UNIVERSITY OF THE PUNJAB

Part-I : Supplementary Examination 2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Mathematics (Old & New Course)
PAPER: I (Real Analysis)

MAX. TIME: 3 Hrs.
MAX. MARKS: 100

NOTE: Attempt any FIVE questions in all selecting at least TWO questions from each section.

Section-I

Q.1(a) Define Euclidean space. With usual notation, for $x, y \in R^k$ show that $\|x.y\| = \|x\|\|y\|$.

(b) If A is bounded set of real numbers and $b > 0$, show that $Inf(bA) = b(Inf(A))$ and $\text{Sup}(bA) = b(\text{Sup}(A))$. (10+10)

Q.2(a) If $s_1 = \sqrt{2}$, $s_{n+1} = \sqrt{2+\sqrt{s_n}}\,; n = 1,2,...$, then prove that the sequence $\{s_n\}$ is convergent.

(b) State and prove the Bolzano-Weierstrass Theorem. (10+10)

Q.3(a) Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$.

(b) If a and c are real numbers with $c > 0$ and f be defined on $[-1,1]$ by $f(x) = \begin{cases} x^a \sin x^{-c} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$,

then f is continuous if and only if $a > 0$.(10+10)

Q.4(a) Show that every differentiable function is continuous but converse is not true in general.

(b) Let f be differentiable on $[a,b]$, $f'(x) = 0$ for all $x \in [a,b]$ if and only if f is constant on $[a,b]$. (10+10)

Q.5(a) Discuss the extreme values of the function $f(x,y,z) = xyz$ subject to the condition $x^2 + y^2 + z^2 = 4$.

(b) If f_x and f_y exist in some neighbourhood of point (a,b) and both are differentiable on (a,b), then show that $f_{xy}(a,b) = f_{yx}(a,b)$. (10+10)

Section-II

Q.6 (a) If $f \in R(\alpha)$ on an interval $I = [a,b]$, and c be any constant, then $cf \in R(\alpha)$ on I and $\int_a^b cf\,d\alpha = c\int_a^b f\,d\alpha$.

(b) Let $f:[a,b] \to R$ be bounded and $\alpha:[\text{a,b}] \to R$ be increasing function. Prove that if $f \geq 0$, then $\int_a^b f\,d\alpha \geq 0$. (10+10)

P.T.O.

Q.7(a) If $f \in R(\alpha_1)$ $f \in R(\alpha_2)$ on $[a,b]$, then prove that $f \in R(\alpha_1+\alpha_2)$ on $[a,b]$ and

$$\int_a^b f\,d\alpha = \int_a^b f\,d\alpha_1 + \int_a^b f\,d\alpha_2 .$$

(b) Let f be continuous on $[a,b]$. If f' exists and is bounded on $]a,b[$, then f is of bounded variation on $[a,b]$. (10+10)

Q.8(a) Test the uniform convergence of $f_n(x) = e^{-n}\cos nx;\ 0 < x < \infty$.

(b) Prove that the sequence of the functions $\{f_n\}$ defined on E, converges uniformly on E if and only if for every $\varepsilon > 0$ there exist an integer N, such that $m,n \geq N, x \in E$ implies that $|f_n(x) - f_m(x)| < \varepsilon$. (10+10)

Q.9(a) Test for the convergence of (i) $\int_0^\infty \frac{x^2}{\sqrt{x^5+1}}dx$ (ii) $\int_0^1 \frac{dx}{x^{1/3}(1+x^2)}dx$.

(b) Discuss the convergence of $\int_0^\infty \frac{\sin x}{x}dx$. (10+10)

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Subject: Mathematics (Old & New Course) **MAX. TIME: 3 Hrs.**

PAPER: II (Algebra) **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions in all selecting at least two questions from each section.

Q.1	(a)	Let G be a finite group and H non-empty subset G. Prove that H is subgroup of G if and only if H is closed.	(10)
	(b)	State and prove Lagrange's Theorem. Also, verify it for all the subgroups of S_3.	(10)
Q.2	(a)	Show that every group of order p^2 is abelian, where p is a prime number. Also, list all non-isomorphic groups of order p^2.	(10)
	(b)	Define normal subgroup, and prove that every subgroup of index 2 is normal.	(10)
Q.3	(a)	Let $f : (\mathbb{Z}, +) \longrightarrow (\mathbb{Z}_{20}, +)$ be a function given by $f(a) = a \ (mod\ 20)$. Show that f is a group homomorphism. Also, compute its kernel and image.	(10)
	(b)	Show that the relation of being conjugate elements in a group G is an equivalence relation. Also, list all the conjugacy classes of S_3.	(10)
Q.4	(a)	Define the derived subgroup G' of a group G. Prove that G' is a normal subgroup of G and the factor group G/G' is abelian. Also, prove that if K is a normal subgroup of G such that G/K is abelian then K contains G'.	(10)
	(b)	State and prove Sylow's first theorem.	(10)
Q.5	(a)	Write down all the elements of the alternating group A_4 of degree 4 and symmetric group S_4. Write down all the subgroups of A_4.	(10)
	(b)	Show that the alternating group A_n of degree n is a normal subgroup of S_n and has order $\frac{1}{2}n!$.	(10)
		SECTION-II	
Q.6	(a)	State and prove Rank - Nullity theorem.	(10)
	(b)	Let A be any n × n with entries from the set of real numbers $\mathbb{R}$. Show that the function $T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ given by $T(x) = Ax$, where $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, is a linear transformation. Also, compute the dimension of Ker (T) for A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.	(10)
Q.7	(a)	Let A be a matrix of order n × n with entries from the set of real numbers $\mathbb{R}$. Show that the given statements are equivalent: (i) A is non-singular matrix, (ii) The rows of A form a basis of $\mathbb{R}^n$, (iii) The columns of A form a basis of $\mathbb{R}^n$.	(10)
	(b)	Prove that a finite integral domain is a field.	(10)
Q.8	(a)	Prove that the ring $\mathbb{Z}_n$ of congruence classes modulo n is a field if and only if n is prime number.	(10)
	(b)	State and prove Cauchy –Schwarz inequality. Give its one application.	(10)
Q.9	(a)	Prove that eigenvectors of a symmetric matrix corresponding to distinct eigenvalues are orthogonal.	(10)
	(b)	Show that every matrix A is a root of its characteristic polynomial. Also, illustrate it with some example.	(10)

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Examination:- M.A./M.Sc.

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Subject: Mathematics (Old & New Course) **MAX. TIME: 3 Hrs.**

PAPER: III (Complex Analysis and Differential Geometry) **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions in all selecting at least TWO questions from each section.

SECTION-I

Q.1. (a) Define analytic function. If $f = u + iv$ is analytic, then prove that

$$\frac{\partial^2 u^2}{\partial x^2} + \frac{\partial^2 u^2}{\partial y^2} = 2|f'|^2.$$

(b) Without verifying Cauchy Riemann equations show that $f(z) = \cos(37x + 35iy)$ is not analytic.

Q.2. (a) Let the transformation be $w = \cosh z$. Show that the line $y = c$, where $2c/\pi$ not an integer, is mapped onto a branch of the hyperbola

$$\frac{U^2}{\cos^2 c} - \frac{V^2}{\sin^2 c} = 1.$$

(b) Given that

$$f(z) = \begin{cases} \dfrac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0, \\ 0, & z = 0. \end{cases}$$

Show that Cauchy Riemann equations are satisfied but the function is not analytic.

Q.3. (a) Prove that an entire bounded function is constant. Also prove that $SinZ$ is unbounded.

(b) Expand $f(Z) = \dfrac{1}{(Z-1)(Z+3)}$ in a Laurents' series valid for the regions

(i) $|Z| < 1$ (ii) $1 < |Z| < 3$ (iii) $|Z| > 3$.

Q.4. (a) Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the straight line from $(0,3)$ to $(2,3)$ and then $(2,3)$ to $(2,4)$.

(b) Evaluate $\displaystyle\int_0^{2\pi} \frac{(1+2\cos\theta)^n \cos n\theta}{3+2\cos\theta}\, d\theta.$

Q.5. (a) State and prove Mittag-Leffler's expansion theorem.

(b) Prove that $\pi \cot \pi Z = \dfrac{1}{Z} + \displaystyle\sum_{n=1}^{\infty} \left[\frac{2Z}{Z^2 - n^2}\right]$ and deduce that $\sin \pi Z = \pi Z \displaystyle\prod_{n=1}^{\infty} \left(1 - \frac{Z^2}{n^2}\right)$.

P.T.O.

SECTION-II

Q.6. (a) Compute curvature of a logarithmic spiral $\gamma(t) = (e^{2t} sint, e^{2}tcost)$. Also find its unit speed parametrization.

(b) The position vector $\vec{r}$ at any point on the surface of revolution is $\vec{r} = (u\cos\theta, u\sin\theta, f(\theta))$. Find Gaussian and Mean curvatures.

Q.7. (a) Prove that ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is a regular surface.

(b) Prove Meusnier's Theorem.

Q.8. (a) If K_n is the normal curvature of a surface in any direction making an angle α with the principal direction, then prove that $K_n = K_a cos^2\psi + K_b sin^2\psi$.

(b) Compute First Fundamental form of the right circular cylinder.

Q.9. (a) If $\vec{r_{11}}$, $\vec{r_{12}}$, $\vec{r_{22}}$ and $\vec{N}$ are linear independent vectors on the surface $\vec{r} = \vec{r}(u, v)$, then derive Weingarten equations.

(b) Compute E_1, G_2 and Γ^2_{12} for the surface $x = u\cos(\theta)$, $y = u\sin(\theta)$, $z = cu$.

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Part-I : Supplementary Examination 2018

Examination:- M.A./M.Sc.

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Subject: Mathematics (Old & New Course) **MAX. TIME: 3 Hrs.**

PAPER: IV (Mechanics) **MAX. MARKS: 100**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section. Each question carries equal marks.

SECTION I

1. (a) Give a definition of $\iint_S \mathbf{F}.\mathbf{n}\, ds$ over a surface S in terms of limit of a sum, where F is a vector function. Suppose that the surface S has projection R on the xy-plane. Show that $\iint_S \mathbf{F}.\mathbf{n}ds = \iint_R \mathbf{F}.\mathbf{n}\frac{dxdy}{|\mathbf{n}.\mathbf{k}|}$.

   (b) Show that $\mathbf{F} = r^2\mathbf{r}$ is conservative force field. Find the scalar potential of this force.

[10+10=20]

2. (a) Evaluate $\iint_S \mathbf{A} \,.\, \mathbf{n}\; dS$ where $\mathbf{A} = z\,\mathbf{i} + x\,\mathbf{j} - 3y^2z\,\mathbf{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

   (b) Evaluate the integral $\oint_C (y - \sin x)dx + \cos x dy$ by the usual integration process and then apply the Green's theorem to show that the results obtained by the two techniques are same. The boundary curve C is the triangle as shown in the adjoining Figure 1.

[10+10=20]

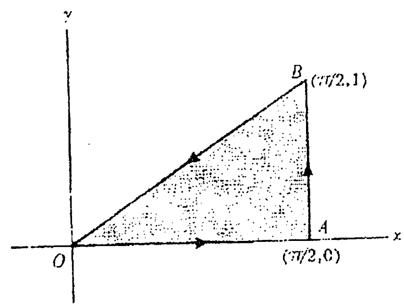

Figure 1: The boundary curve C as stated in Question No. 2(b).

3. (a) What is divergence theorem? Using divergence theorem, evaluate $\iint_S \mathbf{F} \,.\, \mathbf{n}\; dS$ where $\mathbf{F} = 4xz\,\mathbf{i} - y^2\,\mathbf{j} + yz\,\mathbf{k}$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$.

   (b) Differentiate between symmetric and skew symmetric tensors. What is the largest number of different components which a symmetric contravariant tensor of rank two can have if

   (a) $N = 2$, (b) $N = 3$, (c) $N = 4$.

   Can the result be generalized for any dimension?

[10+10=20]

P.T.O

4. (a) What are orthogonal curvilinear coordinates? What is the difference between an orthogonal and a non-orthogonal curvilinear coordinate system? Show that a cylindrical coordinate system is orthogonal.

   (b) Given a transformation $x = (u^2 - v^2)/2$, $y = uv$, $z = z$ $(-\infty < u < \infty, v \geq 0, -\infty < z < \infty)$ from Cartesian coordinates $(x^a) = (x, y, z)$ to parabolic cylindrical coordinates $(\bar{x}^a) = (u, v, z)$ in R^3. Find the transformation matrices $[\partial x^a/\partial \bar{x}^b]$ and $[\partial \bar{x}^a/\partial x^b]$ expressing them both in primed coordinates and their determinants **J** and $\bar{\mathbf{J}}$.

[10+10=20]

5. (a) A quantity $A(p, q, r)$ in the coordinate system x^i satisfies the relation $A(p, q, r)B_r^{qs} = C_p^s$, where B_r^{qs} and C_p^s are tensors of given ranks. Show that $A(p, q, r)$ is a tensor. What is the rank of the tensor $A(p, q, r)$?

   (b) What are symmetric and skew symmetric tensors? Show that every tensor can be expressed as the sum of two tensors, one of which is symmetric and the other skew-symmetric in a pair of covariant or contravariant indices.

[10+10=20]

SECTION II

6. (a) Consider the two observers in the two coordinate systems $Oxyz$ and $OXYZ$ having the common origin O. The coordinate system $Oxyz$ is in rotation with respect to the coordinate system $OXYZ$ taken as fixed in space. Find the 2nd order time-derivative of a vector $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$ as seen by the two observers. What is the acceleration of a moving particle as observed by the two observers?

   (b) An xyz coordinate system is rotating with angular velocity $\omega = 5\mathbf{i} - 4\mathbf{j} - 10\mathbf{k}$ relative to a fixed coordinate system XYZ having the same origin. Find the expression for the velocity and the acceleration of a particle fixed in the xyz system at the point $(3, 1, -2)$ as seen by an observer in the XYZ.

[10+10=20]

7. (a) State and prove perpendicular axis theorem for a discrete distribution of mass in a plane. Apply this result to find the moment of inertia of an elliptic disc of semi-major and semi-minor axes of lengths a and b, about an axis perpendicular to the plane of elliptic disc passing through its center.

   (b) Show that a hoop of mass m and radius $a/\sqrt{2}$ is equimomental with a circular plate of mass m and radius a.

[10+10=20]

8. (a) Show that the total angular momentum of a system of particles about any point O is equal to the angular momentum of the total mass assumed to be located at the center of mass plus the angular momentum about the center of mass.

   (b) Find the expression for moment of inertia of a rigid body about an arbitrary line making angles α, β, γ with the x, y and z coordinate axes.

[10+10=20]

9. (a) Find the expression for momental ellipsoid of an ellipsoid at its center.

   (b) Derive Euler's equations of motion and establish the constancy of kinetic energy and angular momentum. Write Euler's equations of motion in case the axes are not principal axes.

[10+10=20]

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Part-I : Supplementary Examination 2018

Examination:- M.A./M.Sc.

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Subject: Mathematics (Old & New Course) **MAX. TIME: 3 Hrs.**

PAPER: V (Topology and Functional Analysis) **MAX. MARKS: 100**

NOTE:Attempt FIVE questions in all, selecting at least TWO from each section.

SECTION-I

Q.1 **(a)** Let (X,τ) be a topological space and $A \subseteq X$. Let A^d denote the derived set of A. If the closure of A, is defined as $A \cup A^d$ and denoted by $\bar{A}$, then prove that $\bar{A}$ is closed. (10)

(b) Define interior, exterior and frontier of a set. Let $X = \mathbb{R}, \tau =$ usual topology on $\mathbb{R}$. Find the interior, exterior and frontier of the subset $\mathbb{Q}$ of all rational numbers. (10)

Q.2 **(a)** Prove that any uncountable set X with co-finite topology is not first countable and so not second countable. (10)

(b) Prove that a space X is a T_1 – space if and only if, for any $a \in X$, $\{a\}$ is closed. (10)

Q.3 **(a)** A Topological space (X,τ) is normal if and only if for any closed set *A* and an open set U containing *A* there is at least one open set V containing A such that $A \subseteq V \subseteq \bar{V} \subseteq U$. (10)

(b) Show that every metric space is regular (10)

Q.4 **(a)** Let X be a topological space. Then prove that any infinite subset of X has a limit point if and only if every countably infinite subset has a limit point. (10)

(b). A topological space (X,τ) is connected if and only if every continuous function $f : X \to D$ (a discrete space) reduces to a constant function. (10)

SECTION-II

Q.5 **(a)** Prove that every closed ball $\bar{B}(a;r)$ in a normed space $(X,\|.\|)$ is closed. (10)

(b) (i) Prove that every Cauchy sequence is bounded. (10)

(ii) Prove that every convergent sequence is a Cauchy sequence. Give an example to show that a bounded set may not be convergent.

Q.6 **(a)** State and prove Cantor's intersection theorem. (10)

(b) Let N be a Normed space and F be a field. Show that (10)

(i) The function $f : N \times N \to N$ defined by $f(x,y) = x + y$ is uniformly continuous.

P.T.O.

(ii) The function $g: F \times N \to N$ defined by $g(c, x) = cx, c$ is constant, is continuous.

(iii) The function $h_a: N \to N$ defined by $h_a(x) = ax, a$ is fixed scalar, is continuous.

Q.7 **(a)** Let S be a closed subspace of a Banach space N. Then Show that the quotient space N/S is a Banach space with the norm defined by $\|x+S\|_1 = \inf_{s \in S} \|x+s\|$. (10)

(b) Let M be a proper closed subspace of a normed space N and $a \in (0,1)$. Then prove that there exists $x_a \in N$ such that $\|x_a\| = 1$ and $\|x - x_a\| \geq a$ for all $x \in M$. (10)

Q.8 **(a)** Prove that every linear operator on a finite dimensional normed space is bounded. (10)

(b) Let N be an n-dimensional normed space. Then prove that its dual N^* is also an n-dimensional. (10)

Q.9 **(a)** Let A be a non-empty complete convex subset of an inner product space V, and $x \in V \setminus A$. Then there is a unique $y \in A$ such that $\|x - y\| = \inf_{y' \in A} \|x - y'\|$. (10)

(b) For any x, y in a complex inner product space V. Prove that (10)

$$\langle x, y \rangle = \frac{1}{4}\left[\left(\|x+y\|^2 - \|x-y\|^2\right) + i\left(\|x+iy\|^2 - i\|x-iy\|^2\right)\right].$$