UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Supply 2020 & Annual – 2021

Roll No. ....

Subject: Mathematics   Paper: I (Advanced Analysis)   Time: 3 Hrs.   Marks: 100

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION – I

Q1.	a) Show that cartesian product $P \times P$ is denumerable, where $P = \{1,2,3, \dots\dots\dots\}$. b) State and prove Bernstein Theorem.	10 10
Q2	a) For cardinal numbers α, β, γ if $\alpha \leq \beta$, then show that $\alpha\gamma \leq \beta\gamma$ b) Suppose $f: S \to T$ is an ordered isomorphism between two ordered sets S and T. Then show that $a \in S$ is a first, last, minimal or maximal elements of S if and only if $f(a)$ is a first, last, minimal or maximal elements of T.	10 10
Q3	a) Let A be a well-ordered set and $S(A)$ be the collection of all initial segments of elements in A, then Show that A is ordered isomorphic to $S(A)$. b) Let A be a well-ordered set and S be a subset of A with the following property; If $a \preccurlyeq b$ and $b \in S$, then $a \in S$. Then show that $A = S$ or S is an initial segment of A.	10 10
Q4	a) Show that for ordinal numbers λ, μ and η, $(\lambda\mu)\eta = \lambda(\mu\eta)$ and $1.\lambda = \lambda$ b) Show that the set of natural number equipped with divisibility is partial ordered set.	10 10

SECTION – II

Q5	a) Show that Lebesgue measure of countable set is zero. b) For any set A and $\epsilon > 0$, show that there is an open set O such that $A \subseteq O$ and $m^*(O) < m^*(A) + \epsilon$.	10 10
Q6	a) Let $\{E_n\}$ be a decreasing sequence of measurable sets and $m(E_1) < \infty$, then show that $m(\bigcap_{i=1}^{\infty} E_i) = m(\bigcap_{n=1}^{\infty} E_n) = \lim_{n\to\infty} m(E_n)$. b) For a subset $E \subseteq \mathbb{R}$ and $\epsilon > 0$ there is a closed set $F \subseteq E$ such that $m^*(E \backslash F) < \epsilon$ if and only if E is measurable.	10 10
Q7	a) Show that the interval [0, 1) contains a non-measurable set. b) Show that every Borel set is measurable. Does converse hold? Justify your answer. c) Show that the characteristic function χ_A defined on measurable set is measurable if and only if A is measurable subset of D.	8 6 6
Q8	a) Let f and g be extended real- valued measurable function which are finite almost everywhere then show that $f+g$ is measurable. b) Show that an extended function f is Borel measurable if and only if for ant open set V in $\bar{R}$, $f^{-1}(V)$ is a Borel set. c) Let f be a non-negative measurable function on E. Then show that $\int f = 0$ if and only if $f=0$ a.e on E.	8 6 6
Q9	a) State and prove Fatou's Lemma. b) Let E be measurable se, $1 \leq p \leq \infty$ and q be the conjugate of p. For $f \in L^p(E)$ and $g \in L^q(E)$, show that $f.g$ is integrable and $\int_E \lvert f.g \rvert \leq \lVert f \rVert_p . \lVert g \rVert_q$. Also show that if $f \neq 0$, the function $f^* = \lVert f \rVert_p^{1-p} sgn(f) \lvert f \rvert^{p-1} \in L^q(X,\mu)$ such that $\int_E f.f^* = \lVert f \rVert_p$ and $\lVert f^* \rVert_q = 1$ c) Define truncation function and calculate Lebesgue integral of $h(x) = \frac{1}{\sqrt{x}}$ on [0,1]	6 8 6

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M.A./M.Sc. Part – II Supply 2020 & Annual – 2021

Subject: Mathematics | Paper: II (Methods of Mathematical Physics) | Time: 3 Hrs. | Marks: 100

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION – I

1(a)	Show that the eigen functions corresponding to periodic SL system are orthogonal.	10
1(b)	Suppose that $u(x)$ and $v(x)$ are two solutions of the SL systems then prove that the Lagrange's identity must hold $uL(v)-vL(u)=\frac{d}{dx}\{p(x)[u(x)v'(x)-u'(x)v(x)]\}$	10
2(a)	Use the method of Frobenius to find a series solution in x of the DE $2xy''+y'-y=0$	10
2(b)	Find linearly independent solutions of the DE $y''-2xy'+y=0$ by the method of power series expansions.	10
3(a)	Solve the boundary value problem $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, 0\le x\le a, 0\le y\le b, u_x(0,y)=0, u_x(a,y)=0, u_y(x,0)=0, u(x,b)=f(x)$	10
3(b)	Find the complete integral surface of the PDE $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ containing the straight line $x+y=0, z=1$	10
4(a)	Find the integral surface of the PDE $(y^2-z^2)z_x-xyz_y=xz$ containing the curve $x=y=z, x>0$	10
4(b)	Prove the integral representation $F_{21}(a,b,c;x)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-a-1}(1-xt)^{-a}\,dt$	10
5(a)	Discuss the orthogonality of Bessel functions and show that $\int_a^b xJ_\mu(\alpha x)J_\mu(\beta x)dx=0$	10
5(b)	Prove the Rodrigues formula $P_n(x)=\frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$	10

SECTION – II

6(a)	(i) Evaluate the inverse Laplace transform of $L^{-1}\left(\frac{e^{-3s}}{(s-2)^3}\right)$. (ii) Use convolution theorem to calculate the Laplace transform of $f(t)=\int_0^t (t-\beta)^3 e^{\beta}\sin\beta\,d\beta$	10
6(b)	Solve the DE by Laplace transform method $y''(t)-2ky'(t)+k^2y(t)=f(t)$	10
7(a)	Calculate Fourier sine transform of the function $f(x)=e^{-x}\cos x$	10
7(b)	Use Fourier transform to solve the potential equation $u_{xx}+u_{yy}=0$ for the potential function $u(x,y)$ in the semi-infinite strip $0<x<c, y>0$ that satisfies the conditions $u(0,y)=0, u_y(x,0)=0, u_x(c,y)=f(y)$	10
8(a)	Construct the Green's function associated with the boundary value problem $x\frac{d^2u}{dx^2}+\frac{du}{dx}+\lambda r(x)u=0$ with $u(1)=0$ and $u(0)$ finite.	10
8(b)	Construct the Green's function associated with the boundary value problem $u''-u+\lambda u=0$ with $u(0)=0$ and $u(1)=0$.	10
9(a)	Find the extremal of the problem $I[y]=\int_0^1 (x+y'^2)dx, y(0)=1, y(1)=2$	10
9(b)	Discuss geodesic problem. A uniform cable is fixed at its ends at the same level in space and is allowed to hang under gravity. Find the final shape of the cable.	10

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M.A./M.Sc. Part – II Supply 2020 & Annual – 2021

Subject: Mathematics | Paper: III (Numerical Analysis) | Time: 3 Hrs. | Marks: 100

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION – I

Q1. (8+12)

(a) Write an algorithm to find an approximate root of the non-linear equation $f(x)=0$ using secant method.

(b) Find a real root of the equation $x^3 + x^2 - 100 = 0$ by fixed point iterative method.

Q2. (10+10)

(a) Solve the system of equations by triangularisation method

$$10x_1 + x_2 + x_3 = 12$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$x_1 + x_2 + 5x_3 = 7$$

(b) Find the Dominant Eigen value and corresponding Eigen vector of the following matrix by power method

$$\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

Q3. (10+10)

(a) Solve $\frac{dy}{dt} = 1 - 2ty; y(0) = 0, h = 0.2\ for\ t = 0.2, 0.4, 0.6$ using Modified Euler's method.

(b) Use Runge-Kutta method of order four to solve the differential equation $\frac{dy}{dt} = \exp(-2t) - 2y$ over $[0, 0.2]$ with $y(0) = \frac{1}{10}$ by taking $h = 0.1$

Q4. (10+10)

(a) Solve the following system by Crout's method

$$10x_1 - 7x_2 + 3x_3 + 5x_4 = 6$$
$$-6x_1 - 8x_2 + x_3 + 4x_4 = 5$$
$$3x_1 - x_2 + 4x_3 + 11x_4 = 2$$
$$5x_1 - 9x_2 + 2x_3 + 4x_4 = 7$$

(b) Prove that

(i) n-th difference of n-th degree polynomial is constant and (n+1)-th difference is zero.

(ii) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

Q5. (10+10)

(a) Derive the Adams-Bashforth predictor and corrector formulae to solve the first order initial value problems.

(b) Solve the following system of differential equations using Taylor's series method of order 3 for x=0.3, 0.6

$$\frac{dy}{dt} = 1 + tz, \frac{dz}{dt} = -ty, y(0) = 0, z(0) = 1.$$

P.T.O

SECTION – II

Q6. (10+10)

(a) Using Newton's divided difference interpolation formula find $f(6)$ and $f(12)$

x	4	5	8	11	13	14
y	46	88	175	294	343	451

(b) Interpolate by means of Gauss's backward interpolation formula, find the sales of a person for the year 1968 from the following data

year	1940	1950	1960	1970	1980	1990
y	3	11	31	69	131	223

Q7. (10+10)

(a) Find the global error of composite Simpson's 3/8 rule.

(b) A solid of revolution is formed by taking about the x-axis, the area between x-axis and line x=0 and x=1 and a curve through the points with the following coordinates

x	0	0.25	0.5	0.75	1
y	1	0.98567	0.95893	0.91436	0.85659

Q8. (10+10)

(a) Find the first derivative of Y at X=35 using Stirling's formula from the following data

X	10	20	30	40	50
Y	0.1023	0.1047	0.1971	0.1096	0.1122

(b) Find the first and second derivatives of $f(x)$ at $x = 1.9$ from the following data

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	3.1234	4.1971	6.2396	7.4322	8.1148

Q9. (10+10)

(a) Solve the following difference equation:

$$y_{n+2} - 3y_{n+1} + 4y_n = \sin 6n + \cos 6n + 7.$$

(b) Solve the following difference equation

$$y_{n+2} - 9y_{n+1} - 52y_n = 13^n(-3n^2 + 1).$$

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M.A./M.Sc. Part – II Supply 2020 & Annual – 2021

Subject: Mathematics | Paper: IV-VI (Opt. i) (Mathematical Statistics) | Time: 3 Hrs. | Marks: 100

NOTE: Attempt FIVE questions in all selecting at least TWO questions from each Section.

SECTION – I

Q.1 (a) If A, B and C are pairwise independent events in a sample space S and A is independent of $B \cup C$. Then prove that A, B and C are mutually independent. **(10)**

(b) Ten vegetables cans, all the same size, have lost their labels. It is known that 5 contains tomatoes and 5 contains corn. If 5 are selected at random, what is the probability that all contain tomatoes? What is the probability that 3 or more contain tomatoes? **(10)**

Q.2 (a) State and prove reproductive property of Poisson distribution. **(10)**

(b) Write down chief characteristics of hyper-geometric, negative binomial, uniform and gamma distribution. **(10)**

Q.3 (a) State and prove Chebyshev's inequality. **(10)**

(b) A coin is tossed 200 times. Find the probability of getting **(10)**

i) between 80 and 120 heads inclusive
ii) less than 90 heads.
iii) Exactly 100 heads.

Q.4 (a) If $X_1, X_2, \dots, X_n$ is a random sample from a normal distribution with mean μ and standard deviation σ, then obtain the sampling distribution of $\bar{X} = \frac{\Sigma x}{n}$ and $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$. **(10)**

(b) In a normal distribution with $\mu = 47.6$ and $\sigma = 16.2$, find **(10)**

i) the probability that a single observation will be larger than 50,
ii) two points such that a single observation has a 97% probability of falling between them,
iii) P_{10}, P_{30} *and* P_{99}

SECTION – II

Q.5 (a) If X_r and X_s are the r^{th}' and s^{th}' random variables of random sample of size n drawn from the finite population $\{C_1, C_2, \ldots C_N\}$. Then **(10)**

$$Cov(X_r, X_s) = \frac{\sigma^2}{N-1}.$$

(b) If X has the standard normal distribution, find the probability density of $Z = X^2$. **(10)**

Q.6 (a) Determine multiple regression equation in terms of linear correlation coefficients. **(10)**

(b) Given the joint density **(10)**

$$f(x,y) = \begin{cases} 2 & \text{for } 0 < y < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Show that $\mu_{Y/x} = \frac{x}{2}$ and $\mu_{X/y} = \frac{1+y}{2}$.

 P.T.O

Q.7 **(a)** Let the random variable X have marginal density $f_1(x) = 1$, $\frac{-1}{2} < x < \frac{1}{2}$ and let the conditional density of Y be **(10)**

$$f(y|x) = 1, x < y < x + 1 \quad \frac{-1}{2} < x < 0$$

$$= 1, -x < y < 1 - x \quad 0 < x < \frac{1}{2}$$

Show that random variables X and Y are uncorrelated.

(b) Compute the coefficient of determination of the following data **(10)**

X	5	11	4	5	3	2
Y	31	40	30	34	25	20

Q.8 **(a)** Let $X_1, X_2, \dots, X_n$ be a random sample of size n taken from a normal population with mean μ and variance σ^2. If $\bar{x}$ and S^2 represent the mean and biased variance of the sample chosen above. Then prove that $\frac{nS^2}{\sigma^2}$ is a Chi-square variate with $n-1$ degree of freedom. **(10)**

(b) If $F \sim F(v_1, v_2)$ then $Y = (1 + \frac{v_1}{v_2} F)^{-1} \sim \beta(\frac{v_1}{2}, \frac{v_2}{2})$. **(10)**

Q.9 **(a)** If n denotes the degrees of freedom of a t-distribution, then show that **(10)**

$$(n - 2r)\,\mu'_{2r} = n(2r - 1)\,\mu'_{2r-2}$$

Where μ' represents moments about origin.

(b) Find moment generating function for χ^2-distribution. Use it to evaluate mean and variance of the distribution. **(10)**

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Roll No. ....

Subject: Mathematics — Paper: IV-VI (Opt. ii) [Computer Applications] — Time: 3 Hrs. — Marks: 50

NOTE: Attempt any FIVE questions by selecting atleast TWO from each section.

SECTION – I

Q.1 (10)

Write a program to find the inverse of a square matrix using Function Subprogram.

Q2. (10)

Write a program to find the roots of a quadratic equation using arithmetic IF-STAMENT.

Q3. (10)

write a program to find the complex conjugate of a complex number.

Q4. (10)

Write a program to find the largest and smallest numbers and their locations in a list of 100 numbers.

SECTION – II

Q5. (10)

Write a program, to print the values of $y(0.1), y(0.2), z(0.1)$ and $z(0.2)$ from

$$\frac{dy}{dx} = x + z, \quad \frac{dz}{dx} = x - y^2 \text{ with } y(0) = 2,\ z(0) = 1$$

by improved Euler's method.

Q6. (10)

Write a program to find a positive root of $e^{-3x} - \cos(\frac{\pi x}{4}) = 0$, correct to four decimal places using the Bisection method.

Q7. (10)

Write a program to solve the following system of equations using Gauss Seidel iterative method:

$$11x - 303y + 200z = 205,\quad 4x + 11y - z = 33,\quad 6x + 3y + 12z = 35$$

Q8. (10)

From the following table write a program to find $f(0.05)$ using Newton's Forward difference formula.

Table

x	-1	0	1	2	3
$f(x)$	2	1	2	9	28

Q9. (10)

Write the Mathematica statements for the following:

1. Plot the following functions, $Sin3x + 0.5$, $Cos2x + 0.2$ and $Secx$ in the same window over the interval, $-\frac{\pi}{2} < x < \frac{+\pi}{2}$
2. Find all the solutions of $x^4 + x^3 - 8x^2 - 5x + 15 = 0$ which are greater than 2
3. Solve for $x : e^{2x} + e^x = 3$
4. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 7 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ find $A+B,\ A-B,\ AB,\ A^{-1}, B^{-1}$
5. Compute the first five derivatives of $f(x) = e^{x^2}$ at $x = 0$

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Subject: Mathematics Paper: IV-VI (opt.xi) [Theory of Approximation & Splines] Time: 3 Hrs. Marks: 100

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

	SECTION-I	
	QUESTION No. 1	**Marks**
(a)	Derive the relation for the reflection of point $P(x, y)$ about the line $y = x$ and $y = -x$.	(10)
(b)	Discuss the scaling transformation in 2D.	(10)
	QUESTION No. 2	
(a)	Derive the relation for the rotation of the point $P(x, y)$ about the point $Q(x_p, y_p)$.	(10)
(b)	Find the image of the circle $x^2 + y^2 = 4$, under the transformation of shear parallel to x-axis by the factor 2.	(10)
	QUESTION No. 3	
(a)	Show that the trigonometric representation of Chebyshev polynomial on $[-1,1]$ is $T_N(x) = cos(Narccos(x))$.	(10)
(b)	Find the least-squares parabola for the four points $(-3,3)$, $(0,1)$, $(2,1)$ and $(4,3)$.	(10)
	QUESTION No. 4	
(a)	Establish the following Padé Approximation: $$e^x \approx R_{2,2}(x) = \frac{12 + 6x + x^2}{12 + 6x + x^2}$$	(10)
(b)	Find the power fits $y = \frac{A}{x}$ and $y = \frac{B}{x^2}$ for the following data and use $E_2(f)$ to determine which curve fits best. x_k: 0.5, 0.8, 1.1, 1.8 y_k: 7.1, 4.4, 3.2, 1.9	(10)
	SECTION-II	
	QUESTION No. 5	
(a)	State and prove de Casteljau algorithm for Bernstein-Bézier curve of degree n.	(10)
(b)	Derive the formula for barycentric coordinate with respect to a triangle $\Delta V_1V_2V_3$, $V_i = (x_i, y_i), i = 1,2,3$.	(10)
	QUESTION No. 6	
(a)	Prove that rational quadratic Bernstein-Bézier curve represents conic section.	(10)
(b)	Prove that the image of Bernstein-Bézier curve $P(\theta)$ under any affine transformation $\emptyset$ is $\emptyset(P(\theta)) = \sum_{i=0}^{n} B_i^n(\theta)\emptyset(b_i)$.	(10)
	QUESTION No. 7	
(a)	Define tensor product surface. Construct the Bernstein Bezier cubic patch.	(10)
(b)	Derive the recursive relation for Bernstein polynomials.	(10)
	QUESTION No. 8	
(a)	Derive the relation for the error bound of cubic Hermite interpolation.	(10)
(b)	Prove that a natural spline of degree $2n - 1$ with knots at the points $a = x_0 < x_1 < \cdots < x_k = b$ has the following representation in terms of truncated power function over the interval $[a, b]$ $$S(x) = \sum_{i=0}^{n-1} a_i x_i + \sum_{i=0}^{k} c_i (x - x_i)_+^{2n-1}$$	(10)
	QUESTION No. 9	
(a)	Define integral B-spline, uniform-spline, periodic B-spline, closed periodic B-spline.	(10)
(b)	If $N_0^{(2)}(t) = \begin{cases} t & 0 \le t \le 1 \\ 2 - t & 1 \le t \le 2 \\ 0 & otherwise \end{cases}$ Determine $N_{-5}^{(2)}(t)$ using direct method.	

Data table for Question 4(b):

x_k	0.5	0.8	1.1	1.8
y_k	7.1	4.4	3.2	1.9