THE ANSWERS MUST BE ATTEMPTED ON THE ANSWER SHEET PROVIDED

Q.1. Solve the following.   **(6x5=30)**

(i) Show that the equation $3x^2 + 8xy - 3y^2 + 29x - 3y + 18 = 0$ represents a pair of straight lines and find an equation of each line.

(ii) Find the points at which the cardioid $r = 1 + \sin\theta$ has vertical tangents.

(iii) Find an equation of the plane through the straight line

$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and parallel to the straight line $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$.

(iv) Identify the surface defined by $z^2 - 4y^2 - 16x - 16y - 2z + 49 = 0$.

(v) Find the area of the region bounded by the loop of the curve $x^4 + y^4 = 2a^2xy$.

(vi) Find the asymptotes of the curve $r = a\sec\theta + b\tan\theta$.

Q.2. Solve the following.   **(3x10=30)**

(i) Find an equation to the cone whose vertex is the origin and directrix is the circle $x = a, y^2 + z^2 = b^2$. Show that the trace of the cone in a plane parallel to the xy-plane is a hyperbola.

(ii) Define envelope and find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where the relation between the parameters a and b is $a^n + b^n = c^n$, c is a constant.

(iii) Show that locus of the feet of the perpendiculars from the foci on any tangent to an ellipse is the auxiliary circle and product of the lengths of perpendiculars is equal to the square of the semi-minor axis.