UNIVERSITY OF THE PUNJAB

Part-II A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Statistics
PAPER: I (Statistical Inference)

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1 a) i) How would you decide that a particular estimator is good; **(12)**
ii) How would you compare asymptotic unbiasedness and consistency?
iii) What considerations should be made to covert an inference into a Statistical inference?

b) Let $X_1, X_2 \sim p(\lambda)$ i.e. distributed as poisson with parameter λ; then prove that $T_1 = X_1 + X_2$ is sufficient statistic for λ and $T_2 = X_1 + 2X_2$ is not sufficient statistic for λ and what you can infer about the unbiasedness and sufficiency of $T_3 = 3X_1 + 2X_2$. Also find the mean and variance of three statistics. **(13)**

Q.2 a) Under what conditions the minimum variance bound estimator is unique? Prove it. **(07)**

b) What do you mean by consistency? What are its types? Explain each of them. **(08)**

c) Let we draw a sample of size 'n' from a Normal distribution, find the variance-covariance matrix of the parameters to judge their efficiencies. **(10)**

Q.3 a) Find the Maximum likelihood estimator and moment estimators of the parameters of log-normal probability distribution. **(10)**

b) Find the most general form of the distribution for which Geometric Mean (grouped form) is the MLE of the parameter. **(06)**

c) With the help of a suitable probability distribution prove that MLEs are invariant and have minimum variance. **(09)**

Q.4 a) Why the moment estimators are considered to be so important in this era of computerization. **(10)**

b) Find the approximate ML estimator of θ in the Cauchy distribution. Also find the asymptotic variance of this estimator. **(12)**

c) Write the properties of Moment estimators. **(03)**

Q.5 a) Based on random sample of size n from Poisson distribution obtain minimum χ^2 estimator (MCSE) and MLE of θ and compare the two estimators. **(07)**

PTO

b) If X is a Poisson variate with parameter λ and if $g(\lambda)=\frac{1}{m!}\left(\frac{m+1}{\lambda_0}\right)^{m+1}\lambda^m e^{\frac{-(m+1)\lambda}{\lambda_0}}$ is prior density for unknown parameter λ, given a random sample of size n, obtain the Baye's estimator for λ **(08)**

c) Based on a random sample of size n from density $f(x;\theta)=1/\theta,\ 0<x<\theta$ with prior distribution g(θ)=1, 0<θ<1. Obtain the Bayes estimator of θ with respect to loss function $l(\theta;t)=(t-\theta)^2/\theta^2$. **(10)**

Q.6 a) Let μ and σ be the location and scale parameters such that $Z_r=(y-\mu)/\sigma$ where y_r is r[th] order statistic. If $E(Z)=\alpha,\quad V(Z)=V$. Find the ordered least square estimators of μ and σ. Also find their variances and co-variances. **(12)**

b) Explain the statistical method for the construction of confidence intervals. **(06)**

c) Let α' *and* β' be the error sizes of sequential probability ratio test defined by k_0' *and* k_1' *where* $k_0'=\frac{\alpha}{1-\beta}$ *and* $k_1'=\frac{1-\alpha}{\beta}$ then prove that $\alpha'+\beta'\le\alpha+\beta$. **(07)**

Q.7 a) Define and explain i) (BCR) ii) power of the test. iii) MP (most powerful) test iv) Uniformly most powerful test and power curve. **(10)**

b) State Neyman-pearson Lemma. **(03)**

c) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from **(12)**

$$f(x;\theta)=\theta^x(1-\theta)^{1-x}I_{(0,1)}(x) \quad where\ \theta=\theta_0\ or\ \theta=\theta_1 \qquad \theta_0<\theta_1$$

Apply the Neyman-Pearson Lemma to test $H_0:\theta=\theta_0$ *versus* $H_1:\theta=\theta_1$

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Part-II A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Statistics
PAPER: II (Regression Analysis and Econometrics)

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1.a) State OLS assumptions and discuss situation arises when they are fail to satisfy.

b) For simple linear regression, $Y = \alpha + \beta X + \in$, such that $\in's \sim NII(0, \sigma^2)$, show that MLE of σ^2 is consistent estimator.

Q.2.a) What is the rationale of using ridge regression? Obtain mean and variance of ridge regression estimator of $\underline{\beta}$ for the model $\underline{Y} = X\,\underline{\beta} + \underline{\in}$.

b) A production function is specified as $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \in$, The data refer to a sample of 23 firms and observations are measured as deviations from the sample means $\Sigma x_1^2 = 12$, $\Sigma x_2^2 = 12$, $\Sigma x_1 x_2 = 8$, $\Sigma y x_1 = 10$, $\Sigma y x_2 = 8$, $\Sigma y^2 = 10$. What is the least square of β_1 and its standard error after imposing the restriction that $\beta_1 + \beta_2 = 1$

Q.3.a) For GLR model when error terms are non-spherical, show that least square estimator are BLUE.

b) Differentiate between distributed lag model and autoregressive model. Describe the Koyck distributed lag model. Discuss the features of the Koyck transformation.

Q.4.a) What is Multicollinearity? Discuss the remedial measures for Multicollinearity.

b) In GLR model, when error terms follow first order autoregression, discuss the consequences in using OLS estimators.

Q.5.a) The $X'X$ matrix of all exogenous variables in a model is

$$X'X = \begin{bmatrix} 7 & 0 & 3 & 1 \\ 0 & 2 & -2 & 0 \\ 3 & -2 & 5 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Only the first of these exogenous variables has a nonzero Co-efficient in a structural equation to be estimated. This equation includes two endogenous variables and the least square estimates of the reduced form Co-efficient for these two variables are

$$\begin{bmatrix} 0 & 1 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

Taking the first endogenous variable as the dependent variable, state and estimate the equation by appropriate method.

b) In simultaneous equations model why OLS estimators of structural parameters are inconsistent?

Q.6.a) Explain the Goldfield-Quandt's test for homoscedasticity. State its assumption.

b) Discuss Bartlett's and Durbin's instrumental Variables.

Q.7. Discuss the following.

(i) Functions of Econometrics
(ii) Dummy variable
(iii) Identification
(iv) Run / Geary test

UNIVERSITY OF THE PUNJAB

Part-II A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Statistics
PAPER: III (Part-A) [Data Processing and Computer Programming]
TIME ALLOWED: 3 hrs.
MAX. MARKS: 75

NOTE: Attempt any FOUR questions.

Q.1(a) Give the block diagram showing the general architecture of a microcomputer system. Also name the main components of a computer and their functions.

(b) Explain the following terms:

(i) Memory Measuring Units.
(ii) Compiler.
(iii) Hardware and Software.
(iv) Modem
(v) Types of Storage devices

(9+10)

Q.2(a) How are the FORTRAN 77 statements coded using FORTRAN coding sheet? Also what is the purpose of putting a ' C ' in column 1 of the FTN coding sheet.

(b) Write a Fortran program that reads and print the employee's salary after paying health premium according to the following plan

$$\text{Premium} = \begin{cases} 50.50 & \textit{if signle} \\ 90.00 & \textit{if married without children.} \\ 125.00 & \textit{if married with children} \end{cases}$$

(c) Write the following expression in Fortran notation.

(i) $(x^2+y+xy)^3\sqrt{2(xy-6)^4}$ (ii) $Cos(\log_{10}(a+3b))$

(iii) $\dfrac{\sqrt{|Sin(a-|b|)|}}{\left[1-\left(\frac{y}{c}\right)^2\right]^{\frac{1}{2}}}$ (iv) $\dfrac{a}{b}+\dfrac{xy}{abc}$

(d) Rewrite the following statements after correcting errors, if any:

(i) Read , (JOLY(I , J) , I = 1 , 40 , J=1 , 40)
(ii) Write , ((J, I =1 , 20) , I , J =1 , 20)
(iii) Write PAY(I) , I= 1 , 20
(iv) Read (* , *) X , Y

(4+7+4+4)

PTO

Q.3(a) Determine the output of the following programs:

```
C  First program                C  Second program
   I=4                             A = 6.4
   K=6                             B = 600.2
   L=K+2*I                         W = 21.3
   I=2*L+I/2                       Z = A+B*W
   K=K/4                           WRITE(*,10) A, B, W
   L=I+K+L                      10 FORMAT(T10,F6.2,//,T10,F6.2,//,T10,F6.2//,T10,F8.2)
   WRITE(*,*) I, K,L            C
   STOP                            STOP
   END                             END
```

(b) Write down the general form of the GOTO and computed GOTO statements.

(c) Write a program that will print out a table heading with 'Student ID' starting in column 5, 'Midterm Exam' in column 20, 'final exam' in column in 35, 'Internal marks' in column 50, total in column 60 and 'grade' in column 68. On the next line underline all of the headings.

(d) Write a program in FTN to compute nPr and nCr.

(6+4+6+3)

Q.4(a) Define the functions of the following FORTRAN statements. Give examples at least two in each case;

(i) INPUT and OUTPUT STATEMENTS
(ii) DO STATEMENT
(iii) STOP STATEMENT END STATEMENT

(b) Write a complete Fortran program to find the Mean, Standard deviation, Simple Linear Regression line, Standard error of estimate, and coefficient of correlation between N pairs of real numbers.

(c) Write a program that reads in a list of values, stores them in an array, and returns the minimum value and its position in the array.

(6+6+7)

Q.5. With particular reference to C-Language:

I. Write the role of header files and pre-processor directives.
II. Differentiate between Structure and Union, give examples.
III. Differentiate between actual and formal arguments of a function.
IV. What is the importance of define directives?
V. What is the objective of header files before main() in c-language?

(4+4+4+3+4)

Q.6(a) What are the commonly used input / output functions in C? How they are assessed?

(b) What is meant by looping? Describe two different forms of looping.

(c) Write programs using switch statement and else-if statement to make a four function calculator.

(d) Write a loop that will calculate the sum of every third integer, beginning with i = 2 (i.e. calculate the sum 2 + 5 + 8 + 11 + ...) for all values of i that are less than 100. Write the loop three different ways.

(i) Using a while statement
(ii) Using a do-while statement
(iii) Using a for statement.

(4+3+6+6)

Q.7(a) What is the difference between **#define Directive** and **#include Directive**?

(b) Write a program in C to drawn a checker's board on the screen.

(c) Suppose that P dollars are borrowed from a bank, with the understanding that A dollars will be repaid each month until the entire loan has been repaid. Part of the monthly payment will be interest, calculated as i percent of the current unpaid balance. The remainder of the monthly payment will be applied toward reducing the unpaid balance.

Write a C program that will determine the following information:

i) The amount of interest paid each month
ii) The amount of money applied toward the unpaid balance each month
iii) The cumulative amount of interest that has been paid at the end of each month
iv) The amount of the loan that is still unpaid at the end of each month
v) The number of monthly payments required to repay the entire loan

(4+7+8)

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Part-II A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Statistics
PAPER: VI (i) [Statistical Quality Control]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q#1 (a)	Explain in brief three important postulates concerning the laws basic to control.	15
(b)	Assuming that the distribution generated by the process described on the resistance of certain electrical parts with the subgroup size 25 is normal approximately. After 30 subgroups $\sum \bar{X} = 2419.2$ and $\sum \sigma = 107.6$, what proportion of the product meets the specifications 85 ± 10?	10
Q#2	Samples each of size n=6 items are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and S values are calculated for each sample. After analyzing 50 samples, we have $$\sum_{i=1}^{50} \bar{x}_i = 1000, \quad \sum_{i=1}^{50} S_i = 75$$ i. Compute the control limits for the $\bar{x}$ and S control charts. ii. Assume that all points on both charts plot within the control limits. What are the natural tolerance limits of the process? iii. If the specification limits are 19 ± 4.0, what are your conclusions regarding the ability of the process to produce items conforming to specifications? iv. Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process now producing? v. Make suggestions as to how the process performance could be improved.	25
Q#3 (a)	Discuss some limitations of control charts for fraction defectives.	10
(b)	A process is being controlled with a fraction non-conforming control chart. The process average has been shown to be 0.07. Three-sigma control limits are used, and the procedure calls for taking daily samples of 400 items. a) Calculate the upper and lower control limits. b) If the process average should suddenly shift to 0.10, what is the probability that the shift would be detected on the first subsequent sample? c) What is the probability that the shift in part (b) would be detected on the first or second sample taken after the shift?	15
Q#4(a)	What is Average Outgoing Quality? How do we interpret Average Outgoing Quality Limit (AOQL) in acceptance sampling programs?	10

PTO

(b)	In a plan, 22 items were tested for 500 hours with replacement and an acceptance number of 2. Construct an OC-curve showing probability of acceptance as a function of mean life.	15
Q#5(a)	What is acceptance sampling? Discuss four major types of acceptance sampling plans.	15
(b)	A double sampling plan is as follows: Select a sample of 2 from a lot of 20 articles. If both articles inspected are good, accept the lot. If both are defective, reject the lot. If one is good and one is defective, take a second sample of one article. If the article in the second sample is good, accept the lot. If it is defective, reject the lot. If a lot having 25% defectives is submitted, what is the probability of acceptance?	10
Q#6	Use the following data to set up short run $\bar{X}$ and $\bar{R}$ charts using the DNOM approach. The nominal dimensions for each part are $T_A = 100, T_B = 60, T_C = 75, T_D = 50$	25

Sample No.	Part No.	M_1	M_2	M_3
1	A	105	102	103
2	A	101	98	100
3	A	103	100	99
4	A	101	104	97
5	A	106	102	100
6	B	57	60	59
7	B	61	64	63
8	B	60	58	62
9	C	73	75	77
10	C	78	75	76
11	C	77	75	74
12	C	75	72	79
13	C	74	75	77
14	C	73	76	75
15	D	50	51	49
16	D	46	50	50
17	D	51	46	50
18	D	49	50	53
19	D	50	52	51
20	D	53	51	50

Q#7	Write a short note on any **Five** of the following: i. International Standard Organization (ISO) ii. Reliability and Life Testing iii. In-control and out-of-control Average Run Lengths (ARLs) iv. Acceptance control chart v. The Bath-tub Curve vi. Fast Initial Response (FIR)	5 each

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Part-II A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Statistics **TIME ALLOWED: 3 hrs.**
PAPER: VI (iii) [Operations Research] **MAX. MARKS: 100**

NOTE: Attempt any FOUR questions. Graph paper will be provided on demand.

Q.1 a) Explain the Phases of OR

b) What is Big M-technique.

c) Write down the scope of Linear Programming.

d) A firm can produce three types of cloth A,B,C. Three kinds of wool are required for it, red wool, green wool and blue wool. One unit length of type A cloth needs 2 yards of red wool and 3 yards of blue wool; one unit length of type B cloth needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue and one unit length of type C cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has only a stock of 8 yards of red wool, 10 yards of green wool and 15 yards of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs. 3.00 of type B cloth is Rs. 5.00 and that of type C cloth is Rs. 4.00. Determine how the firm should use the available material, so as to maximize the total income from the finished cloth.
15+10

Q. 2 (a) Show unboundedness graphically.

(b) Find optimal solution by M-technique.
Max $X_o = 2X_1 + 3X_2 - 5X_3$
Subject to $X_1 + X_2 + X_3 = 7$; $2X_1 - 5X_2 + X_3 \geq 10$; $X_1, X_2, X_3 \geq 0$ 10+15

Q.3.a) What is transportation modal? Explain its variants.

b)Find optimal solution of the following transportation modal:

	1	2	3	4	Supply
1	10	0	20	11	15
2	12	7	9	20	25
3	0	14	16	18	5
Demand	5	15	15	10	45

10+15

Q.4 a) Explain the following terms.

i) Minimax criteria

ii) Dominance property method

Turnover Leaf

(P.T.O.)

b) Solve the following payoff matrix.

	Player A		
Player B	1	2	3
1	-2	3	0
2	3	1	-1
3	-3	4	2
4	5	-2	-4

12+13

Q.5.a) What is generalized inventory model? Explain its main components.

b) A manufacturer has to supply his customer with 24000 units of his product per year. This demand is fixed and known. Since the unit is used by the customer is an assembly line operation and the customer has no storage space for the units, the manufacturer must ship a day's supply each day. If the manufacturer fails to supply the required units, he will lose the account and probably his business. Hence the cost of shortage is assumed to be infinite, and consequently, non will be tolerated. The inventory holding cost amounts to .59 per unit per month, and setup cost per run is Rs. 350. Find the optimum lot size and the length of optimum production run. 10+15

Q.6.a) What do you understand by Network Analysis? Write its objectives.

b) Distinguish between the CPM Modal and PERT modals.

c) The Following time-cost table (time in week and cost in rupees) applied to a project. Use it to arrive at the network associated with completing the project in minimum Time with minimum cost.

Activity	Normal		Crash	
	Time	Cost	Time	Cost
1-2	2	800	1	1400
1-3	5	1000	2	2000
1-4	5	1000	3	1800
2-4	1	500	1	500
2-5	5	1500	3	2100
3-4	4	2000	3	3000
3-5	6	1200	4	1600
4-5	5	900	3	1600

5+6+14

Q 7 Write note on the following:

(i) Standard Form of LP
(ii) Canonical Form of LP
(iii) Degenerate Solution
(iv) Kinds of game
(v) Main component of Queueing 25

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Part-II A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Statistics **TIME ALLOWED: 3 hrs.**
PAPER: VI (iv) [Part-A-Survey and Report Writing] **MAX. MARKS: 50**

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1. Define Sampling Survey. Explain various types of survey and the main steps involved in the execution of sample survey.

Q.2. Explain, in detail, the errors that affect accuracy and the ways they can be minimized.

Q.3. Write a note on the types of Sampling Designs. Why are probability sampling generally preferred than non-probability sampling design?

Q.4. A consumer durable company is planning to launch a new type of washing machine. The company would like to have information about how consumers select a brand of washing machine.

i. Identify the research objectives for the above situation.
ii. Prepare a sample questionnaire to collect relevant data from consumers.

Q.5. What do you understand by the term questionnaire? Explain the two most common types of survey questions and the situation in which they are used.

Q.6. Explain the significance of survey report and narrate the various steps involved in writing such a report.

Q.7. Explain any three of the following:

a) Overloaded and double barreled questions
b) Concept of data analysis
c) Interpretation of data
d) Use of coding
e) Pretesting of a questionnaire

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Part-II A/2018
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Statistics
PAPER: VII (i) [Time Series Analysis]

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt any FOUR questions.

Q.1.a) Define the following terms. **(5)**
   i. Seasonal and Non-seasonal differencing
   ii. Logarithmic transformation

b) Show that the autocorrelation is bounded within [-1,1] and is symmetric with respect to lag. **(10)**

c) Express the following two MA(1) processes in AR representation **(10)**

Model-1: $Y_t = Z_t + 0.3Z_{t-1}$

Model-2: $Y_t = Z_t + 1.3Z_{t-1}$

Check the invertibility of the above MA processes and discuss the AR representation in relation to the invertibility status.

Q.2.a) If $U_t = \phi U_{t-1} + w_t$ and $V_t = \phi V_{t-1} + z_t$ are two independent AR(1) processes then show that $Y_t = U_t + V_t$ is an AR(1) process, where w_t and z_t are independent purely random processes. **(5)**

b) Derive the mean, variance, autocorrelation function of an AR(1) process: $Y_t = \varphi Y_{t-1} + Z_t$, where $\{Z_t\}$ is a purely random process having zero mean and finite variance. **(10)**

c) Show that the AR(2) process given by **(10)**

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + Z_t$$

is stationary provided that

$$\phi_1 + \phi_2 < 1$$
$$\phi_2 - \phi_1 < 1$$
$$-1 < \phi_2 < 1$$

Q.3.a) Derive the autocorrelation function (lag 1, 2 and 3) of an MA(2) process: **(10)**

$$Y_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$$

Based on your result, discuss the behavior of the autocorrelation function of an MA(q) process.

b) Find the autocorrelation function of the qth order moving average process given by **(10)**

$$Y_t = \frac{1}{q+1} \sum_{i=0}^{q} Z_{t-i}$$

where $\{Z_t\}$ is a purely random process having zero mean and finite variance.

c) Considering an MA(1) process given by $Y_t = \mu + Z_t + \theta Z_{t-1}$, where μ is a constant. Show that its autocorrelation function depends on θ but not on μ. **(5)**

Q.4.a) Show that for an ARMA(p,q) process $\varphi(B)Y_t = \theta(B)Z_t$, we have **(10)**

$$\varphi(B)\rho_k = 0$$

for $k \geq (q+1)$, where q is the order of MA component.

(P.T.O.)

b) For the ARMA(1,1) model $(1+0.3B)Y_t = (1-0.2B)Z_t$, evaluate the first three weights in MA representation and AR representation. **(10)**

c) Show that model $Y_t = 0.3Y_{t-1} - 0.1Y_{t-2} + Z_t + 0.5\,Z_{t-1}$ is over-parameterized. Finds its equivalent simpler form. **(5)**

Q.5.a) If $\{Y_t\}_{t=1}^{n}$ follows an AR(p) process $Y_t = \sum_{i=1}^{p}\phi_i Y_{t-i} + z_t$ where $\{z_t\}$ is independently and normally distributed process with zero mean and finite variance σ_z^2 then show that log-likelihood function is **(15)**

$$\ln L = const. - \frac{n}{2}\ln\sigma_z^2 + \frac{1}{2}\ln\left|M_p\right| - \frac{1}{2\sigma_z^2}\left(\sum_{t=p+1}^{n}\left(Y_t - \sum_{i=1}^{p}\varphi_i Y_{t-i}\right)^2 + Y^T M_p Y\right)$$

where $Y^T = (Y_1, Y_2, \cdots, Y_p) \sim N_p(0, \Sigma_p)$ and $M_p = \sigma_z^2\Sigma_p^{-1}$. Also suggest procedure to find the approximate maximum likelihood estimates of AR parameters.

b) Suggest the method to obtain the least squares estimates of an MA(1) process with non-zero mean. **(10)**

Q.6.a) Derive the forecast of Y_{n+l} based on minimum mean squared error forecast assuming $Y_t = \psi(B)z_t$ where $\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \ldots$ and $\{\psi_j$: j=1,2,...$\}$ are the weights of moving average representation. **(15)**

b) Show that for an ARIMA(1,1,1) process: $(1-\phi B)\nabla Y_t = (1+\theta B)Z_t$, the forecast at origin t with lead time l is given by **(10)**

$$Y_t(l) = Y_t + \frac{\phi(1-\phi^l)}{1-\phi}(Y_t - Y_{t-1}) + \frac{\theta(1-\phi^l)}{1-\phi}Z_t$$

Q.7.a) Show that the forecast errors for lead time 1 with different forecast origins are always uncorrelated. **(10)**

b) An AR(1) model $(1+0.5B)(Y_t - 50) = Z_t$ is fitted to an observed time series of 80 observations. Now a practitioner wants to use this model to forecast Y_{81} and Y_{82}. Compute the desired MMSE forecasts and 95% forecast interval if $Y_{79} = 59, Y_{80} = 52, \sigma_z^2 = 4$. Update the forecast of Y_{82} if it is observed that $Y_{81} = 62$. **(15)**

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Part-II **A/2018**
Examination:- M.A./M.Sc.

Roll No. ....

Subject: Statistics **TIME ALLOWED: 3 hrs.**
PAPER: VII (ii) [Multivariate Analysis] **MAX. MARKS: 100**

NOTE: Attempt any FOUR questions.

Q1.		Write short note on any **FIVE** of the following carrying equal marks: i. Assumptions under Factor Analysis ii. Spectral Decomposition of the Matrices iii. Generalized Variance. iv. Discriminant Analysis v. Canonical Correlations Analysis vi. Principal Component Analysis vii. Multivariate Analysis viii. Positive Definite and Semi-positive Definite Matrices	*(25)*
Q2.	(a)	If $\mathbf{X}$ denotes a $(p\times 1)$ column vector of random variables follow a Multivariate Normal Distribution with mean vector μ and variance covariance matrix Σ then find the $E(\mathbf{X})$ and $Var(\mathbf{X})$.	*(15)*
	(b)	Show that the matrix for the following quadratic form is positive-definite: $3x_1^2+2x_2^2-2\sqrt{2}\,x_1x_2$	*(10)*
Q3.	(a)	From a sample of 20 observations the following mean vector and covariance matrix found: $\bar{\mathbf{X}}=\begin{bmatrix}81.6\\120.1\\90.0\end{bmatrix},\ \mathbf{S}=\begin{bmatrix}10&5&1\\&8&2\\&&10\end{bmatrix}$ (a) Test at 5% level that $\mu'=[90\quad 100\quad 98]$ (b) Find 95% simultaneous confidence intervals for $\mu_1,\ \mu_2,\ \mu_3$. (c) Find 95% Bonferroni Confidence Intervals for $\mu_1,\ \mu_2,\ \mu_3$.	*(18)*
	(b)	Let $\mathbf{W}\sim\mathbf{W}_p(\mathbf{f},\Sigma,\mathbf{M})$. If $\mathbf{C}$ is any $(p\times 1)$ vector of constants, then show that $\mathbf{C'WC}\sim\sigma^2\chi^2(f,\delta^2)$.	*(07)*
Q4		Suppose $\mathbf{X}'=[X_1\quad X_2\quad \dots\quad X_p]$ is a p-component random vector having a certain multivariate normal distribution then derive the principal components of $\mathbf{X}$.	*(25)*

(P.T.O.)

Q5.	(a)	Explain the method of factor analysis, indicating the assumptions involved.	(15)
	(b)	Perform an appropriate factor analysis for the following matrix : $\rho = \begin{bmatrix} 1.0 & 0.83 & 0.78 \\ & 1 & 0.67 \\ & & 1.0 \end{bmatrix}$	(10)
Q6.	(a)	Suppose that population 1 $P_1 \sim N(\mu_1, \sigma_1^2)$ and population 2 $P_2 \sim N(\mu_2, \sigma_2^2)$. Discuss the maximum likelihood discriminant rule.	(10)
	(b)	Consider the three bivariate normal populations with same covariance matrix, given by: $\boldsymbol{\mu}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \boldsymbol{\mu}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \boldsymbol{\mu}_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma} = \begin{bmatrix} 5 & 2 \\ & 1 \end{bmatrix}$ Suppose that the three populations have prior probabilities $1/2, 1/3, 1/6$. Allocate $\mathbf{X}' = \begin{bmatrix} 0.25 & 1.34 \end{bmatrix}$ to one of these populations.	(15)
Q7.	(a)	What is Canonical Correlation? Derive the canonical correlations and canonical variables.	(15)
	(b)	The covariance matrix for four standardized variables Z_1, Z_2, Z_3, Z_4 is, $\rho = \begin{bmatrix} 1 & 0.4 & 0.5 & 0.6 \\ & 1 & 0.3 & 0.4 \\ & & 1 & 0.2 \\ & & & 1 \end{bmatrix}$ Let $\mathbf{Z}_1' = \begin{bmatrix} Z_1 & Z_2 \end{bmatrix}$ and $\mathbf{Z}_2' = \begin{bmatrix} Z_3 & Z_4 \end{bmatrix}$. Find canonical correlation between $\mathbf{Z}_1' \& \mathbf{Z}_2'$. Also find the first pair of canonical variates.	(10)