UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. ……………

Subject: Statistics **Paper: I (Statistical Inference)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1 a) How would you decide that a particular estimator is good? Write a comprehensive note by comparing all the properties of a good estimator. **(12)**

b) Let $Y_1<Y_2<Y_3$ be the order statistics of random sample of size 3 from uniform distribution over the interval $[0,\theta]$. Check Y_1, Y_2 and Y_3 for unbiasedness about θ and if they are not, then how they can be made unbiased. **(13)**

Q.2 a) What do you mean by consistency? Write different forms of consistency. **(07)**

b) State and prove the Cramer-Rao inequality for minimum variance. Also explain under which condition(s) it fails to give minimum variance bound; state the condition(s) concerned. **(10)**

c) In random samples from gamma distribution with p.d.f. **(08)**

$f(x:\theta,p) = \frac{1}{\Gamma p\theta^p} x^{p-1} e^{-x/\theta}, 0 < x \leq \infty, p > 0$ find the MVB estimator of θ for known p.

Q.3 a) Find the maximum likelihood estimator and moment estimators of the parameters of a log-normal probability distribution. **(12)**

b) If $X \sim N(\mu,\sigma^2)$ for a random sample of size n then find MLE of a point A, such that $\int_A^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx = 0.05$ also prove that minimum variance unbiased estimator of A is $\bar{x} + \left[\frac{1.645\Gamma((n-1)/2)\sqrt{n}}{2^{1/2}\Gamma(n/2)}\right] E\sigma^2$. **(13)**

Q.4 a) Find the moment estimators of the parameters of log-normal distribution. **(10)**

b) Let there are 3n observations $x_1, x_2.....x_n$, $y_1,y_2....y_n$ and $z_1,z_2....z_n$ with same unknown variance σ^2. The mean values of the observations are given by $E(X_i) = \theta_1 + 2\theta_2 + 3\theta_3$, $E(Y_i) = 2\theta_1 + 3\theta_2 + \theta_3$, $E(Z_i) = 3\theta_1 + \theta_2 + 2\theta_3$ i=1,2,3...n. where $\theta_1,\theta_2,\theta_3$ are unknown parameters. Apply the least square method to derive estimates of contrasts $(\theta_1-\theta_2), (\theta_2-\theta_3), (\theta_3-\theta_1)$ by using $\theta_1+\theta_2+\theta_3=0$ and obtain the unbiased estimate of σ^2. Also prove that $V(\hat{\theta}_i) = 14\sigma^2/9n$. Find the variance of each contrast. What do you conclude? **(15)**

Q.5	a)	Compare χ^2 (chi-square), minimum chi-square and modified chi-square methods in statistical inference; Under which situations they are applied?	**(07)**
	b)	Based on a random sample of size 'n' from the density $f(x/\theta)=1/\theta$ with prior distribution as $g(\theta)=1,\ \ 0<\theta<1$ obtain the Bayes' estimator for θ with respect to the loss function $\ell(\theta,t)=(t-\theta)^2/\theta^2$	**(10)**
	c)	What is the difference between Bayesian and Classical inference? explain. What is Bayes' estimator? Differentiate between the prior and posterior density.	**(08)**
Q.6	a)	Prove that the approximate values of k_0 and k_1 in SPRT(Sequential probability ratio test) are $\alpha_a/(1-\beta_a)$ *and* $(1-\alpha_a)/\beta_a$	**(08)**
	b)	Let $x_1, x_2, \ldots\ldots, x_n$ denote a random sample from a distribution which has p.d.f. $f(x_i)$ that is positive on only non-negative integers. It is desired to test the simple hypothesis $H_o: f(x)=e^{-1}/x!,\quad x=0,1,2,\ldots\ldots$ against alternative simple hypothesis $H_1: f(x)=(1/2)^{x+1}\quad x=0,1,2,\ldots\ldots$ Derive the expression for BCR (Best critical region). Consider the case of n=1 and k=1, k being any positive integer in the expression $\dfrac{L(\theta', x_1, x_2, \ldots\ldots, x_n)}{L(\theta'', x_1, x_2, \ldots\ldots, x_n)} \leq k$ where $H_o: \theta=\theta',\quad H_1=\theta=\theta''$. Find the power of the test for this combination of n and k when (i) H_o is true (ii) H_1 is true	**(12)**
	c)	What do you mean by 95% confidence interval? What is confidence region?	**(05)**
Q.7	a)	What do you mean by the Pivotal Quantity? Explain with the help of any suitable example.	**(08)**
	b)	Construct a large sample confidence interval for unknown parameter of Poisson distribution.	**(08)**
	c)	Write the short notes on i) Most power test ii) confidence belt iii) shortest confidence interval	**(09)**

UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. …………….

Subject: Statistics **Paper: II (Regression Analysis and Econometrics)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1. Differentiate between
- i) Spherical and Non-spherical error terms
- ii) Autoregressive and distributed lag models.
- iii) Order and Rank conditions of identification.
- iv) ANOVA and ANOCOV models
- v) Under fitting and over fitting a model

Q.2.a) Let $\underline{Y} = X\underline{\beta} + \underline{u}$, such that $\underline{u} \sim N(0, \sigma^2 I)$ show that least squares estimators of $\underline{\beta}$ are BLUE.

b) What is dummy variable trap? Discuss the effect of dummy variables on a function.

Q.3.a) In two variable regression model, suppose the error variance follow, $\text{var}(u_i) \propto [E(y_i)]^2$ How would you transform the model so as to achieve homoscedastic error terms? How would you estimate the model?

b) What are consequences when regressors are correlated?

Q.4.a) What do you know about serial correlation and autocorrelation? Explain the Geary test for autocorrelation?

b) For partitioned GLR model, obtain full regression and stepwise regression estimates. When they are identical?

Q.5.a) Consider the model $\underline{Y} = X\underline{\beta} + \underline{u}$, such that $\underline{u} \sim N(\underline{0}, \sigma^2 I)$ *and* $R'\underline{\beta} = \underline{\gamma}$ obtain least squares estimate of $\underline{\beta}$ and its variance covariance matrix.

b) Consider the regression model $Y = \beta_1 X_1 + \beta_2 X_2 + \ldots\ldots\ldots\ldots \beta_K X_K + u$, which fallows all classical assumptions. Develop the appropriate test statistics for $H_0 = \underline{\beta_1} = \underline{\beta_2} = \ldots\ldots\ldots\ldots \underline{\beta_K} = 0$,

Q.6. For the model

$$y_1 = \alpha_1 x_1 + \alpha_2 y_2 + u_1, \qquad y_2 = \beta_1 y_1 + \beta_2 x_2 + \beta_3 x_3 + u_2$$

Following calculations are available

$$\hat{y}_1 = 5x_1 + 10x_2 + 2x_3, \qquad \hat{y}_2 = 10x_1 + 10x_2 + 5x_3$$

$$\sum x_1^2 = 2, \quad \sum x_2^2 = 4, \quad \sum x_3^2 = 20, \quad \sum x_1 x_2 = \sum x_2 x_3 = \sum x_1 x_3 = 0$$

Estimate the structural parameters, where possible, by appropriate method.

Q.7.a) Why and how instrumental variables are selected? Show that instrumental variable estimate is consistent.

b) When Ridge Regression is used? Define ridge regression estimate and find bias, if any, and variance of this estimate.

UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. …………….

Subject: Statistics
Paper: III (Part-A) (Data Processing and Computer Programming) **Time: 3 Hrs.** **Marks: 75**

NOTE: Attempt any FOUR questions.

Q.1. **(a)** Describe the functions of the following components of a digital computer:

(i) Types of storage devices (ii) Compiler and Interpreter

(b) Write algorithm and flow chart to print out the even numbers between 1 to 'n' and their squares.

(c) Define the following DOS commands its uses and examples:

(i) XCOPY (ii) DIR (iii) CHKDSK

(6+7+6)

Q.2. **(a)** Write the following mathematical expression into Fortran expressions.

i) $\left[\frac{x}{2y}\right]^{n-1}$ ii) $\frac{Sin\ x}{|y| + Cos\ Z}$

iii) $e^{|a|} - \frac{b^2}{|c|}$ iv) $Sin\,(x-2y) + e^{xy} - |x^2 - y^2|$

v) $\left|\sqrt{x - y^3} - \frac{z^3}{\cos(a+b)}\right|$

(b) Suppose J = 10, K = 20 & L = 30. Find the value of logical expression.
i) 2 * J . EQ . K . AND . K. LE . L
ii) K. EQ . 10 . AND . . NOT . J . LT . L – 15

(c) Write a FTN program to calculate the Sum of following series

$$1-\frac{2^2}{3^2}+\frac{4^2}{5^2}-\frac{6^2}{7^2}+.........+\frac{(2N)^2}{(2N+1)^2}$$

(10+4+5)

Q.3. **(a)** Define the LOOPS and its types with structure in FTN.

(b) Write a FTN program to read in an integer n > 2 and determine if n is a prime number.

(c) Write a FTN program using nested DO-loops which calculates z from the polynomial $z = x^3 - y^3$ for values of x and y from -4 to 4 in steps of 0.5.

(4+8+7)

Q.4.(a) Find error, if any, in each of the following arithmetic statement.

```
X * Y       = Z
PI          = 3.14
9ABC        = ALPHA / BETA
DISTANCE    = VELOC * TIME
```

(b) Suppose the rate of interest on the amount of loan is 7 percent if amount ≤ Rs. 10000.00, but is 6 percent if the amount exceeds Rs. 10000.00. Write a FTN program to find the interest on the amount of loan.

(c) Given a real matrix A of size (M X N) write a FTN program to print the position

(7+7+5)

Q.5. (a) Write the following expressions into C++ language:

i. $\sqrt[3]{yz} + \frac{e^{\frac{1}{2}(x^2)}}{x+y+z}$ ii. $a^x + \frac{2}{3}(x+y)^{3/4}$

(b) What is meant by looping? Describe two different forms of looping.

(c) Write a C programme that generate Fibonacci series

1, 1, 2, 3, 5, 8, 13-----------.

(6+5+8)

Q.6. (a) Explain with examples the format and function of the following C statements.

(i) getch() (ii) scanf() (iii) printf()

(b) Write a program in C which asks the user for the radius of a circle and then calls a function named AREA() to calculate the area of the circle.

(c) Write C program that find the sum of the series using array.

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \dots \dots \dots \dots \dots + \frac{1}{n}$$

(6+6+7)

Q.7. (a) Distinguish between a switch statement and else-if statement.

(b) Write a program in C which read all elements of two matrices A and B of any order, find the product of these two matrices and prints their product.

(c) Write a program to calculate overtime rate of 10 employees using do-while(). Overtime is paid at the rate of R. 250 per hour for over 40 working hours.

(4+8+7)

UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. ....

Subject: Statistics | Paper: VI (i) [Statistical Quality Control] | Time: 3 Hrs. | Marks: 100

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q#1 (a)	Describe in brief what to do if we need to construct a control chart with variable sample sizes	10
(b)	Show how assignable causes of variation are identified on $\bar{x}$ and R chart?	15
Q#2	Samples of n=6 items are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and S values are calculated for each sample. After analyzing 50 subgroups, we have $$\sum_{i=1}^{50}\bar{x}_i = 1000, \quad \sum_{i=1}^{50} S_i = 75$$ 1. Compute the control limits for the $\bar{x}$ and S control charts. 2. Assume that all points on both charts plot within the control limits. What are the natural tolerance limits of the process? 3. If the specification limits are 19 ± 4.0, what are your conclusions regarding the ability of the process to produce items conforming to specifications? 4. Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process now producing? If the process was centered at $\mu = 19.0$, what would be the effect on percent scrap and rework?	25
Q#3 (a)	What is meant by process capability? How will you determine the same?	10
(b)	The following data represent the number of nonconformities per 1000 meters in telephone cable. From analysis of these data, would you conclude that the process is in statistical control? What control procedure would you recommend for future production?	15

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	1	12	6
2	1	13	9
3	3	14	11
4	7	15	15
5	8	16	8
6	10	17	3
7	5	18	6
8	13	19	7
9	0	20	4
10	19	21	9
11	24	22	20

Q#4(a)	What is an item-by-item sequential sampling plan? Also describe its procedure.	10
(b)	Define in control and out of control Average Run Lengths (ARLs). How do we interpret?	15
Q#5	Take a sampling plan with $n_1 = 100, c_1 = 0, n_2 = 100, c_2 = 4$ If the incoming lots have fraction nonconforming $p = 0.05$ then what is the probability of acceptance? What is the probability of rejection on first sample?	25
Q#6(a)	In a plan, 22 items were tested for 500 hours with replacement and an acceptance number of 2. Construct an OC-curve showing probability of acceptance as a function of mean life.	10
(b)	State some modern definitions of reliability and life testing.	15
Q#7	Write a short note on any **Five** of the following: i. Lot Tolerance Percent Defective (LTPD) ii. Bath-tub Curve iii. Producer's and Consumer's Risks iv. Rectifying Inspection v. Average Time to Signal (ATS) vi. Dodge-Romig Sampling Plans	5 each

UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. ……………..

Subject: Statistics **Paper: VI (iv) (Part-A-Survey and Report Writing)** **Time: 3 Hrs.** **Marks: 50**

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q.1. Define and differentiate Coverage and Non-response errors. Also discuss the methods to control these errors.

Q.2. Describe Assessment Surveys and method of their evaluation.

Q.3. Discuss briefly various methods of conducting a survey.

Q.4. What is the role of sample size in a successful survey? How does it affect the precision and accuracy of survey results?

Q.5. What are the major components of a survey report? Explain.

Q.6. Discuss and give examples to explain under what kind of situation you would use the following sampling schemes.

a) Stratified Random Sampling

b) Simple Random Sampling

c) Purposive Sampling

Q.7. Discuss the validity of a questionnaire. What are the specific features of a questionnaire for a telephonic survey?

UNIVERSITY OF THE PUNJAB

M.A./M.Sc. Part – II Annual Examination – 2022

Roll No. ....

Subject: Statistics **Paper: VII (ii) (Multivariate Analysis)** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt any FOUR questions. All questions carry equal marks.

Q1. The following are the five measurements on the three variables: (5+20)

X_1	9	2	6	5	8
X_2	12	8	6	4	10
X_3	3	4	0	2	1

a) Write down data matrix. What is the order of the data matrix?

b) Write down the sample mean vector, sample covariance matrix and sample correlation matrix for the data above.

Q2. ~~Write down~~ the singular value decomposition of the matrix below. Also show that the matrix is a positive definite matrix. (15+10)

$$A = \begin{bmatrix} 3 & -\sqrt{2} \\ -\sqrt{2} & 2 \end{bmatrix}$$

Q3. Show that mean and variance of the multivariate normal distribution are $\underline{\mu}$ and Σ respectively. (25)

Q4. Explain the difference between central and non-central Wishart distribution. Derive the mean of Wishart random matrix. (12+13)

Q5. a) Let $X' = [X_1 \ \ X_2]$ be random vector with $\mu' = [\mu_1 \ \ \mu_2]$ with variance covariance matrix $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$ find the mean vector and covariance matrix for the following linear combination.

$$Z_1 = X_1 + X_2$$

$$Z_2 = X_1 - X_2$$

(15+10)

b) Show that linear combination of Wishart matrices follows Wishart distribution.

Q6. Carryout principal component analysis for the matrix below. (25)

$$\Sigma = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$$

Q7. ~~The~~ covariance matrix for four standardized variables Z_1, Z_2, Z_3, Z_4 is, (25)

$$\rho = \begin{bmatrix} 1 & 0.4 & 0.5 & 0.6 \\ & 1 & 0.3 & 0.4 \\ & & 1 & 0.2 \\ & & & 1 \end{bmatrix}$$

Let $\mathbf{z_1} = [Z_1 \quad Z_2]^T$ and $\mathbf{z_2} = [Z_3 \quad Z_4]^T$. Obtain first pair of canonical variates and canonical correlation.