UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : Fifth Semester – Fall 2021

Roll No. ....

Paper: Mathematical Methods of Physics-I **Course Code: PHY-302** **Time: 3 Hrs.** **Marks: 60**

Q.1. Solve the following: **(6x5=30)**

1. With $\hat{\mathbf{e}}_1$ a unit vector in the direction of increasing u_1, show that

$$\nabla.\hat{\mathbf{e}}_1 = \frac{1}{h_1 h_2 h_3}\frac{\partial}{\partial u_1}(h_2 h_3)$$

2. Show that $\mathbf{r} = \rho\hat{\mathbf{e}}_\rho + z\hat{\mathbf{e}}_z$. Working entirely in circular cylinderical coordinates also prove that $\nabla.\mathbf{r} = 3$ and $\nabla \times \mathbf{r} = 0$. (Note: $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$)

3. Given $A_k = \frac{1}{2}\varepsilon_{ijk}B^{ij}$, with $B^{ij} = -B^{ji}$, antisymmetric tensor, show that

$$B^{mn} = \varepsilon^{mnk}A_k.$$

4. Show that

$$\frac{1}{2\pi i}\oint_C z^{m-n-1}dz, \qquad m \text{ and } n \text{ integers}$$

(with the contour encircling the origin once counterclockwise) is a representation of the Kronecker δ_{mn}.

5. Evaluate

$$\oint_{|z-2i|=4}\frac{z}{z^2+9}dz,$$

by using Cauchy's integral formula.

6. Use the divergence theorem to establish Green's identity

$$\iint_S (f\nabla g).\mathbf{n}dS = \iiint_D \left(f\nabla^2 g + \nabla f.\nabla g\right)dV,$$

here f and g are scalar functions with continuous second partial derivatives.

Q.2. Solve the following. **(5x6=30)**

1. Evaluate the Cauchy principal value of

$$\int_{-\infty}^{+\infty}\frac{dx}{(x^2+a^2)^2}(a>0).$$

2. The derivative of the second-order tensor $\mathbf{T}$ with respect to the coordinate u^k, find an expression for the covariant derivative $T^{ij}_{;k}$ of its contravariant components.

3. Calculate the elements g_{ij} of the metric tensor for spherical polar coordinate. Also calculate the Γ^k_{ij} for spherical polar coordinates. (Note: $x = r\cos\phi\sin\theta$, $y = \rho\sin\phi\sin\theta$, $z = r\cos\theta$)

4. Use the Cauchy's residue theorem to evaluate

$$\oint_{|z-1|=2}\frac{\tan z}{z}dz.$$

5. Find the spherical coordinate components of the velocity and acceleration of a moving particle. ($x = r\cos\phi\sin\theta$, $y = \rho\sin\phi\sin\theta$, $z = r\cos\theta$)