Q.1. Solve the following: (6x5=30)

(i) Solve by any appropriate method $2xy\dfrac{dy}{dx} = y^2 - x^2.$

(ii) Solve the initial value problem $(x^4 + y^2)dx = xydy,\quad y(2) = 1.$

(iii) Find the particular solution $\dfrac{dr}{d\theta} = b\left[\cos\theta\, \dfrac{dr}{d\theta} + r\sin\theta\right],\ r\left(\dfrac{\pi}{2}\right) = \pi, 0 < b < 1.$

(iv) Find a general solution $\dfrac{d^2y}{dx^2} + 9y = 18x + 36\sin 3x.$

(v) Solve the nonhomogeneous ODE $\dfrac{d^2y}{dx^2} + y = \sec x.$

(vi) Find the general solution $\dfrac{dy}{dx} + x^2 y = e^{-x^3}\sinh x.$

Q.2. Solve the following: (3x10=30)

(i) Solve the nonhomogeneous Euler-Cauchy equation

$$x^3\frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} - 6y = x^4\ln x, \quad x > 0.$$

(ii) Find an integrating factor and solve the problem

$$(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0, \quad y(0) = -1.$$

(iii) Solve the initial value problem

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{0.5x} + 40\cos 10x - 190\sin 10x,$$

$$y(0) = 0.16, \quad \frac{dy}{dx}(x = 0) = 40.08.$$