UNIVERSITY OF THE PUNJAB

Third Semester – 2019

Examination: B.S. 4 Years Program

Roll No. in Fig. ....

Roll No. in Words. ....

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PAPER: Mathematics A-III **MAX. TIME: 30 Min.**

Course Code: MATH-201/MTH-21309 Part–I(Compulsory) **MAX. MARKS: 10**

.... **Signature of Supdt.:**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Division of marks is given in front of each question.

This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the right answer, cutting and overwriting is not allowed. (1x10=10)

(i)	If $\left(\overline{A}\right)' = -A$ then A is called **(a)** Symmetric matrix **(b)** Skew symmetric matrix **(c)** Hermitian matrix **(d)** Skew Hermitian matrix
(ii)	If W is a linear subspace of V then **(a)** $\dim(W) \le \dim(V)$ **(b)** $\dim(W) \ge \dim(V)$ **(c)** $\dim(W) = \dim(V)$ **(d)** None of these
(iii)	A set of linear equations is represented by the matrix equation Ax = b. The necessary condition for the existence of a solution for this system is **(a)** A must be invertible **(b)** b must be linearly depended on the columns of A **(c)** b must be linearly independent on the columns of A **(d)** none of these
(iv)	A unit vector orthogonal to both $(1, 1, 2)$ and $(0, 1, 3)$ in R^3 is ---------- **(a)** $\left(\frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ **(b)** $\left(\frac{-1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ **(c)** $\left(\frac{2}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}\right)$ **(d)** $\left(\frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$
(v)	The characteristic polynomial of the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ is............... **(a)** $\dim(W) = \dim(V)$ **(b)** $p(\lambda) = (2-\lambda)(3-\lambda)$ **(c)** $p(\lambda) = 0$ **(d)** None of these
(vi)	The property $\forall$ a, b$\in$R then $a+b=b+a=b \in R$ then a is called **(a)** Identity **(b)** Inverse **(c)** Conjugate **(d)** None of these
(vii)	The subspace of R^3 spanned by the vector (a, b, c) is ---------- **(a)** $x = t, y = bt, z = ct$ **(b)** $x = -at, y = -bt, z = -ct$ **(c)** $x = at, y = bt, z = ct$ **(d)** None of these
(viii)	Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T: R^3 \to R^3$ defined by T(x,y,z)=(x,y,z) is **(a)** Identity **(b)** Not Linear **(c)** Rational **(d)** None of these
(ix)	A linear transformation T that is both one-one and onto is has Ker(T) **(a)** 0 **(b)** 1 **(c)** 2 **(d)** None of these
(x)	Let V be vector space of finite dimension n. Then any $n+1$ or more vectors in V is --------- **(a)** linearly dependent **(b)** linearly independent **(c)** both (a) and (b) **(d)** none of these.

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PAPER: Mathematics A-III — **MAX. TIME: 2 Hrs. 30 Min.**

Course Code: MATH-201/MTH-21309 Part – II — **MAX. MARKS: 50**

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q.2. Questions with Short Answers. (5x4=20)

(i)	Decide yes or no about the following statement, if no justify by counter example. If A is 2×2 matrix such that $A^2 = \mathbf{0}$ Then A must be a null matrix-	(4)
(ii)	If possible express $\vec{v} = (2,-5,3)$ in R^3 as a linear combination of the vectors $\vec{u}_1 = (1,-3,2)$, $\vec{u}_2 = (2,-4-1)$, $\vec{u}_3 = (1,-5,7)$.	(4)
(iii)	Check whether W is a subspace of V or not. $V = \{f : \; f : R \to R\}$, $W = \{f \in V : \; f(1) = 0\}$.	(4)
(iv)	Show that $\det \begin{bmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{bmatrix} = (a-1)^3(a+3)$	(4)
(v)	If A and B are invertible matrices of same order and AB is invertible then show that $(AB)^{-1} = B^{-1}A^{-1}$.	(4)

P.T.O.

LONG QUESTIONS	(5x6=30)	
Q.3	Solve the following system of linear equations by using an appropriate method. $x_1 - 2x_2 + 3x_3 = 3$ $2x_1 + x_2 - x_3 = 9$ $-3x_1 - 4x_2 + 5x_3 = -15.$	(6)
Q.4	Define $T: R^3 \to R^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is T one-to-one?	(6)
Q.5	If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ determine the value of $A^2 - 4A - 5I$.	(6)
Q.6	Find the real orthogonal matrix P for which $P^{-1}AP$ is orthogonal where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$	(6)
Q.7	Express matrix M as a linear combination of the matrices A, B and C where $M = \begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}.$	(6)

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Q.1. Encircle the right answer, cutting and overwriting is not allowed. (1x10=10)

(i)	Rank of A is --------- Rank of A^T (a) equal (b) greater than (c) less than (d) None of these
(ii)	If A and B are *symmetric* matrices. Then AB is also (a) symmetric (b) skew-symmetric (c) Hermitian (d) None of these
(iii)	If A is a matrix of order 3×3 and $\det(A) = -1$, then the value of $\det(2A)$ is -------- (a) -8 (b) -6 (c) 8 (d) 18
(iv)	The dimension of $KerT$ is called .... (a) Rank (b) Nullity (c) basis (d) none of these
(v)	The characteristic polynomial of the matrix $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$ is............... (a) $p(\lambda) = (1-\lambda)^2$ (b) $p(\lambda) = (5-\lambda)(1-\lambda)$ (c) $p(\lambda) = 0$ (d) None of these
(vi)	If $(\overline{A})' = -A$ then A is called...................matrix (a) Symmetric (b) Skew symmetric (c) Hermitian (d) Skew Hermitian
(vii)	The property $\forall$ a, b $\in$R then $(a+b) = (b+a)$ is called (a) Closure property (b) Transitive property (c) Commutative property (d) Associative property
(viii)	A linear transformation $T: U \to V$ is one-to-one if and only if ---------- (a) $N(T) = \{0\}$ (b) $N(T) \neq \{0\}$ (c) $N(T) = \{1\}$ (d) $N(T) = \{-1\}$
(ix)	Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ is (a) Linear (b) Not Linear (c) Rational (d) None of these
(x)	A symmetric matrix of order n has -------- eigen values. (a) n-1 (b) n+1 (c) n (d) 0

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Q. 2	SHORT QUESTIONS	
(i)	Show that the functions $f(t) = \cos t,\ g(t) = \sin t,\ h(t) = t$ from R to R are linearly independent.	(4)
(ii)	Without evaluating, verify that $$\det\begin{bmatrix} b+c & b & c \\ c & c+a & a \\ b & a & a+b \end{bmatrix} = 2a(b^2+c^2).$$	(4)
(iii)	Show that the matrix $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is involutery.	(4)
(iv)	Check whether W is a subspace of V or not. $V = R^2$ and W is the set of first quadrant vectors of R^2.	(4)
(v)	For the given matrix find the basis of Column space. $$\begin{bmatrix} 1 & 2 & 2 & -1 \\ 3 & 6 & 5 & 0 \\ 1 & 2 & 1 & 0 \end{bmatrix}$$	(4)

P.T.O.

LONG QUESTIONS		
Q.3	Solve the system of linear equations by Gaussian elimination method $x+y+2z=9, \quad 2x+4y-3z=1, \quad 3x+6y-5z=0$	(6)
Q.4	If possible, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$	(6)
Q.5	For a real number k, consider the following system. For what value k does the system have a solution. $$\begin{aligned} x_1+x_2+kx_3+x_4 &= k \\ -x_2+x_3+2x_4 &= 0 \\ x_1+2x_2+x_3-x_4 &= -k. \end{aligned}$$	(6)
Q.6	Determine whether the vectors are linearly independent or not? $v_1=(1,-2,3), v_2=(5,6,-1), v_3=(3,2,1)$.	(6)
Q.7	If possible find a matrix P that diagonalize the following matrix. $$A=\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$	(6)