UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Fourth Semester – 2019

Roll No. in Fig. ....

Roll No. in Words. ....

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Signature of Supdt.:

Paper: Linear Algebra

Course Code: MATH-224 / MTH-22120 Part–I (Compulsory) Time: 30 Min. Marks: 10

ATTEMPT THIS PAPER ON THIS QUESTION SHEET ONLY.

Division of marks is given in front of each question.

This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the correct choice. **(1x10=10)**

(i)	Every non-zero vector in a vector space over a field is ... a) Linearly independent b) Linearly dependent c) Basis d) Subspace
(ii)	A complete Normed space is called... a) Normed space b) Bancach space c) Hilbert space d) Not Given
(iii)	The number of subgroups of the cyclic group of order 49 is ... a) 1 b) 2 c) 3 d) Not Given
(iv)	The number of generators in a cyclic group of order 15 is ... (a) 8 (b) 10 (c) 3 (d) 12 e) Not Given
(v)	The order of the smallest Abelian group is... a) 3 b) 2 c) 1 d) Not Given
(vi)	Let $T: R^{15} \to R^{15}$ be a linear transformation. If the dimension of the null space of T is 12, then the dimension of $R(T)$ is a) 15 b) 5 c) 3 d) Not Given
(vii)	The trace of a matrix $\begin{bmatrix} -11 & 3 & 9 \\ 3 & 8 & 4 \\ 6 & 1 & 13 \end{bmatrix}$ is... (a) 8 (b) 4 (c) 10 d) Not Given
(viii)	In the group (Z, o) of all integers, where $aob = \frac{1}{2}ab$ for $a, b \in Z$, the inverse of 4 is a) 0 b) c) 3 d) 4 e) Not given
(ix)	The group $C_2 \times C_2$ is isomorphic to a) C_4 b) V_4 c) S_3 d) Not given
(x)	The di ension of the vector space of reals over complex numbers, namely $R(C)$ is... a) 1 b) 2 c) infinite d) Not Given

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B.S. 4 Years Program / Fourth Semester – 2019

Roll No. ....

Paper: Linear Algebra
Course Code: MATH-224 / MTH-22120 Part – II **Time: 2 Hrs. 30 Min. Marks: 50**

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDE

Q. 2	Short Questions (4x5 = 20 Marks)
(i)	Prove that every finite dimensional vector space contains a basis.
(ii)	Let $G = < a, b \mid a^3 = b^2 = (ab)^2 = e >$. Find all subgroups of G.
(iii)	Define reducible and irreducible representations and state Schurs Lemma without proof.
(iv)	Prove that $Q(\sqrt{2}) = \{a + \sqrt{2}b : a, b \in R\}$ is a vector space over R, where R is the set of real numbers.
(v)	State and explain Gram-Schmidt method with one example.
	Long Questions (6x5 = 30 Marks)
Q.3	Solve the following system of equations $$\begin{aligned} 3x - 2y + 3z &= 8 \\ x + 3y + 6z &= -3 \\ 2x + 6y + 12z &= -6 \end{aligned}$$
Q.4	Prove that in an n-dimensional vector space, any set of n+1 or more vectors is linearly dependent.
Q.5	For a system of linear equations in n unknowns, if augmented matrix $M = [A, B]$, then prove that: (a) The system has a solution if and only if rank (A) = rank (M). (b) The solution is unique if and only if rank (A) = rank (M) = n.
Q.6	Diagonalize the matrix (if possible) $$\begin{bmatrix} 4 & -3 & -3 \\ 3 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix}$$
Q.7	Prove that the set of all bijective mappings on a set $G = \{1,2,3\}$ to G form a group .