UNIVERSITY OF THE PUNJAB

Fifth Semester – 2019

Examination: B.S. 4 Years Program

Roll No. in Fig. .....

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Signature of Supdt.:

PAPER: Classical Mechanics **MAX. TIME: 15 Min.**

Course Code: PHY-301 Part–I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.
Please encircle the correct option. Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the right answer, cutting and overwriting is not allowed. (1x10=10)

(i) The degree of freedom of a two paticles system moving freely in space is

(a) 4
(b) 2
(c) 3
(d) 6

(ii) Hamiltonian of a system describes its

(a) time evolution
(b) space evolution
(c) state
(d) constraints

(iii) The brachistochorone problem is the

(a) least area problem
(b) least distance problem
(c) least time problem
(d) least energy problem

(iv) If the Lagrangian does not involve a particular coordinate q_i such coordinate is called

(a) Angle coordinate.
(b) Ficticious coordinate
(c) Complete coordinate
(d) Cyclic coordinate.

(v) Equation of conics $r = \frac{h}{1+e\cos\theta}$ draws a parabola when

(a) $e = 1$
(b) $e > 1$
(c) $e < 1$
(d) $e = 0$

P.T.O.

(vi) The canonical transformations preserve

(a) Lagrange equations
(b) Hamilton's equations
(c) Poisson equations
(d) None

(vii) The shortest distance between two points on a curved surface is

(a) a straight line
(b) a geodesic
(c) a tangent
(d) a semi-circle

(viii) If the Lagrangian is cyclic in q_i , then:

(a) p_i is not conserved.
(b) p_i is conserved.
(c) q_i appears in the Lagrangian
(d) $\dot{q}_i$ does not appear in the Lagrangian

(ix) Scleronomous constraints have

(a) explicit time dependence
(b) no explicit time dependence
(c) explicit force dependence
(d) no explicit force dependence

(x) Kepler's second Law of planetry motion directly follows from

(a) Conservation of linear momentum
(b) Conservation of angular momentum
(c) Homogeniety of time
(d) Homogeneity of space

UNIVERSITY OF THE PUNJAB

Fifth Semester – 2019

Examination: B.S. 4 Years Program

Roll No. .....

PAPER: Classical Mechanics
Course Code: PHY-301 Part – II

MAX. TIME: 2 Hrs. 45 Min.
MAX. MARKS: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q 2. Show explicitly that (5)

$$\frac{\partial \mathbf{x}}{\partial q_i} = \frac{\partial \dot{\mathbf{x}}}{\partial \dot{q}_i}$$

where $\mathbf{x} = \mathbf{x}(q_1, q_2, \cdots, q_n, t)$.

Q 3. State Hamilton's principle of least action and use it to derive (5)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0.$$

Q 4. If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange equation of motion, show by direct substitution that (5)

$$L' = L + \frac{d}{dt}F(q_1, \cdots, q_n; t),$$

also satisfies the Lagrange's equation of motion where F is an arbitrary differentiable function of its argument.

Q 5. Consider a one parameter family of transformations (5)

$$q_i(t) \to Q_i(s, t) \qquad s \in \mathbb{R}$$

such that $Q_i(0, t) = q_i(t)$. Show that if the Lagrangian is invariant under this transformation, then there exists a conserved quantity (Noether's Theorem)

Q 6. Consider the motion of a particle in a central force field (10)

$$V(r) = -\frac{k}{r}.$$

Write down the Lagrangian in polar coordinates and integrate the equation of motion to derive

$$\theta(r) = \int \frac{l\, dr}{r^2\sqrt{2\mu\left(E + \frac{k}{r} - \frac{l^2}{2\mu\, r^2}\right)}} + \text{constant},$$

P.T.O.

where E is the total energy and l is the angular momentum. Now change variables as $u = \frac{l}{r}$ to derive the equation of a conic section

$$\frac{\alpha}{r} = 1 + \varepsilon \cos\theta.$$

Q 7. (a) Show that the transformation (5)

$$\begin{aligned} q &= PQ^2 \\ p &= \frac{1}{Q}, \end{aligned}$$

is canonical and also show that the corresponding generating function is

$$F = \frac{q}{Q}$$

(b) Find the force law for a central force field that allows a particle to move in a logarithmic spiral orbit given by (5)

$$r = k\exp(\alpha\theta)$$

where k and α are constants

Q 8. (a) Show that, if a transformation from (q, p) to (Q, P) be canonical then the bilinear form (5)

$$\sum_i \left(\delta p_i dq_i - \delta q_i dp_i\right),$$

is invariant under the canonical transformation.

(b) Show that the equation of a curve for which surface area is minimum is a catenary (5)

$$x = a\cosh\frac{y-b}{a}$$

where a and b are constants.