UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. ……………..

Subject: Mathematics **Paper: I (Advanced Analysis)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION – I

Q1.	a)	Check that the countable union of finite sets is countable or not.	10
	b)	Show that every infinite set has a denumerable subset.	10
Q2	a)	Show that the set of all points of the circumference of an ellipse has the cardinal $\aleph$.	10
	b)	If S is ordered isomorphic to T. Then prove that S is equipotent to T. Does converse hold? Justify your answer.	10
Q3	a)	State and prove principle of transfinite induction.	10
	b)	Prove that set of real numbers [0, 1] is non denumerable.	10
Q4	a)	Show that the axiom of choice is equivalent to Zermelo's postulate.	10
	b)	Show for ordinal numbers λ, μ and η , $(\lambda\mu)\eta = \lambda(\mu\eta)$ and $1.\lambda = \lambda$	10

SECTION – II

Q5	a)	Let $\{f_n\}$ be a sequence of measurable functions on R_n. Show that the set $\{x \in R^n: \lim_{n\to\infty} f_n(x)\ exist\}$ *is* measurable.	10
	b)	Let f be a measurable function on [0; 1] with $\|f(x)\| < \infty$ for $a.e.\ x$. Prove that there exists a sequence of continuous functions g_n on $[0,1]$ such that $g_n \to f$ for $a.e.\ x \in [0,1]$.	10
Q6	a)	Let $E \subseteq \mathbb{R}$ such that $m^*(E) < \infty$. Then show that E is measurable if and only if for given $\epsilon > 0$, there is a finite union H of finite open intervals such that $m^*(E\Delta H) < \epsilon$.	10
	b)	Show that the interval (a,∞) is measurable.	10
Q7	a)	Construct the cantor set C, is it countable or not and also calculate its Lebesgue measure.	10
	b)	In R^2, show that the set $\{A = (x,y): 0 \le x \le 2,\ \ 0 \le y \le 2,\}$ is measurable and its measure is 4.	10

Q8	a) Show that the step function with measurable domain is measurable. b) Let $f(x) = c, x \in R,$ and $g(x) = \begin{cases} 0, & x \in Q \\ c, & x \in Q' \end{cases}$ tell either $f = g$ a.e. or not. And also tell about the measurability of both functions.	10 10
Q9	a) Prove that the following Lebesgue integrals exist: (i) $\int_0^1 (x \log x)^2 \, dx$ (ii) $\int_0^1 \log x \log(1-x)^2 \, dx$ b) For which values of $p \in R$, the Lebesgue integral $\int_0^\infty x^p \sin x^2 \, dx$ exists	10 10

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M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. ....

Subject: Mathematics **Paper: II (Methods of Mathematical Physics)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION – I

1(a)	If $u_m(x)$ and $u_n(x)$ are solutions of a regular or periodic SL system corresponding to distinct eigen values λ_m and λ_n of the parameter λ, then $u_m(x)$ and $u_n(x)$ are orthogonal w.r.t the weight function $r(x)$ i.e. $\int_a^b u_m(x)u_n(x)r(x)dx, \quad m \neq n$	**10**
1(b)	Determine eigen solutions and eigen values of the system $(xy')' + \frac{\lambda}{x}y = 0,\ 1 \le x \le e$ with end point $y(1) = 0,\ y(e) = 0$	**10**
2(a)	Obtain two linearly independent solutions by the Frobenius method for the differential equation, near the singular point $x = \infty$. $x^3\frac{d^2y}{dx^2} + x(1-x)\frac{dy}{dx} + y = 0$	**10**
2(b)	Use the method of power series to obtain general solution of the Airy equation $y'' - xy = 0$, $-\infty < x < \infty$	**10**
3(a)	Solve the boundary value problem $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ u(x,0) = 0, u(x,b) = 0, u(0,y) = g_1(y),\ u(a,y) = g_2(y)$	**10**
3(b)	Find the integral surface of the quasi-linear PDE $xz_x - yz_y = z$, which contains the circle $x^2 + y^2 = 1, z = 1$.	**10**
4(a)	Find the integral surface when the Cauchy data are given. $xz_x + yz_y = 2xy, z = 2$ on $y = x^2$.	**10**
4(b)	Show that the Legendre polynomial $P_n(x)$ can be regarded as a special case of hypergeometric function as ${}_2F_1\left(-n, n+1, 1; \frac{1-x}{2}\right) = P_n(x)$	**10**
5(a)	Prove the following for Bessel functions $J_n(x)$ (i) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ (ii) $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$	**10**
5(b)	Formulate and solve the problem of steady flow of heat in a circular disc where the boundary of the disc is maintained at temperature $f(\theta)$.	**10**

SECTION – II

6(a)	Solve the DE by Laplace transform method $y''(t) + 3y'(t) + 2y(t) = te^{-t}$, $y(0) = 1$, $y'(0) = 0$	**10**

6(b)	Show that $L^{-1}\left(\frac{F(s)}{s}\right)=\int_0^t f(\tau)d\tau$.	10
7(a)	Calculate Fourier sine transform of the function $f(x)=\frac{1}{x^2+4}$	10
7(b)	State and prove convolution theorem of Fourier transform.	10
8(a)	Find the Green's function associated with the boundary value problem $y''+\frac{1}{4}y=f(x), y(0)=0, y(\pi)=0$. Also solve the problem when $f(x)=\sin 2x$ and $f(x)=x/2$	10
8(b)	Construct the modified Green's function associated with the boundary value problem $y''+\lambda y=0$, $y(0)=y(1)$ and $y'(0)=y'(1)$.	10
9(a)	Find the extremal of the problem $I[y]=\int_0^\pi (2yz-2y^2+y'^2-z'^2)dx \to$ minimum, subject to $y(0)=0, y(\pi)=1, z(0)=0, z(\pi)=-1$.	10
9(b)	Show that a solid of revolution which for a given surface area has maximum volume is a sphere.	10

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Roll No. ....

Subject: Mathematics **Paper: III (Numerical Analysis)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION – I

Q1.

(10+10)

(a) Write an algorithm for Regula falsi method to find an approximate root of the non-linear equation $f(x)=0$.

(b) Find the solution of $sinx-1-x^3=0$ by Newton Raphson's method.

Q2.

(a) **(10+10)**

Solve the system of equations by Choleski Factorization method

$$2x_1+3x_2+5x_3=18$$
$$3x_1+4x_2+2x_3=13$$
$$5x_1+2x_2+3x_3=14$$

(b) Find the Dominant Eigen value and corresponding Eigen vector of the following matrix by power method

$$\begin{bmatrix}4 & 9 & -2\\ 0 & 3 & -1\\ 2 & 1 & 5\end{bmatrix}$$

Q3.

(10+10)

(a) Solve $\frac{dy}{dx} = 1 - y; y(0) = 0, h = 0.1\ for\ x = 0.1, 0.2, 0.3$ using Modified Euler's method.

(b) Use Runge-Kutta method of order three to solve the differential equation $\frac{dy}{dx} = y + e^x; y(0) = 0, h = 0.2$ for $y(0.2)$ and $y(0.4)$.

Q4. (12+08)

(a) Solve the following system of equations by Guass Seidel method

$$10x + y - 2z = 7.74$$
$$3x + 4y + 15z = 54.8$$
$$x + 12y + 3z = 39.66$$

(b) Prove that n^{th} divided difference of polynomial of n^{th} degree is constant.

Q5. (08+12)

(a) Derive the Runge-Kutta method of order two.

(b) Solve the following differential equation using Runge-Kutta method of order 4 for x=0.1, 0.2

$y'' - xy + 4 = 0,\quad y(0) = 3,\ y'(0) = 0.$

SECTION – II

Q6.

(10+10)

(a) Derive the Newton's divided difference interpolation formula.
(b) Find a polynomial satisfied by the following table

x	-4	-1	0	2	5
y	3	11	31	69	131

Q7. (10+10)

(a) A river is 80 feet wide. Depth d in feet at a distance of x feet from one bank is given by the following table.

X	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of cross-section.

(b) Find the global error of Boole's rule.

Q8. (10+10)

(a)A slider in a machine moves along a fixed straight rod. Its distance x (cm) along the rod is given below for various values of time t seconds. Find the velocity of the slider and its acceleration when t=0.3.

t	0	0.1	0.2	0.3	0.4	0.5	0.6
x	30.13	31.62	32.87	33.64	33.95	33.81	33.24

(b)Find the first, second and third derivatives of $f(x)$ at $x = 2$ from the following data using Newton's forward difference interpolation formula

x	2	4	6	8	10	12
$f(x)$	4.95	5.1047	8.9714	9.1096	6.1122	7.98

Q9. (10+10)

(a) Solve the following difference equation:

$$y_{k+2} - 6y_{k+1} + 8y_k = 2.3^k + sin3k.$$

(b) Solve the following difference equation

$$y_{n+3} - 4y_{n+2} + 6y_{n+1} + 8y_n = (5n^2 - 7).$$

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Subject: Mathematics **Paper: iv-1-N / IV-VI (Opt. i) (Mathematical Statistics)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION – I

Q.1	**(a)**	Find first three moments about mean for a Binomial distribution.	**(10)**
	(b)	If A is independent of B, $B \cap C$ and $B \cup C$, then show that A is independent of C.	**(10)**
Q.2	**(a)**	An urn contains 4 black, 3 red and 2 green balls and 2 balls are selected at random from it. If X denotes the number of red balls and Y denotes the number of green balls selected, then find **(i)** the joint probability function $f(x, y)$ **(ii)** $P(X + Y \leq 1)$.	**(10)**
	(b)	A continuous r.v has p.d.f $f(x) = \begin{cases} k(2-x)(x-5), & 2 \leq x \leq 5 \\ 0, & elsewhere \end{cases}$ Calculate the distribution function, and estimate the probabilities $P(\lvert X-\mu\rvert \leq 1.6)$ and $P(\lvert X-\mu\rvert \geq 1.6)$.	**(10)**
Q.3	**(a)**	Prove that if X and Y are independent Gamma variates, with parameters l and m respectively, then $\frac{X}{X+Y}$ is a $\beta_1(l, m)$ variate.	**(10)**
	(b)	State and prove Baye's Theorem.	**(10)**
Q.4	**(a)**	An athlete finds that in a high jump he can clear a height of 1.64m once in five attempts and a height of 1.53m nine times out of ten attempts. Assuming the heights he can clear in various jumps form a normal distribution, estimate the mean and standard deviation of the distribution.	**(10)**
	(b)	For the normal distribution find mean deviation of the distribution	**(10)**

SECTION – II

Q.5	**(a)**	If $\overline{X}$ is the mean of a random sample of size n from a finite population of size N with mean μ and variance σ^2 then show that $E(\overline{X}) = \mu_{\overline{X}} = \mu$ and $Var(\overline{X}) = S_{\overline{X}}^2 = \frac{\sigma^2}{n} \times (\frac{N-n}{N-1})$	**(10)**
	(b)	If X_r and X_s are the rth' and sth' random variable of random sample of size n drawn from the finite population $\{C_1, C_2, ...C_N\}$. Then $Cov(X_r, X_s) = \frac{\sigma^2}{N-1}$	**(10)**

Q.6	**(a)**	Derive the formula for coefficient of Rank Correlation between two variables X and Y.	(10)
	(b)	If the joint probability density of X_1 and X_2 is given by $f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)} & for\ x_1 > 0\ and\ x_2 > 0 \\ 0 & elsewhere \end{cases}$ Find the probability density of $Y = \frac{X_1}{X_1 + X_2}$.	(10)
Q.7	**(a)**	Find the measure of symmetric and kurtic of the Chi-square distribution (χ^2 - distribution).	(10)
	(b)	Derive probability density function of student's t-distribution.	(10)
Q.8	**(a)**	Prove that $R_{3.12} = \sqrt{\frac{r_{31}^2 + r_{32}^2 - 2r_{13}r_{23}r_{12}}{1 - r_{12}^2}}$	(10)
	(b)	If X and Y are two uncorrelated variables and if $U = X + Y$ and $V = X - Y$, find the correlation between U and V in terms of σ_x and σ_y where σ_x and σ_y are standard deviations of X and Y.	(10)
Q.9	**(a)**	If F *follows* $F(\upsilon_1, \upsilon_2)$ then $y = (1 + \frac{\upsilon_1}{\upsilon_2} F)^{-1}$ *follows* $\beta(\frac{\upsilon_1}{2}, \frac{\upsilon_2}{2})$ Where υ_1, υ_2 are the degrees of freedom for F-distribution.	(10)
	(b)	If X has the standard normal distribution, find the probability density of X^2.	(10)

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Roll No. ……………

Subject: Mathematics **Paper: IV-2-N / IV-VI (Opt. ii) [Computer Applications]** **Time: 3 Hrs.** **Marks: 50**

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section. All Questions carry equal marks.

SECTION – I

Q1. Write a program to find the inverse of a square matrix.

Q2. Write a program to read a list of n numbers and print it out in ascending order.

Q3, Write a program to calculate the value of PI approximately by summing up the first 500 terms of the series

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} \cdots$$

Q4. Write a Function subprogram, calculate the Mean of a linear array A with N elements.

SECTION – II

Q5. Write a program to solve, with a numerical technique

$$\frac{dy}{dx} = y - \frac{2x}{y},\ y(0) = 1, \text{ over the interval } [0,2].$$

Q6. Write a program to find the value of the following integral

$$\int_0^1 (x^2 + x + 1)dx$$

with Simpson's Rule and Trapezoidal Rule. Compare the approximate values with the exact value; identify which method is superior.

Q7. From the following table write a program to find $f(0.5)$

Table

x	-1	0	1	2	3
$f(x)$	-1	0	1	8	27

Q8. Write a program to find a positive real root correct to 4 decimal places of the following nonlinear equation:

$$e^{-x^2} - sin(x) - 0.005 = 0$$

with Newton Raphson Method.

Q9. Write the Mathematica statements for the following:

1. Evaluate $\frac{d}{dx}\left(\log_3^{x^2+4x}\right)$.
2. Evaluate $\int_0^1 \sec x dx$.
3. Plot the graph of $\tan x$, $Sec(x)$, $-\frac{\pi}{4} < x < \frac{+\pi}{4}$.
4. Find the conjugate of $z = x + y + iz$
5. Find the sum of the series $2 + 4 + 6 + 8 + \cdots + 2^n$.

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Roll No. …………….

Subject: Mathematics Paper: IV-3-N / IV-VI (Opt. iii) (Group Theory) Time: 3 Hrs. Marks: 100

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

SECTION-I

Q 1. (a) Let G = $\langle x, y : x^5 = y^2 = e, x^2 y = yx \rangle$. Explore G and find Order Structure of G. Find a group which is isomorphic to G.

(b) Define Holomorph of a group. Find all Semi-directs products of C_3 by C_3.

Q 2. (a) The group $S_3 \times Z_2$ is isomorphic to one of the following groups: Z_{12}, $Z_6 \times Z_2$, A_4 or D_6. Determine the group by elimination.

(b) Define simple group. Cheek wheatear the group of order 2540 is simple or not?

Q 3. (a) Let G be a group of order p^2q where p, q are primes such that $q < p$ and $p^2 \not\equiv 1(\text{mod } q)$. Then show that G is abelian.

(b) Classify all non-isomorphic abelian groups of the following orders:
(I) 2022 (II) 2222

Q 4. (a) Let $G = B \times A$. Then show that the factor group G/A is isomorphic to B.

(b) State and prove Orbit-Stablizer Theorem.

SECTION-II

Q 5. (a) Define a chief series in a group G. Prove that a group G has a chief series if and only if every ascending and descending normal chains of subgroups of G break off.

(b) Define composition series. Prove that any two composition series of a group G are isomorphic.

Q 6. (a) Let N be normal subgroup of a group G. Show that G is solvable if and only if both N and G/N are solvable.

(b) Prove that every finite group-p group is solvable.

Q 7. (a) A group G, with identity 1, is nilpotent of class k if and only if

$$\left[\cdots\left[\left[[g_1, g_2], g_3\right], g_4\right]\cdots, g_{k+1}\right] = 1.$$

(b) Prove that the direct product of a finite number of nilpotent groups is nilpotent.

Q 8. (a) Define cyclic and central extensions. Find all extensions of a cyclic group of order 3 by a cyclic group of order 3.

(b) Prove that $GL(n,q)/SL(n,q)$ is isomorphic to the group $GF(q)^{\times} = F_q^{*}$.

Q 9. (a) Prove that a finite group G is nilpotent if and only if G is the direct product of its sylow-p subgroups.

(b) Show that a normal subgroup H of G is contained in the Frattini subgroup of G if and only if H has no partial complement in G.

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M.A./M.Sc. Part – II Annual Examination – 2022

Subject: Mathematics | **Paper: IV-4-N / (IV-VI) (Opt. iv) [Rings & Modules]** | **Time: 3 Hrs.** | **Marks: 100**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section I

Q 1. a) Let $a, b \in F[x]$, $b \neq 0$ and F is a field. Then there exist $q, r \in F[x]$ such that $a = bq + r$ with $r = 0$ or $\deg(r) < \deg(b)$. Moreover q and r are unique.

b) Let R be an integral domain, R is Unique Factorization Domain if and only if R is a factorization domain and every irreducible element is prime. **10+10**

Q 2. a) Let R be an integral domain, show that

(i) $x|y$ if and only if $yR \subseteq xR$

(ii) u is a unit of R if and only if $uR = R$

(iii) the set of all units of R is an abelian group.

b) The polynomial $x - c$ is a factor of a polynomial $p(x) \in K[x]$ if and only if $x = c$ is a root of $p(x) = 0$. **10+10**

Q 3. a) Let R be an integral domain, $p \in R\backslash\{0\}$. Then p is prime if and only if $R/_{pR}$ is an integral domain.

b) Prove that every integral domain can be embedded in a field. **10+10**

Q 4. a) Find $[\mathbb{Q}(\sqrt{3}, \sqrt[5]{7}) : \mathbb{Q}]$.

b) Define extension of a field. Find the smallest extension of $\mathbb{Q}$ having a root of $x^3 - 2 \in \mathbb{Q}[x]$. **10+10**

Q 5. a) Find the degree of minimal splitting field of $x^6 + 1$ over $\mathbb{Q}$.

b) Let K be a field, an element $a \in K$ is algebraic over F if and only if $[F(a): a]$ is finite. **10+10**

Section II

Q 6. a) Let M and Nbe two R-modules, $f: M \to N$ and $g: N \to M$ be two module homomorphisms such that $gof = I_M$ (identity map on M). Show that $N = Kerg \oplus Imf$.

b) Let A and B be submodules of an R- module M. Then prove that

$$(A + B)/_B \cong A/_{A \cap B}.$$

10+10

Q 7. a) Every FG – R-module is homomorphic image of a free module.

b) Let R be a ring with identity and M be irreducible R-module, then M is cyclic. 10+10

Q 8. a) Show that in $\mathbb{Z}[\sqrt{-5}]$ the elements 6 and $2(1 + i\sqrt{5})$ do not have a HCF.

b) Prove if R –module $M = M_1 \oplus M_2$ and M can be generated by "p" elements. Then M_1 can be generated by "p" elements. 10+10

Q 9. Let M be a module over PID R and suppose that M is freely generated by a finite set of k elements. Prove that every basis of M contains exactly k elements. 20

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M.A./M.Sc. Part – II Annual Examination – 2022

Subject: Mathematics | **Paper: IV-VI (opt. ix) (Electromagnetic Theory)** | **Time: 3 Hrs.** | **Marks: 100**

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

SECTION-I

1. (a) Discuss electric potential, electrostatic induction and electric field.

   (b) Find the total charge in an infinite sheet of surface charge with density $\sigma(x,y)=\dfrac{\sigma_0 e^{-|x|/a}}{\left[1+(y/b)^2\right]}$ where a is constant.

2. An infinitely long hollow semi-cylinder of radius R carries a uniform surface charge distribution σ_0

   (a) What is the electric field along the axis of the cylinder?

   (b) Use the results of (a) to find the electric field along the axis due to a semi-cylinder of volume charge ρ_0.

3. (a) Discuss the conservation of charge.

   (b) Calculate the magnitude of the given point charge so that the electric field 64 cm distant is 3.14 N/C.

4. (a) Determine the Lorentz condition and discuss it for an electrostatic field.

   (b) A parallel plate capacitor has a capacitance of 112 pF, a plate area of 96.5 cm^2, and a mica dielectric with constant 5.40. At a 55 V potential difference, calculate the magnitude of the induced surface charge.

5. (a) Workout the relationship between conductivity and resistance.

   (b) An infinite line charge produces a field of $4.52\times10^4 N/C$ at a distance of 1.96 m. Calculate the linear charge density.

SECTION-II

6. (a) Formulate the Maxwell's equations inside matter in general form.

   (b) Find the magnetic field between the circular plates of a parallel-plate capacitor that is charging using the Ampere-Maxwell law. The plates have a radius of R. The bordering field should be ignored.

7. (a) Discuss the propagation of plane electromagnetic waves in conducting media.

   (b) A radio station transmits a *10-kW* signal at a frequency of *100 MHz*. For simplicity, assume that it radiates as a point source. At a distance of *1 km* from the antenna, find: (i) the amplitude of the electric and magnetic field strengths, and (ii) the energy incident normally on a square plate of side *10 cm* in *5 min*.

8. (a) Show that the electric and magnetic field vectors satisfy the wave equation in free space.

   (b) Show that the flux of poynting vector through any closed surface gives the energy flow due to a plane wave through an imaginary cylinder.

9. (a) In a hollow cylindrical waveguide, explain the transverse electric waves.

   (b) Show that there must be conservation of energy by formulating the coefficient of reflection and transmission at an interface when the incident wave is polarized with its $\vec{E}$ vector normal to the plane of incident.

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Roll No. ....

Subject: Mathematics **Paper: IV-VI (Opt. v) (Number Theory)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

SECTION – I

Q.1 **(a)** Let a and b be any two integers with at least one of them is non-zero. Prove that $\gcd(a,b)$ exists and is unique. (7)

(b) (i) State and prove the Fundamental Theorem of Arithmetic. (7+6)

(ii) In base 12 name the integers $1,2,3,...9,10=\alpha,11=\beta$ then evaluate $(\alpha\beta 9)+(\beta 7\alpha)$ and $(\alpha\beta 9)(\beta 7\alpha)$

Q.2 **(a)** Solve $2x \equiv 1 mod 5$, $3x \equiv 9 mod 6$, $4x \equiv 1 mod 7$. (2+8)

(b) Find all solutions of (10)

$$f(x)=x^3+x+57\equiv 0 \pmod{125}$$

Q.3 **(a)** Define a multiplicative arithmetic function. Let n be an integer > 1. Show that $\sigma(n)$ is odd if and only if n is a perfect square or twice a perfect square. (2+8)

(b) (i) State and Prove Wilson Theorem. (6+4)

(ii) Let $(a,42)=1$. Prove that $a^6 \equiv 1 \pmod{168}$.

Q.4 **(a)** Let m and n be integers both > 1, such that every prime divisor of n is a prime divisor of m. Prove that (10)

$$\phi(mn)=n\phi(m)$$

(b) (i) Find last two digits of 13^{35} in its decimal representation. (5+5)

(ii) Find exponent of 11 in 200!

Q.5 **(a)** Let a be primitive root modulo n and b,c be integers, then show that $Ind\ bc \equiv Ind\, b + Ind\, c \pmod{\varphi(m)}$. Hence solve the congruence $11x^3 \equiv 2 \pmod{23}$ using indices. (5+5)

(b) Let m, n be integers both > 2 and $(m,n)=1$. Show that there exist no primitive root modulo mn (10)

SECTION – II

Q.6 **(a)** Let p be an odd prime. Prove that (10)

$$\left(\frac{2}{p}\right)=\begin{cases}1 \text{ if } p\equiv \pm 1 (\text{mod } 8)\\ -1 \text{ if } p\equiv \pm 3 (\text{mod } 8)\end{cases}$$

(b) Define quadratic residue and quadratic non-residues of a prime number. Prove that the product of two quadratic residues or quadratic non-residues of a prime number p is always a quadratic residue of p. (10)

Q.7 **(a)** Define a primitive polynomial. State and prove Gauss Lemma of primitive polynomials. (10)

(b) Let p be an odd prime. Prove that the polynomials (7+3)

$\frac{x^p-1}{x-1}$ and $\frac{x^{p^2}-1}{x^p-1}$ are irreducible.

Q.8 **(a)** Let $f(x)$ and $g(x)$ be non-zero polynomials over F, relatively prime over F. Prove that there exist polynomials $s_0(x)$ and $t_0(x)$ over F, such that $s_0(x)f(x)+t_0(x)g(x)=1$. (10)

(b) If θ algebraic over F, so is every element of $F(\theta)$. (10)

Q.9 **(a)** For $\alpha,\beta\in R[\theta]$, Show that $N\alpha\beta=N\alpha N\beta$, where N is the norm of the algebraic number (10)

(b) Show that an element $\alpha\in R(\theta)$ is a unit if and only if its norm is ± 1. (10)

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M.A./M.Sc. Part – II Annual Examination – 2022

Subject: Mathematics **Paper: IV-6-N / IV-VI (opt.vi) (Fluid Mechanics)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions by selecting atleast TWO questions from each section. All questions carry equal marks.

Section I

Q.1(a)	Define viscosity, what is Newton's law of viscosity? Explain briefly.	(10)
(b)	Glycerin has density of 1260 kg/m^3 and a kinematic viscosity of 0.001183 m^2/s. What shear stress is required to deform this element at a strain rate of $10^4\ s^{-1}$?	(10)
Q.2(a)	State and prove Kelvin's minimum energy theorem.	(10)
(b)	Derive Bernoulli's equation for steady inviscid flow under conservative forces.	(10)
Q.3(a)	Find the Cartesian equation of the streamlines when the fluid is streaming from three equal sources situated at the corners of an equilateral triangle.	(10)
(b)	The velocity components of a certain two-dimensional fluid flow are given by $u = 3x + y,\ v = 2x - 3y.$ Show that the continuity equation is satisfied. Is the flow irrotational? Calculate the circulation around the circle $(x + 1)^2 + (y - 6)^2 = 4.$	(10)
Q.4(a)	What is a boundary surface? Derive the condition for the surface $F\ (x, y, z, t)=0$ to be a boundary surface.	(10)
(b)	For $u = -\omega y,\ v = \omega x,\ w = 0,$ show that the surfaces intersecting the streamlines orthogonally exist and are planes through z-axis, although the velocity potential does not exist.	(10)
Q. 5	The velocity potential for a certain two-dimensional incompressible irrotational fluid flow is $\varphi = x + \frac{1}{2}\ln(x^2 + y^2)$, by converting it in polar coordinates, evaluate the following: (i) Stream function (ii) Complex velocity potential (iii) Speed and Stagnation points (iv) Equipotential line and Streamlines	(20)

Section II

Q.6(a)	Use the method of images to prove that if there be a source of strength m at the point z_0 in a fluid bounded by the lines $= 0$ *and* $\theta = {}^{\pi}/_{3}$, the solution is $\varphi + i\psi = -\frac{m}{2\pi}\ln(z^3 - z_0^3)(z^3 - z_0'^3)$, where $z_0 = x_0 + iy_0$ and $z_0' = x_0 - iy_0$.	(10)
(b)	Discuss briefly the superposition of Poiseuille and Couette flows. Is this superposition is dynamically admissible? If yes, why? Which components of velocity are nonzero for it? Evaluate the expression for velocity field and pressure for the same flow.	(10)
Q.7(a)	How a vortex, doublet and uniform stream are combined? Describe the resulting flow in detail.	(10)
(b)	State and Prove Kutta and Joukowski theorem.	(10)
Q.8 (a)	Under which configuration of vortices the time required to move a distance a is $\frac{a}{u}$? Prove it. Also define the interval for which the above configuration remains unchanged.	(10)
(b)	Derive the Navier-Stokes equations for an incompressible viscous fluid. Show that, in the case of Couette flow, the velocity of the fluid is given by $u = \frac{U}{h}y - \frac{h^2}{2\mu}\frac{dp}{dx}\frac{y}{h}(1 - \frac{y}{h})$.	(10)
Q.9(a)	The velocity distribution for the steady, laminar and fully developed flow between two fixed parallel plates is given by $u = -\frac{1}{2\mu}\frac{dp}{dx}(hy - y^2)$, *where* $\frac{dp}{dx}$ is the pressure gradient, h is the gap width between the plates and y is the distance measured upward from the lower plate. Determine the volumetric flow rate, maximum velocity and the shearing stress at both the plates.	(10)
(b)	What is the Stokes first problem? Write its mathematical formulation and solve it by Laplace transform for the velocity field	(10)

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M.A./M.Sc. Part – II Annual Examination – 2022

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Subject: Mathematics **Paper: IV-7-N / IV-VI (opt.vii) [Quantum Mechanics]** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

Section-I

1. (a) State Heisenberg uncertainty principle and obtain uncertainty Δx for the wavefunction given by (10)

$$\psi(x,t) = \sqrt{\frac{1}{a\sqrt{2\pi}}}\exp\left[-\frac{(x-x_0)^2}{4a^2}\right]\exp\left(\frac{ip_0x}{\hbar}\right)\exp\left(-i\omega_0 t\right)$$

(b) Discuss fundamental difference between wave and particle nature using double slit experiment. Also explain de Broglies hypothesis regarding matter waves. (10)

2. (a) An operator $\hat{A}$, for an observable A, has two normalized eigenstates ψ_1 and ψ_2 with respective eigenvalues a_1 and a_2. Similarly, An operator $\hat{B}$, for an observable B, has two normalized eigenstates ϕ_1 and ϕ_2 with respective eigenvalues b_1 and b_2. The eigenstates are related by (10)

$$\psi_1 = \frac{3\phi_1 + 4\phi_2}{5}, \qquad \psi_2 = \frac{4\phi_1 - 3\phi_2}{5}.$$

- Observable A is measured and the value a_1 is obtained. What is the state of the system immediately after this measurement?
- If B is now measured, what are the possible states and what are the respective probabilities.
- Right after the measurement of B, A is measured again. What are the the probabilities of getting a_1 and a_2?
- Also determine whether the operators commute or not.

(b) Show that $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + \left[\hat{A},\hat{B}\right] + \frac{1}{2!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] + \frac{1}{3!}\left[\hat{A},\left[\hat{A},\left[\hat{A},\hat{B}\right]\right]\right] + \ldots.$ (10)

3. (a) Let $\left[\hat{A},\hat{B}\right] = i\hat{C}$, show that $\Delta A\Delta B \geq \frac{\hbar}{2}\langle C\rangle$. (10)

(b) Express the function $\psi = e^{ikx}$ in terms of the normalized basis $\phi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$ of Hilbert space $\mathcal{H}$, where $\mathcal{H}$ consists of L_2 integrable functions over the interval $[0,a]$. (10)

4. (a) Derive reflection and transmission coefficient for rectangular barrier scattering where the energy E of the particles in the incident beam is greater than the height V of potential barrier. (10)

(b) Show that the time evolution of expectation value of an observable A is given by (10)
$\frac{d\langle A\rangle}{dt} = \langle\frac{i}{\hbar}\left[\hat{H},\hat{A}\right] + \frac{\partial\hat{A}}{\partial t}\rangle$.

Section-II

5. (a) Derive an expression for $\hat{L}_z$ in spherical polar co-ordinates. (10)

   (b) Find the first order corrections, $E_n^{(1)}$, to the eigen energies of Anharmonic oscillator whose perturbation Hamiltonian is given by $\hat{H}' = K'x^4$, where $K' \ll 1$ and $\hat{x} = \frac{1}{\sqrt{2}}\left(\hat{a} + \hat{a}^{\dagger}\right)$. (10)

6. (a) Determine and justify whether the perturbation problem for two dimensional harmonic oscillator belongs to degenerate or non-degenerate perturbation theory. (10)

   (b) Explain scattering amplitude for 3-D scattering and obtain general expression relating it to scattering cross section.

7. (a) Find expressions for first order corrections to the eigenvalues and eigenfunctions for very small perturbation to the time independent, non-degenerate Hamiltonian $\hat{H}_0$. (10)

   (b) Prove the following statements. (10)

   - $[\hat{L}_+, \hat{L}_-] = 2\hbar\hat{L}_z$, where $\hat{L}_\pm$ are ladder operators.
   - If ϕ_m is an eigenfunction of $\hat{L}_z$ for eigenvalue $\hbar m$, then show that $\hat{L}_+\phi_m$ is also an eigenfunction of $\hat{L}_z$ corresponding to eigenvalue $\hbar(m+1)$.

8. (a) Obtain the radial part of the Scrhödinger's wave equation. Also find the allowed eigen energies for infinite spherical well given by

$$V(r) = \begin{cases} 0, & r \leq a; \\ \infty, & r > a. \end{cases}$$

(10)

   (b) Using Born's approximation, compute the phase shift δ_1 for scattering in a centrally symmetric field. (10)

9. (a) Find the eigenfunctions and corresponding eigenvalues of the Hamiltonian operator for one dimensional quantum harmonic oscillator.

   (b) Compute the expression for ionization potential of hydrogen, helium and lithium atoms. (10)

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M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. …………….

Subject: Mathematics

Paper: IV-8-N / IV-VI (Opt.viii) (Special Theory of Relativity and Analytical Dynamics) Time: 3 Hrs. Marks: 100

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

SECTION I

1. (A) Explain the Michelson-Morley experiment for its intended objective and the significance of the null result of the Michelson-Morley experiment.

(B) Discuss the general Lorentz transformation for relative arbitrary velocities and hence deduce the results for time dilation, length contraction and general velocity addition formula. [10+10=20]

2. (A) Prove that the first two equations of special Lorentz transformation can be written in the form $ct' = -x\sinh\phi + ct\cosh\phi$, $x' = x\cosh\phi - ct\sinh\phi$, where the rapidity $\phi = \tan^{-1}\left(\frac{v}{c}\right)$. Also establish the following versions of these equations: $ct' + x' = e^{-\phi}(ct + x)$, $ct' - x' = e^{\phi}(ct - x)$, $e^{2\phi} = \left(1 + \frac{v}{c}\right)/\left(1 - \frac{v}{c}\right)$

(B) Show that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformation.

[10+10=20]

3. (A) For the following special cases of Lorentz transformations:
(a) space-time rotations, (b) time-reversal transformations, (c) space reflections, and (d) space-time translations, give appropriate details of these transformations.

(B) What is the difference between a definite and an indefinite metric. Identify the vectors $A^a = (-1, 4, 0, 1)$, $B^a = (2, 0, -1, 1)$ and $C^a = (2, 0, -2, 0)$ as time like, space like or light like for the Minkowski metric $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$.

[10+10=20]

4. (A) Let us assume that a particle (projectile particle of mass m_P) moving with certain velocity strikes another particle (target particle of mass m_T). The two particles move off. Assume that the collision of two particles generates a new particle. Find the expression for minimum kinetic energy (threshold energy T_0) required for production of this new particle (of rest mass m_N).

(B) Derive the expression for relativistic Compton shift and find the expression for the recoiling electrons energy and momentum.

[10+10=20]

5. (A) What is a four-vector potential? Express Maxwell's field equations in the four-vector form.

(B) Derive Maxwell's equations from Maxwell's Tensor $F^{\mu\nu}$. [10+10=20]

SECTION II

6. (A) What is the difference between usual derivative d/dt and δ−variation of position vector $\mathbf{r}_\nu = \mathbf{r}_\nu(t, q_s), \nu = 1, 2, ...N, s = 1, 2, ...n$ of a particle? Prove that $d\ \delta = \delta\ d$ with the usual notation, where d denotes the usual derivative.

   (B) Consider the motion of a particle of mass m moving in space. Choosing the spherical polar co-ordinates (r, θ, ϕ) as the generalized co-ordinates, calculate the components of the generalized force if a force $\mathbf{F}$ acts on it.

[10+10=20]

7. (A) What is a simple harmonic oscillator (SHO)? Prove that the force $\mathbf{F} = -k\ x\ \mathbf{i}$ acting on the SHO is conservative. Set up the Lagrangian and discuss the motion.

   (B) What is generalized momentum $\mathbf{p}_j$? For generalized momentum $\mathbf{p}_j$, establish the relation $\dot{\mathbf{p}}_j = \frac{\partial L}{\partial q_j}$. [10+10=20]

8. (A) Explain the canonical transformation from a set of coordinates q, p to a new set of coordinates Q, P in the Hamilton theory. What are four possible different generating functions of a canonical transformation.

   (B) If $F(q, p, t)$ and $G(q, p, t)$ are two integrals of motion, then show that the Poisson bracket $[F, G]$ is also an integral of motion.

[10+10=20]

9. (A) Discuss Hamilton-Jacobi Method and apply it to the problem of the Harmonic Oscillator.

   (B) Let (F, G) be the Poisson bracket then find the expression for the Poisson brackets (a) (F_1F_2, G), (b) $\partial(F, G)/\partial t$, (c) $(F_1, (F_2, F_3))$ and (d) (F, H) where H is the Hamiltonian and F, F_1, F_2, F_3, G are arbitrary functions depending on q_i, p_i and t. [10+10=20]

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M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. ....

Subject: Mathematics **Paper: IV-10-N / IV-VI (opt. x) [Operations Research]** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

SECTION 1

Q.1 Ozark farm uses at least 800 lb of special feed daily. The special feed is a mixture of corn and soyabean meal with the following composition.

lb per lb of feed stuff

Feed stuff	Protein	Fiber	Cost($/lb)
Corn	0.09	0.02	0.30
Soyabean meal	0.60	0.06	0.90

The diet requirement of the special feed is at least 30% protein and at most 5% fiber. Ozark farm wishes to determine the daily minimum cost of feed mix.

(a) Formulate the problem as a linear programming model. (10)

(b) Solve the problem using graphical method. (10)

Q.2 **(a)** Write the differences between M-technique and two-phase method. (8)

(b) Apply simplex method to solve the following problem: (12)

Maximize $z = 5x_1 + 4x_2$

subject to

$$6x_1 + 4x_2 \le 24$$
$$x_1 + 2x_2 \le 6$$
$$-x_1 + x_2 \le 1$$
$$x_2 \le 2$$
$$x_1, x_2 \ge 0$$

Q.3 **(a)** Consider the problem (10)

Minimize $z = 3x_1 + 2x_2 + 3x_3$

subject to

$$x_1 + 4x_2 + x_3 \ge 7$$
$$2x_1 + x_2 + x_4 \ge 10$$
$$x_1, x_2, x_3, x_4 \ge 0$$

Solve the problem using x_3 and x_4 for the starting basic feasible solution. (Do not use any artificial variables.)

(b) Use dual simplex method to solve the following problem. (10)

Minimize $z = 5x_1 + 6x_2$

subject to

$$x_1 + x_2 \ge 20$$
$$4x_1 + x_2 \ge 40$$
$$x_1, x_2 \ge 0$$

Q.4 **(a)** Write a note on transshipment model. (8)

(b) JoShop needs to assign four jobs to four workers. The cost of performing a job is a function of the skills of the workers. The following table summarizes the cost of the assignments. Worker 1 cannot do job 3, and worker 3 cannot do job 4. Determine the optimal assignment using the Hungarian method. (12)

	Job 1	Job 2	Job 3	Job 4
Worker 1	$50	$50	---	$20
Worker 2	$70	$40	$20	$30
Worker 3	$90	$30	$50	---
Worker 4	$70	$20	$60	$70

Find the optimal assignment.

Q.5

(a) Write a note on Vogel's approximation method. (8)

(b) Solve the following transportation model using method of multipliers. (12)

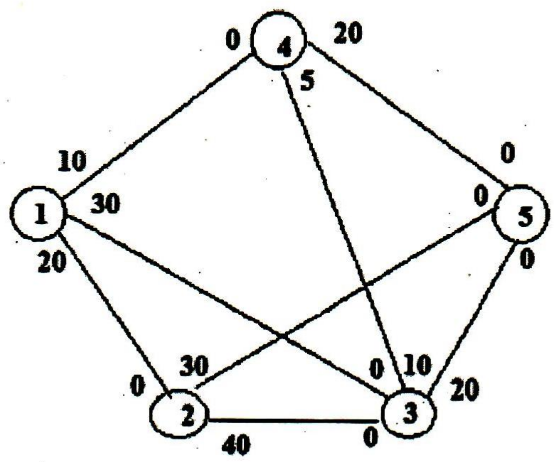
---	3	5	4
7	4	9	7
1	8	6	19
5	6	19	

SECTION 2

Q.6.

(a) Define the terms network, directed network, path and cycle. (8)

(b) Determine the maximal flow from node 1 to node 5 in the network using the maximal flow algorithm. (12)



Q.7

(a) Use the revised simplex method to solve the following problem. (10)

Maximize $z = 2x_1 + x_2$

subject to

$$3x_1 + 4x_2 \leq 6$$
$$6x_1 + x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

(b) Use dynamic programming to solve the following problem. (10)

Maximize $z = x_1 x_2 x_3$

subject to

$$x_1 + x_2 + x_3 = 10$$
$$x_1, x_2, x_3 \geq 0$$

Q.8

Use branch and bound method to solve the following problem. Select x_1 as the branching variable at node 0. (20)

Maximize $z = 2x_1 + 3x_2$

subject to

$$7x_1 + 5x_2 \leq 36$$
$$4x_1 + 9x_2 \leq 35$$

x_1, x_2 are nonnegative integers.

Q.9

Use parametric linear programming to solve the following problem. (20)

Maximize $z = (3 - 2t)x_1 + (2 + t)x_2 + (5 + 2t)x_3$

subject to

$$x_1 + 2x_2 + x_3 \leq 40$$
$$3x_1 + 2x_3 \leq 60$$
$$x_1 + 4x_2 \leq 30$$
$$x_1, x_2, x_3, t \geq 0$$

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M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. ....

Subject: Mathematics **Paper: IV-VI (opt.xi) [Theory of Approximation & Splines]** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

	SECTION-I	
	Question No. 1	**Marks**
(a)	Find the power fits $y = Ax^2$ and $y = Bx^3$ for the following data and use $E_2(f)$ to determine which curve fits best. x_k: 2.0, 2.3, 2.6, 2.9 y_k: 5.2, 7.1, 10.1, 14.1	(10)
(b)	Find the least-squares plane for the given data (1,1,7), (1,2,9), (2,1,10), (2,2,11), (2,3,12).	(10)
	Question No. 2	
(a)	For $f(x) = \sin(x)$, on $\left[0, \frac{\pi}{4}\right]$, find the Chebyshev nodes and the error bound for the Lagrange polynomial $P_5(x)$.	(10)
(b)	Establish the Pade approximation for $\cos(x) \approx R_{4,4}(x) = \dfrac{15120 - 6900x^2 + 313x^4}{15120 + 660x^2 + 13x^4}$	(10)
	Question No. 3	
(a)	Determine the image of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$, under the reflection transformation through the angle $\frac{\pi}{8}$.	(10)
(b)	Derive the matrix of transformation for 2D shear transformation.	(10)
	Question No. 4	
(a)	Derive the matrix of for 2D rotation transformation about an arbitrary point.	(10)
(b)	Discuss the scaling transformation in 3D.	(10)
	SECTION-II	
	Question No. 5	
(a)	Using de Casteljau's algorithm, evaluate the value of Bernstein-Bézier curve $P(\theta) = \sum_{i=0}^{3} B_i^3(\theta)\underline{b}_i$ at $\theta = \frac{1}{3}$.	(10)
(b)	Find out new control points of Bernstein-Bézier quartic form from Bernstein Bézier cubic form. Show it geometrically as well.	(10)

	Question No. 6	
(a)	Prove that Bernstein-Bézier curve of degree n satisfies affine invariance property.	(10)
(b)	Derive the formula for the control point form of the cubic curve. Derive the Timmer form and Ball form of the cubic curve as well.	(10)
	Question No. 7	
(a)	Find the natural cubic spline $S(x)$ that passes through (0,0.0), (1,0.5), (2,2.0) and (3,1.5) with the boundary conditions $S''(0) = 0$, $S''(3) = 0$.	(10)
(b)	Show that for $x \in [x_0, x_k[$, the spline S defined by $S(x) = \sum_{i=0}^{n} a_i x^i + \sum_{i=0}^{k} c_i (x - x_i)_+^n$ has representation of the form $S(x) = \sum_{i=0}^{n} b_i x^i + \sum_{i=1}^{k-1} c_i (x - x_i)_+^n$	(10)
	Question No. 8	
(a)	Compute the error bound of cubic Hermite interpolation.	(10)
(b)	Discuss the following end conditions for cubic spline interpolation: (i) Not a knot end condition (ii) Clamped end condition (iii) periodic end condition	(10)
	Question No. 9	
(a)	Discuss tensor product surfaces and Bernstein-Bézier cubic patch.	(10)
(b)	State and prove de Boor algorithm for the B-spline curve of degree n.	(10)

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M.A./M.Sc. Part – II Annual Examination – 2022

Subject: Mathematics **Paper: IV-12-N / (IV-VI) (Opt. xii) (Advanced Functional Analysis)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FIVE questions in all selecting at least TWO questions from each section.

SECTION – I

Q.1 (a) What is completeness Geometrically. Find 2 nontrivial proper complete subspaces of R^3.

(b) Let $X = (X, \|\cdot\|)$ be normed space. Prove that there exists a Banach space $\widehat{X}$ and and Isometry A from X onto a subspace W of $\widehat{X}$ which is dense in $\widehat{X}$. Furthermore, $\widehat{X}$ is unique up to an isometry.

Q.2 (a) Consider the set of all integers Z and the set of all rational numbers Q in R with usual metric. Determine which set is everywhere dense and which set is nowhere dense. Also describe the category of each of these sets in R.

(b) What is l^7. Discuss its separability and completeness. Give example of a subspace of l^2 which is not closed.

Q.3(a)Let M be an orthonormal set in a Hilbert space H. Show that M is total in H if and only if for all $x \in H$ the Parseval identity holds.

(b)Prove that an orthonormal set M in a separable Hilbert space H is at most countable.

Q.4 (a) Define Hilbert spaces. Give three examples of Hilbert Spaces.

(b)Define Linear manifolds with examples.

SECTION – II

Q.5 (a)State and prove Hahn-Banach Theorem for normed spaces.

(b) If p_1 and p_2 are sublinear functionals on a vector space X and c_1 and c_2 are positive constants, show that $p = c_1p_1 + c_2p_2$ is sublinear on X.

Q.6 (a) Give some applications of Principle of uniform boundedness.

(b) State and prove Riesz's representation theorem for bounded linear operator on a Hilbert Space.

Q.7 (a) Define first and second dual of a vector space. Give two examples.

(b) Prove necessary and sufficient conditions for weak convergence of sequence in a normed space.

Q.8 (a) Compute Conjugate space of $C[0,1]$.

(b)Define second conjugate space. Compute this space for l_p-spaces.

Q.9 (a) Let Y be subspace of $X = C[0,1]$ which consists if all functions $f \in X$ which have a continuous derivative. Then show that the differential operator

$$T : Y \to X$$

defined as $T(f) = f'$, is closed.

(b) If let N be a normed space and S be a non empty subset of N. Then S is bounded if and only if $f(S)$ is bounded for each bounded linear functional f defined on N.