



UNIVERSITY OF THE PUNJAB

B.S. in Computer Science First Year : Annual-2021

Roll No.

Subject: Calculus I

Paper: 1

Time: 2 Hrs. 30 Min. Marks: 80

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Question No. 2:

- a. Write any four properties of absolute values. (4 marks)
- b. Solve an inequality below and express the solution set as an interval or union of the intervals. (8 marks)

$$x^{-2} - 4x^{-1} + 4 > 0$$

- c. Say whether the functions are even, odd or neither. (8 marks)

$$f(x) = 3, f(x) = \frac{1}{x^2-1}, f(x) = \frac{1}{x-1} \text{ and } f(h) = |h^3|,$$

Question# 3

- a. Write an equation for the line through (-2, -1) and (3, 4). (4 marks)
- b. Find the average rate of change of the functions over the given interval. (8 marks)

i. $f(x) = x^3 + 1,$ $[-1, 1]$

ii. $f(t) = \cot t,$ $[\frac{\pi}{4}, \frac{3\pi}{4}]$

Find $\lim_{x \rightarrow 1} \cos\left(\frac{x^2-1}{x-1}\right)$

- c. Find the discontinuities of $f(x) = \cos \frac{x}{x-\pi}$, if any. Determine

$f(t) = \frac{\ln \tan^{-1} x}{x^2-9}$ is continuous. (8 marks)

Question No 4

- a. Confirm that the stated formula is the local linear approximation of f at $x_0 = 1$, where $\Delta x = x - 1$. (4 marks)

$f(x) = x^4;$ $(1 + \Delta x)^4 \approx 1 + 4\Delta x$

- b. Find the limit $\lim_{x \rightarrow 0^+} x^{\sin x}$. (8 marks)

- c.
 - i. Find an interval $[a, b]$ on which $f(x) = x^4 + x^3 - x^2 + x - 2$ satisfies the hypotheses of Rolle's Theorem.
 - ii. Generate the graph of $f'(x)$, and use it to make rough estimates of all values of c in the interval obtained in part (i) that satisfy the conclusion of Rolle's Theorem.
 - iii. Use Newton's Method to improve on the rough estimates obtained in part (ii). (8 marks)

Question No 5

- a. Write any four properties of indefinite integral. (4 marks)
- b. Evaluate (8 marks)
- i. $\int \left(\frac{1}{x} + \sec^2 \pi x \right) dx$
- ii. $\int \sin^2 x \cos x dx$
- c. Appropriate formulas from geometry to evaluate the integrals (8 marks)
- i. $\int_{-1}^3 (4 - 5x) dx.$
- ii. $\int_{-3}^0 (2 + \sqrt{9 - x^2}) dx.$

Question No 6

- a. Define linear equation in terms of differential equation with its type and examples. (4 marks)
- b. A tank with a 1000 gal capacity initially contains 500 gal of water that is polluted with 50 lb of particulate matter. At time $t = 0$, pure water is added at a rate of 20 gal/min and the mixed solution is drained off at a rate of 10 gal/min. How much particulate matter is in the tank when it reaches the point of overflowing? (8 marks)
- c. Solve the differential equation $(x^2 + 1) \frac{dy}{dx} + xy = 0$ by the method of integrating factors. Solve the initial-value problem (8 marks)

$$x \frac{dy}{dx} + y = x, y(1) = 2.$$

Question No 7

- a. Sketch the graphs of the ellipses $\frac{x^2}{9} + \frac{y^2}{16} = 1.$ (4 marks)
- b. Describe the graph of the equation (8 marks)
- $$y^2 - 8x - 6y - 23 = 0$$
- c. Sketch the hyperbola, and label the vertices, foci, and asymptotes. (8 marks)
- $$16x^2 - y^2 - 32x - 6y = 57$$