## Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Division of marks is given in front of each question. This Paper will be collected back after expiry of time limit mentioned above.
Q.1. Fill in the Blanks or answer True / False.

1. The process of contraction of an $N$ th-order tensor produces another tensor, of order $N-4$. (True/False)
2. A vector field A is solenoidal if $\nabla \cdot \mathrm{A}=0, \quad$ (True/False)
3. $\oint_{|z|=2}\left(z+\frac{1}{z^{2}}\right) d z=$
4. $f(z)=\frac{2 z+5}{(z-1)(z+5)(z-2)^{4}}$ has simple poles at $z=1$ and $z=-5$, and a pole of order 4 at $z=2$. (True/False)
5. $z=0$ is a removable singularity of $f(z)=\frac{e^{2 x}-1}{z}$.
6. The quantities $g_{i j}=\mathrm{e}_{i} \cdot \mathrm{e}_{j}$ form the covariant components of a second-order tensor. (True/False)
7. The Christoffel symbol $\Gamma_{i j}^{m}$ in term of metric tensor is given by $\Gamma_{i j}^{m}=$ $\qquad$
8. $\operatorname{Re}\left((1+i)^{10}\right)=\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$..................... $\operatorname{Im}\left((1+i)^{10}\right)=$
9. If $f(z)=a_{0} z^{0}+a_{1} z+a_{2} z^{2}+\ldots+a_{n} z^{n}$ and $C$ is a simple closed contour, then $\oint_{C} f(z) d z=$
10. $\oint_{|z|=3} \frac{z}{z^{2}-\pi^{2}} d z=$

## Q.2. Questions with Short Answers.

1. If $\hat{e}_{1}$ a unit vector in the direction of increasing $u_{1}$, show that

$$
\nabla \cdot \hat{e}_{1}=\frac{1}{h_{1} h_{2} h_{3}} \frac{\partial\left(h_{2} h_{3}\right)}{\partial u_{1}} .
$$

2. Use the divergence theorem to establish following identities

$$
\iint_{S}(f \nabla g) \cdot n d S=\iiint_{D}\left(f \nabla^{2} g+\nabla f \cdot \nabla g\right) d V .
$$

here $f$ and $g$ are scalar functions with continuous second partial derivatives.
3. Given $A_{k}=\frac{1}{2} \varepsilon_{i j k} B^{i j}$, with $B^{i j}=-B^{j i}$, antisymmetric tensor, show that

$$
B^{m n}=\varepsilon^{m n k} A_{k}
$$

4. Determine the order of the poles of $f(z)=\frac{\cot (\pi z)}{z^{2}}$.
5. Suppose $z_{0}$ is any constant complex number interior to any simple closed contour $C$. Show that

$$
\oint_{C} \frac{d z}{\left(z-z_{0}\right)^{n}}=\left\{\begin{array}{c}
2 \pi i, \quad n=1 \\
0, n \text { is a positive integer } \neq 1
\end{array}\right.
$$

## Q.3. Questions with Brief Answers.

1. Show that

$$
\int_{0}^{\pi} \frac{d \theta}{(a+b \cos \theta)^{2}}=\frac{\pi a}{\left(a^{2}-1\right)^{\frac{3}{2}}}
$$

2. Show that

$$
\Gamma_{i j}^{m}=\frac{1}{2} g^{m k k}\left(\frac{\partial g_{j k}}{\partial u^{i}}+\frac{\partial g_{k i}}{\partial u^{j}}-\frac{\partial g_{i j}}{\partial u^{k}}\right)
$$

3. Using Cauchy's integral formula for derivatives, evaluate

$$
\oint \frac{z+1}{z^{4}+4 z^{3}} d z
$$

where $C$ is the circle $|z|=1$
4. Evaluate

$$
\int_{-\infty}^{+\infty} \frac{x^{2}}{1+x^{4}} d x
$$

5. Find the spherical coordinate components of the velocity and acceleration of a moving particle. ( $x=$ $r \cos \phi \sin \theta, y=\rho \sin \phi \sin \theta, z=r \cos \theta)$
