UNIVERSITY OF THE PUNJAB

Roll No. in Fig. ....

B.S. 4 Years Program / Fourth Semester – 2019

Roll No. in Words. ....

Paper: Differential Equations-II
Course Code: MATH-223 / MTH-22334 Part–I (Compulsory) Time: 30 Min. Marks: 10

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Signature of Supdt.:

ATTEMPT THIS PAPER ON THIS QUESTION SHEET ONLY.
Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Fill in the blank or answer True / False. **(10x1=10)**

1. $\mathcal{L}\{\frac{df(t)}{dt}\} =$ ....
2. $\mathcal{L}\{\delta(t-t_0)\} =$ ....
3. $\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} =$ ....
4. $x =$ ....is a regular singular point of the second order linear differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - p^2)y = 0$.
5. $J_{1/2}(x) =$ ....
6. $P_2(x) = \frac{1}{2}\left(3x^2 - 1\right)$. (True/False)
7. $\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} =$ ....
8. The equation $\frac{d^2y}{dx^2} + \sin y = 0$ is a 2nd order linear differential equation. (True/False)
9. $y = 0$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 4y = 0$. (True/False)
10. The general solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (k^2x^2 - \nu^2)y = 0$ is $y = c_1J_\nu(x) + c_2J_{-\nu}(x)$, where ν is an integer. (True/False)

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Fourth Semester – 2019

Roll No. ....

Paper: Differential Equations-II
Course Code: MATH-223 / MTH-22334 Part – II

Time: 2 Hrs. 30 Min. Marks: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Section-I (Short Questions) Marks=20

1. Show that

$$J_0'(x) = J_{-1}(x) = J_{-1}(x).$$

2. Show that

$$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}.$$

3. Determine the singular points of the differential equation $x\left(x^2+1\right)^2 \frac{d^2y}{dx^2} + y = 0$. Classify each singular point as regular or irregular.

4. Use the Laplace transformation to solve the differential equation.

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} = 6e^{3t} - 3e^{-t}, \qquad y(0) = 1, \qquad y'(0) = -1.$$

5. Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\}$.

Section-II Marks=30

1. Use the substitution $y(x) = x^{1/2}z(x)$, find the general solution of differential equation $x^2\frac{d^2y}{dx^2} + (a^2x^2 - \nu^2 + \frac{1}{4})y = 0$.

2. Find the general solution of the differential equation

$$x\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + xy = 0.$$

3. Use the Laplace transform to solve the given integral equation

$$f(t) = e^t + e^t\int_0^t f(\tau)e^{-\tau}d\tau, \text{ for } f(t).$$

4. Evaluate the Laplace transform

$$\mathcal{L}\left\{t\int_0^t \sin\tau\, d\tau\right\}.$$

Note: Do not evaluate the integral before transforming.

5. Find a one-parameter family of solutions for the differential equation

$$\frac{dy}{dx} = y^2 - \frac{1}{x}y - \frac{4}{x^2},$$

where $y_1(x) = \frac{2}{x}$ is a known solution of the equation.