UNIVERSITY OF THE PUNJAB

Part-I A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics (Old & New Course)
PAPER: I (Real Analysis)

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Note: Attempt FIVE questions in all, selecting at least TWO from each Section

Section-I

Q.1(a) Define Supremum and infimum of a set. Find the Supremum and infimum of the set $E=\{x\in R: x^2\le 5\}$.

(b) Define an ordered field. Show that in every ordered field F, $x,y,z\in F$; we have

(i) If $x>0$ and $y<z$, then $xy<xz$.

(ii) If $x<0$ and $y<z$, then $xy>xz$. (10+10)

Q.2(a) If $p>0$, then $\lim\limits_{n\to\infty}(p)^{1/n}=1$.

(b) Discuss the convergence of the series $\sum\limits_{n=1}^{\infty}\frac{\sin nx}{1+n^2}$. (10+10)

Q.3(a) State and prove the Cauchy condensation test.

(b) Examine the continuity of $f(x)$ at $x=1$ $f(x)=\begin{cases}2x & if\ \ 0\le x<1\\ 3 & if\ \ x=1\\ 2x & if\ \ 1<x\le 2\end{cases}$ (10+10)

Q.4(a) If f and g are continuous real functions on $[a,b]$ which are differentiable on $]a,b[$, then there exists a point $x\in(a,b)$ at which $[f(b)-f(a)]g'(x)=[g(b)-g(a)]f'(x)$.

(b) Discuss the differentiability at $x=0$ and $x=2$ of the function $f(x)=|x|+|x-2|$. (10+10)

Q.5(a) Let $f(x)=\begin{cases}x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right) & if\ (x,y)\ne(0,0)\\ 0 & if\ (x,y)=(0,0)\end{cases}$. Show that $f_{yx}(0,0)=f_{xy}(0,0)$.

(b) Find the extreme values of $f(x,y)=3xy$, subject to the constraint $x^2+2y^2-4=0$. (10+10)

Section-II

Q.6(a) If $f\in R(\alpha)$ on $I=[a,b]$, then show that $|f|\in R(\alpha)$ on I and then $\left|\int\limits_a^b f\,d\alpha\right|\le\int\limits_a^b|f|\,d\alpha$

(b) If $f_1\in R(\alpha)$ $f_2\in R(\alpha)$ on $[a,b]$, then prove that $f_1+f_2\in R(\alpha)$ and $\int\limits_a^b(f_1+f_2)\,d\alpha=\int\limits_a^b f_1\,d\alpha+\int\limits_a^b f_2\,d\alpha$. (10+10)

Q.7(a) Let $f\in R(\alpha)$ on $[a,b]$. Prove that if there is a differential function on $[a,b]$ such that $F'=f$, then $\int\limits_a^b f(x)dx=F(b)-F(a)$.

(b) If f and g are of bounded variation on $[a,b]$, then $f+g$ and $f.g$ are also of bounded variation On $[a,b]$. (10+10)

Q.8(a) Test for the uniform convergence of the series $\sum\limits_{n=1}^{\infty}\frac{e^{-nx}}{2n^2-1}; 0<x<\infty$.

(b) Show that the series $\sum\limits_{k=1}^{\infty}(xe^{-x})^k$ converges uniformly on the interval $[0,1]$. (10+10)

Q.9(a) Show that $\int\limits_2^{\infty}\frac{\cos x}{\log x}dx$ is conditionally convergent.

(b) Show that $\int\limits_0^1(x-1)^{m-1}x^{n-1}dx$ is convergent if $m>0, n>0$. (10+10)

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(b) If $f_1 \in R(\alpha)$ $f_2 \in R(\alpha)$ on $[a,b]$, then prove that $f_1+f_2 \in R(\alpha)$ and $\int_a^b (f_1+f_2)\,d\alpha = \int_a^b f_1\,d\alpha + \int_a^b f_2\,d\alpha$. (10+10)

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Part-I A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics (Old & New Course) **TIME ALLOWED: 3 hrs.**
PAPER: II (Algebra) **MAX. MARKS: 100**

***NOTE:* Attempt any FIVE questions in all selecting at least two questions from each section.**

		SECTION-I	Marks
Q.1	(a)	Show that any two cyclic groups of the same order are isomorphic.	(10)
	(b)	Let G be a group of prime order p. Show that G is cyclic group. Prove that a group of order 15 is cyclic.	(10)
Q.2	(a)	Prove that any group G can be embedded in a group of bijective mappings of a certain set.	(10)
	(b)	Define (i) Z (G), Centre of a group G (ii) N_G(H), and Normalizer of subset H of G. Show that centre of group is always its normal subgroup. Also, give an example of a group G and its subset H whose N_G(H) is not normal subgroup.	(10)
Q.3	(a)	Let G and G' be two groups and $f : G \longrightarrow G'$ be a group isomorphism. Show that if G is cyclic group, then G' is also cyclic.	(10)
	(b)	Prove that relation of being conjugate elements in group G is an equivalence relation on G. Also, write the class equation of the group $\mathbb{Z}_{15}$.	(10)
Q.4	(a)	Let G be a group. Show that the derived group G' is normal subgroup of G. Also, compute the derived subgroup of S_3 and quotient group S_3 / S_3' .	(10)
	(b)	Let G be a finite group and p be a prime dividing $\|G\|$, then show that G has an element of order p.	(10)
Q.5	(a)	State and prove Sylow's second theorem.	(10)
	(b)	Show that S_n is generated by 2-cycles (1 2), (1 3), ..., (1 n). Show that S_n can also be generated by 2-cycles (1 2), (2 3), (3 4), ...,(n −1 n).	(10)
		SECTION-II	
Q.6	(a)	Let V (F) be a finite dimensional vector space with basis $\{v_1, v_2, v_3, \dots, v_n\}$. If $T : V(F) \longrightarrow W(F)$ be a bijective linear transformation, then show that the set $\{T(v_1), T(v_2), T(v_3), \dots, T(v_n)\}$ forms a basis of W.	(10)
	(b)	Define field. Let R be a commutative ring with multiplicative identity then prove that M is a maximal ideal of a ring R if and only if R/M is a field.	(10)
Q.7	(a)	Let V be a finite dimensional vector over a field F and U be its subspace. Show that dim (V/U) = dim V − dim U .	(10)
	(b)	Prove that the ring $\mathbb{Z}$ of integers is a Principal ideal domain.	(10)
Q.8	(a)	Give the definition of zero divisor in a ring. Also, show that a non-zero element $\bar{a} \in \mathbb{Z}_n = \{\bar{1}, \bar{2}, \dots, \overline{n-1}\}$ is a zero divisor if and only if gcd $(a, n) \neq 1$.	(10)
	(b)	Let R be a finite commutative ring with 1. Show that every prime ideal of R is maximal ideal.	(10)
Q.9	(a)	State and prove Cayley – Hamilton theorem.	(10)
	(b)	Find a real orthogonal matrix P for which $P^{-1}AP$ is diagonal where $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.	(10)

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Part-I **A/2018**
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics (Old & New Course) **TIME ALLOWED: 3 hrs.**
PAPER: III (Complex Analysis and Differential Geometry) **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions by selecting at least TWO questions from each section.

SECTION-I

Q.1. (a) Prove that $v(x, y) = \exp(x)[x\sin(y) + y\cos(y)]$, is a harmonic function. Find an analytic function whose real part is v.

(b) Derive Cauchy Riemann equations in polar form from Cartesian form.

Q.2. (a) Show that a mapping $W = f(z) = \cos(z)$ is conformal at points $z_1 = i$, $z_2 = 1$, $z_3 = \pi + i$ and determine the angle of rotation by $\alpha = Arg(f'(z_0))$ at all points.

(b) Given that

$$f(z) = \begin{cases} \dfrac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0, \\ 0, & z = 0. \end{cases}$$

Show that Cauchy Riemann equations are satisfied but the function is not analytic.

Q.3. (a) Prove that $\cosh\left\{c(Z + \frac{1}{z})\right\} = \sum\limits_{n=-\infty}^{\infty} a_n Z^n$ where

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} \cosh(2c\cos\theta)\cos n\theta d\theta$$

for any finite value of $Z \neq 0$.

(b) Find z, when $\cos(z) = i$.

PTO

Q.4. (a) Evaluate $\int_0^{\infty} \frac{x^2}{x^6+1}\, dx$.

(b) State and prove Fundamental theorem of algebra. Also prove that all real cubic polynomials have atleast one x-intercept.

Q.5. (a) Prove that $\sin \pi Z = \pi Z \prod_{n=1}^{\infty} \left(1 - \frac{Z^2}{n^2}\right)$

and deduce that

$$\frac{\pi}{2} = \frac{2\,.\,2\,.\,4\,.\,4\,.\,6\,.\,6\,.\,8\,.\,8}{1\,.\,3\,.\,3\,.\,5\,.\,5\,.\,7\,.\,7\,.\,9} \cdot \frac{2n\,.\,2n}{(2n-1)(2n+1)}.$$

(b) Evaluate $\int_C \frac{e^{2z}}{\cosh(\pi z)} dz$, where C is the circle $|z| = 1$.

SECTION-II

Q.6. (a) If K_n is the normal curvature of a surface in any direction making an angle α with the principal direction, then prove that $K_n = K_a cos^2\psi + K_b sin^2\psi$. Also what is ψ in case of a unit sphere.

(b) Prove that the surface is developable if and only if its Gaussian curvature is zero.

Q.7. (a) Let σ be a surface patch then prove that $\| \sigma_u \times \sigma_v \| = (EG - F^2)^{\frac{1}{2}}$.

(b) Calculate second curvature for the surface $x = u\cos(\theta)$, $y = u\sin(\theta)$, $z = f(\theta)$.

Q.8. (a) Find the envelop of $lx + my + nz = p$ when $a^2l^2 + m^2b^2 + n^2c^2 = p^2$.

(b) Find the principal curvature and lines of curvature for the surface generated by the binormal to a twisted curve.

Q.9. (a) If $\overrightarrow{r_{11}}$, $\overrightarrow{r_{12}}$, $\overrightarrow{r_{22}}$ and $\overrightarrow{N}$ are linear independent vectors on the surface $\overrightarrow{r} = \overrightarrow{r}(u, v)$, then derive Weingarten equations.

(b) Compute second fundamental form for the surface $x = f(u)\cos(v)$, $f(u) = u\sin(v)$, $z = g(u)$ where profile curve $u \rightarrow (f(u), 0, g(u))$ is unit speed curve.

UNIVERSITY OF THE PUNJAB

Part-I A/2018
Examination:- M.A./M.Sc.

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Subject: Mathematics (Old & New Course) **TIME ALLOWED: 3 hrs.**
PAPER: IV (Mechanics) **MAX. MARKS: 100**

SECTION I

Note:-Attempt any FIVE questions selecting at least two questions from each section. Each question carries equal marks.

1. (a) Show that a vector field F is irrotational if and only if it is a conservative vector field.
   (b) Evaluate directly the integral $\iint_S \mathbf{A} \cdot \mathbf{n}\, dS$ where $\mathbf{A} = 4xz\ \mathbf{i} - y^2\ \mathbf{j} + yz\ \mathbf{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

[10+10=20]

2. (a) What is divergence theorem? Demonstrate the divergence theorem physically. Using divergence theorem, evaluate $\iint_S \mathbf{r} \cdot \mathbf{n}\, dS$, where S is a closed surface.
   (b) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$, where C is a closed curve of the region bounded by $y = x$ and $y = x^2$.

[10+10=20]

3. (a) Find the expression for the unit vectors in cylindrical coordinates and hence calculate the time-derivative of these unit vectors.
   (b) Evaluate $\iiint_V \sqrt{x^2 + y^2}\, dxdydz$, where V is the region bounded by $z = x^2 + y^2$ and $z = 8 - (x^2 + y^2)$.

[10+10=20]

4. (a) Given a transformation $x = a\cosh u \cos v,\ y = a \sinh u \sin v,\ z = z\ (u \geq 0,\ 0 \leq v < 2\pi,\ -\infty < z < \infty)$ from Cartesian coordinates $(x^a) = (x, y, z)$ to elliptic cylindrical coordinates $(\bar{x}^a) = (u, v, z)$ in $\mathbb{R}^3$. Find the square of the element of arc length and identify the scale factors for this transformation.
   (b) Covariant components of a tensor in rectangular coordinates are $2x - z,\ x^2y,\ yz$. Find its contravariant components in cylindrical coordinates ρ, ϕ, z.

[10+10=20]

5. (a) Show that a scalar field $\Phi = a_{jk}A^jA^k$ can always be written as $\Phi = b_{jk}A^jA^k$ where b_{jk} is symmetric covariant tensor of rank 2 .
   (b) What are curvilinear coordinates? Orthogonal curvilinear coordinates? Coordinate curves? Coordinate surfaces? Define unit vectors e_1, e_2, e_3 tangent to the coordinate curves and E_1, E_2, E_3 normal to the coordinate surfaces. What is the relationship between them?

[10+10=20]

P.T.O.

SECTION II

6. (a) Let xyz–coordinate system be rotating with respect to XYZ–coordinate system fixed in space. Derive expression for the time derivative of vector **A** for the observers in two coordinate systems. Prove that the angular acceleration is the same in the two coordinate systems.

   (b) An xyz coordinate system is rotating with angular velocity $\omega = 5\mathbf{i} - 4\mathbf{j} - 10\mathbf{k}$ relative to a fixed coordinate system XYZ having the same origin. Find the expression for the velocity and the acceleration of a particle fixed in the xyz system at the point $(3, 1, -2)$ as seen by an observer in the XYZ.

   [10+10=20]

7. (a) Find moment of inertia of a rectangular lamina about an axis passing through its center and (i) parallel to one of its edges, (ii) perpendicular to the plane of lamina.

   (b) State and prove parallel axes theorem for the inertia matrix for a discrete distribution of mass.

   [10+10=20]

8. (a) Suppose that the moments and products of inertia of a rigid body with respect to an xyz coordinate system intersecting at the origin O are $I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{yz}, I_{zx}$ respectively. Find the moment of inertia of the rigid body about a line making angles α, β, γ with the x, y and z corrdinate axes.

   (b) Calculate the rate of change of K.E. with respect to time using Euler's equation.

   [10+10=20]

9. (a) A rigid body which is symmetric about an axis has one point fixed on this axis. Discuss the rotational motion of the body, assuming that there are no forces acting other than the reaction force at the fixed point. Hence calculate the precession frequency in the case of earth rotating about its axis.

   (b) What is an equimomental system? Find an equimomental system of a uniform triangle of mass M.

   [10+10=20]

UNIVERSITY OF THE PUNJAB

Part-I A/2018
Examination:- M.A./M.Sc.

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Subject: Mathematics (Old & New Course) **TIME ALLOWED: 3 hrs.**
PAPER: V (Topology and Functional Analysis) **MAX. MARKS: 100**

NOTE: Attempt FIVE questions in all, selecting at least TWO from each section.

SECTION-I

Q.1 (a) (i) Define metric topology and metrizable topological space. (10)

(ii) Let $X = \{1,2,3\}$, $\tau = \{\phi, \{1,2\}, \{3\}, X\}$ be a topological space. Find the interior, exterior and frontier of the set $A = \{1,3\}$.

(b) (i) Define co-finite topology. Let $X = \mathbb{N}, \tau =$ co-finite topology on $\mathbb{N}$ and $A_n = \{2,3,4,\ldots,n+1\}, n \in \mathbb{N}$. Then prove that $\overline{\bigcup_{n\in\mathbb{N}} A_n} \neq \bigcup_{n\in\mathbb{N}} \overline{A}$. (10)

(ii) Show that every neighbourhood in a co-finite topological space is an open set.

Q.2 (a) Prove that every second countable space is separable, that is, has a countable dense subset. (10)

(b) For any space X, if $C(X,\mathbb{R})$ separates points of X then X is Hausdorff space. (10)

Q.3 (a) Prove that every metric space is a Hausdorff space. (10)

(b) Prove that every compact Hausdorff space is normal. (10)

Q.4 (a) Prove that every closed subset of a compact space is compact. (10)

(b) Let $X = \bigcup_{\alpha\in\Omega} X_\alpha$ where each X_α is connected and $\bigcap_{\alpha\in\Omega} X_\alpha \neq \phi$. Then prove that X is connected. (10)

SECTION-II

Q.5 (a). Let (X,d) be a metric space. Then prove that (10)

(i) ϕ and X are open. (ii) The union of finite number of closed sets is closed.

(iii) The intersection of any number of closed sets is closed.

(b) Prove that a Cauchy sequence in (X,d) converges if and only if it has a convergent subsequence. (10)

PTO

Q.6 (a) (i) Prove that $I:(X,d)\rightarrow(X,d)$ defined by $I(x)=x,\forall x\in X$ is continuous. (10)

(ii) Prove that $f:\mathbb{R}\rightarrow\mathbb{R}$ with usual metric on $\mathbb{R}$, defined by $f(x)=cx,\forall x\in\mathbb{R}$ is uniformly continuous.

(b) Prove that the space $C[a,b]$ is a Banach space. (10)

Q.7 (a) Let M be a subset of a finite dimensional normed space N. Then M is compact if and only if M is closed and bounded. (10)

(b) Suppose $\|.\|$ and $\|.\|_0$ are equivalent norms on N_1, let N be a finite dimensional subspace of $(N_1,\|.\|_0)$, then prove that N is complete subspace of $(N_1,\|.\|_0)$. (10)

In particular $(N,\|.\|_0)$ is complete.

Q.8 (a) Prove that every linear operator on a finite dimensional normed space is bounded. (10)

(b) For any $a=(a_1,a_2,...a_n)\in R^n$ define $f_a:R^n\rightarrow R$ by (10)

$f_a(x)=\sum_{i=1}^{n}a_ix_i, x=(x_1,x_2,...,x_n)\in R^n$. Then prove that

(i) f_a is linear functional (ii) f_a is bounded (iii) $\|f_a\|=\|a\|$.

Q.9 (a) Let $\{e_1,e_2,...,e_n,...\}$ be orthonormal system in an inner product space. Then prove that the minimum value of the expression $\left\|x-\sum_{k=1}^{n}a_ke_k\right\|^2$ assumes for $a_k=c_k=\langle x,e_k\rangle$. (10)

Hence establish the Bessel's inequality $\sum_{k=1}^{\infty}|c_k|^2\leq\|x\|^2$.

(b) Let A be a non-empty complete convex subset of an inner product space V, and $x\in V\setminus A$. Then there is a unique $y\in A$ such that $\|x-y\|=\inf_{y'\in A}\|x-y'\|$ (10)