# UNIVERSITY OF THE PUNJAB

**B.S. 4 Years Program : Fifth Semester – Fall 2021**

**Roll No. ……………**

**Paper: Classical Mechanics** | **Course Code: PHY-301** | **Time: 3 Hrs.** | **Marks: 60**

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**Q.1 Solve the following:** **(6x5=30)**

1. Show that the total angular momentum of a system of particles is equal to the sum of angular momentum of center of mass and the angular momentum of the particles about the center of mass.
2. State D'Alembert's Principle and use it to derive the Lagrange's equation of motion
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i\ .$$
3. The spherical pendulum has Lagrangian
$$L = \frac{1}{2}ml^2\left(\dot{\theta}^2 + \sin^2\theta\ \dot{\varphi}^2\right) - mgl\cos\theta$$
Write down the Euler-Lagrange equations of motion.
4. Show that the Poisson bracket of two integrals of motion is again an integral of motion.
5. A sphere of radius $\rho$ is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of inside radius $R$. Determine the Lagrange function, the equation of constraint and Lagrange's equation of motion. Find the frequency of small oscillation.
6. If $L$ is a Lagrangian for a system of $n$ degrees of freedom satisfying Lagrange equation of motion, show by direct substitution that
$$L' = L + \frac{d}{dt}F(q_1, \cdots, q_n; t),$$
also satisfies the Lagrange's equation of motion where $F$ is an arbitrary differentiable function of its argument.

**Q.2. Solve the following.** **(3x10=30)**

1. Show that the Poisson bracket satisfies the Jacobi identity $\{f,\{g,h\}\} + \{g,\{h,f\}\} + \{h,\{f,g\}\} = 0.$ (10)
2. Consider the motion of a particle in a central force field
$$V(r) = -\frac{k}{r}.$$
Write down the Lagrangian in polar coordinates and integrate the equation of motion to derive
$$\theta(r) = \int \frac{l\ dr}{r^2\sqrt{2\mu\left(E + \frac{k}{r} - \frac{l^2}{2\mu\ r^2}\right)}} + \text{constant},$$
where $E$ is the total energy and $l$ is the angular momentum. Now change variables as $u = \frac{l}{r}$ to derive the equation of a conic section
$$\frac{\alpha}{r} = 1 + \varepsilon\cos\theta.$$
3. Show that the transformation
$$P = q\cot p,$$
$$Q = \ln\left(\frac{\sin p}{q}\right),$$
is canonical and also show that the corresponding generating function is
$$F = e^{-Q}\left(1 - q^2e^{2Q}\right)^{\frac{1}{2}} + q\sin^{-1}\left(qe^{Q}\right)$$