UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2019

Paper: Mathematical Methods of Physics-II
Course Code: PHY-307 Part – I (Compulsory) Time: 15 Min. Marks: 10

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ATTEMPT THIS PAPER ON THIS QUESTION SHEET ONLY.
Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Fill in the blanks or answer True / False. (1x10=10)

1. $\left(\frac{d}{dx}\right)^{1/2} + k^2$ is a linear operator. (True/False)
2. $(k-1)! = \Gamma(k)$ (True/False)
3. $\mathcal{F}\left[\frac{d^2}{dt^2} f(t)\right] = -\omega^2 g(\omega)$. (True/False)
4. If χ is a solution of Laplace's equation $\nabla^2\chi = 0$, then $\frac{\partial\chi}{\partial x}$ is not a solution of Laplace's equation. (True/False)
5. Legendre equation $(1-x^2)y'' - 2xy' + 2a(a+1)y = 0$ has singularity at $x = \pm 1$. (True/False)
6. $\Gamma(z) = \lim_{n\to\infty} \frac{1.2.3....n}{z(z+1)(z+2)...(z+n)} n^z, \quad n \neq 0, -1, -2, ...,$. (True/False)
7. $\delta(1-x) = \sum\limits_{n=0}^{\infty} \frac{2n+1}{2} P_n(x)$. (True/False)
8. $\Gamma(5/2) =$ ....
9. $\mathcal{L}\left\{f'''(t)\right\} =$ ....
10. $\mathcal{L}\left\{\cos(at)\right\} =$ ....

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2019

Roll No. ....

Paper: Mathematical Methods of Physics-II
Course Code: PHY-307 Part – II

Time: 2 Hrs. 45 Min. Marks: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Section-II (Short Questions) Marks=20

1. Linear operator A has n distinct eigenvalues and n corresponding eigenfunctions: $A\psi_i = \lambda_i\ \psi_i$. Show that the n eigenfunctions are linearly independent. Do not assume A to be Hermitian.

2. A triangular wave is represented by

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ -x, & -\pi < x < 0. \end{cases}$$

Represent $f(x)$ by a Fourier series.

3. Show that

$$\mathcal{F}\left[f^*(-t)\right] = F^*(\omega),$$

where $F(\omega) = \mathcal{F}\left[f(t)\right]$.

4. Using

$$\Gamma(z) = \lim_{n\to\infty} \frac{1.2.3....n}{z\,(z+1)\,(z+2)\,...\,(z+n)} n^z, \qquad n \neq 0, -1, -2, ...;$$

show that

$$\Gamma(z+1) = z\Gamma(z).$$

5. Evalute $\int_{-\infty}^{+\infty} x^3 e^{-x^2} H_n(x) H_n(x) dx$.

Section-III Marks=30

1. Show that

$$G(x,t) = \begin{cases} x, & 0 \le x \le t, \\ t, & t \le x \le 1, \end{cases}$$

is the Green's function for the operator $L = -\frac{d^2y}{dx^2}$ and the boundary conditions $y(0) = 0, y'(1) = 0$.

2. Verify the Dirac delta function expansions

$$\delta\,(1-x) = \sum_{n=0}^{\infty} \frac{2n+1}{2} P_n(x),$$
$$\delta\,(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{2n+1}{2} P_n(x).$$

3. Expand x^r in a series of associated Laguerre's polynomials $L_n^{(k)}$, with k fixed and n ranging from 0 to r (or to ∞ if r is not an integer).

4. Show, by direct differentiation, that,

$$J_\nu(x) = \sum_{s=0}^{\infty} \frac{(-1)^s}{s!\Gamma(s+\nu+1)} \left(\frac{x}{2}\right)^{\nu+2s},$$

satisfies the two recurrence relations

$$J_{\nu-1}(x) + J_{\nu+1}(x) = \frac{2\nu}{x} J_\nu(x),$$
$$J_{\nu-1}(x) - J_{\nu+1}(x) = 2\frac{dJ_\nu(x)}{dx}.$$

5. Evaluate

$$\int_{-1}^{1} (1+x)^a\,(1-x)^b\,dx,$$

in terms of the beta function.