UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Third Semester – Fall 2021

Roll No. ....

Paper: Pure Mathematics **Course Code: MATH-222** **Time: 3 Hrs. Marks: 60**

Q.1. Solve the following: (15x2=30)

i) For sets A, B and C, if $A \cap B = A \cap C$, determine whether $B = C$ or not? Justify your answer.

ii) If $f: A \to B$ is a function, then determine when $f \circ f$ can be defined? Also find $f \circ f$ for function $f: R \to R$ defined by $f(x) = x^2 + x + 2$.

iii) Determine domain and range of the function $f(x) = \frac{1+x}{x-5}$

iv) Define composition of two functions with example.

v) Find the sum of an infinite GP 3, 1, 1/3, 1/9…?

vi) Give an example to show that $A \cup (B \backslash C) \neq (A \cup B) \backslash (A \cup C)$

vii) State the rule of inference used in the argument ``Alice is a mathematics major. Therefore, Alice is either a mathematician major or a computer science major``.

viii) Define discrete topological space and construct this space on the set $A = \{1, 2, 3\}$.

ix) Define sub-base of a topological space with example.

x) What are the closed and open sets of indiscrete topological space?

xi) Let $X = \{a, b, c, d, e\}$. Determine whether or not the set $\tau_1 = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ is a topology. Justify your answer.

xii) Find the open balls $B(a; r)$ in discrete metric space X for $r \leq 1$,

xiii) For the set of real numbers R, write a mapping $d: R \times R \to R$ which is not a metric on R.

xiv) When a relation is a function?

xv) Construct the truth table for $(p \vee q) \to (p \wedge q)$.

Q.2. Solve the following. (5x6=30)

i) For sets A, B and C show that $\overline{A \cap B} = \bar{A} \cup \bar{B}$ and $(A - B) \cap (C - B) = \phi$ by
   (a) Showing each side is a subset of the other side.
   (b) Using membership table.

ii) Verify the distributive law $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

iii) Show that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$

iv) Show that the function $d(p, q) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$, where $p = (a_1, b_1)$ and $q = (a_2, b_2)$ is a metric on plane R^2

v) If $f: X \to Y$ is constant function defined as $f(x) = p \in Y, \forall x \in X$. Then prove that f is continuous relative to any topology τ on X and any topology τ^* on Y.