                                                                                                            |
| <b>Linear Programming</b>                                 | Ingredients of linear Programming. Graphical approach, simplex method, economic application of linear programming. Concept of                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |

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|--------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                | primal & dual. Duality theorems. Solving of Primal via dual. Economic interpretation of a dual.                                                                                                                                                                                                                                                                                       |
| <b>Non-Linear Programming:</b> | The Nature of Non Linear Programming Non-Linearities in Economics. Kuhn Tucker Condition. Interpretation of Kuhn Tucker Condition. Kuhn Tucker Sufficiency Theorem: Concave Programming. Arrow Enthoven Sufficiency Theorem: Quasiconcave Programming. Economic Application-Utility Maximization, Least Cost Combination. Solving a Nonlinear Program via the Kuhn-Tucker Conditions. |

**Teaching Methodology:**

- To deliver lectures on topics included in course outline
- To require each student to solve independent assignments on topics included in the course.

**Evaluation Criteria:**

| <b>Evaluation Method</b> |  |
|--------------------------|--|
| Quizzes/Assignments      |  |
| Mid-Term Exam            |  |
| Final-Term Exam          |  |

**Recommended Books:**

- Chiang A.C. Fundamental Methods of Mathematical Economics McGraw Hill –Latest Edition.
- Weber E. Jean, Mathematical Analysis, Business and Economic Application (latest edition).
- Bradley, T. Essential mathematics for economics and business. Latest edition. John Wiley & Sons.
- Dowling, E. T. Theory and Problems of Introduction to Mathematical Economics. Latest Edition, McGraw-Hill, New York.
- Hoffman, L. D., & Bradley, G. L. Calculus for business, economics, and the social and life sciences. McGraw-Hill. Latest edition.
- Kolman, B., & Beck, R. E. (1995). Elementary linear programming with applications. Academic Press.
- Sydsæter, K. and Hammond, P. Essential Mathematics for Economic Analysis. Latest edition., Pearson Education Limited, England.