UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I
Annual Examination - 2017
(Special Exam)

Roll No. ....

Subject: Mathematics General-I **TIME ALLOWED: 3 hrs.**
PAPER: Calculus (Differential and Integral Calculus) **MAX. MARKS: 100**

Attempt any **SIX** questions by selecting **TWO** questions from **Section-I, TWO** questions from **Section-II, ONE** question from **Section-III** and **ONE** question from **Section-IV**.

Section-I

Q. 1. (a) 4+4

i. Solve the inequality $|2x+5| > |2-5x|$

ii. Find the points of discontinuity of the function:

$$f(x) = \begin{cases} x+2 & if\ 0 \le x < 1 \\ x & if\ 1 \le x < 2 \\ x+5 & if\ 2 \le x < 3 \end{cases}$$

(b) 4+5

i. If $x,y \in R$, then prove that $|x+y| \le |x| + |y|,\ \forall\ x,y \in R$.

ii. Examine the continuity of

$$f(x) = \begin{cases} \frac{x^2}{a} - a & if\ 0 < x < a \\ 0 & if\ x = a \\ a - \frac{a^2}{a} & if\ x > a \end{cases} \quad \text{at } x=a$$

Q. 2. (a) If $y = \arctan x$. Show that 8+9

$(1+x^2)\, y'' + 2xy' = 0$. Hence find the value of $y^{(n)}$ at $x=0$.

(b) Show that $f(x) = |x| + |x-1|$ is continuous for every real value of x but is not differentiable at $x = 0$ and $x = 1$.

Q. 3. (a) Differentiate $(arcsinx)^{\frac{1}{x^x}}$ with respect to 'x'. 8+9

(b) Discuss the validity of Rolle's Theorem. Find c if possible such that $f'(c) = 0$.

$f(x) = x\,(x+3)e^{-\frac{x}{2}}$ on $[-3,0]$

Q. 4. (a) Show that, under certain conditions to be stated, 8

$f(a+h) = f(a) + h\, f'(a+\theta h)$ where $0<\theta<1$. Prove that the limiting value of θ, when h decreases indefinitely, is $\frac{1}{2}$.

(b) Evaluate the given Limits: 4+5

i. $\lim_{x \to 0} \left(\frac{\sinh x}{x}\right)^{\frac{1}{x^2}}$

ii. $\lim_{x \to 0} \frac{x\cos x - \ln(1+x)}{x^2}$

PTO

Section-II

Q. 5. (a) Find a reduction formula for $\int x^n \,\text{Sin}ax\, dx$ where n>1 is an integer. Hence evaluate $\int x^4 \text{Sin}4x\, dx$. 8+9

(b) Evaluate $\int \frac{secx}{1+cosecx} dx$.

Q. 6. (a) Show that $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx = \frac{\pi}{8} \ln 2$. 8+9

(b) Use the trapezoidal rule with n=4 to approximate $I = \int_0^4 \sqrt{x^2+1}\, dx$. Also find the exact value of the integral.

Q. 7. (a) If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, with Centre C, meets the major and minor axes in T and t. Prove that $\frac{a^2}{CT^2}+\frac{b^2}{Ct^2}=1$. 8+9

(b) Sketch the graph of the curve. $r = a\cos 3\theta$, $a>0$

Q. 8. (a) Find the measure of the angle of intersection of the curves $r=\cos 2\theta$ and $r=\sin\theta$ at $(\frac{1}{2},\frac{\pi}{6})$. 8+9

(b) Prove that $x\sin t - y\cos t + a\cos 2t = 0$ is an equation of the normal to the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, t being parameter.

Section-III

Q. 9. (a) Locate the points of relative extrema of the curve $f(x) = \sin x \cos 2x$. 8+8

(b) Find the area of the region bounded by the loop of the curve

$$x^4+y^4=2a^2xy$$

Q. 10. (a) Find equations of the asymptotes of the curve. 8+8

$$x^2(x-y)^2+a^2(x^2-y^2)=a^2xy$$

(b) Calculate the perimeter of the limacon $r = a+b\cos\theta$, if $\frac{b}{a}$ is small.

Section-IV

Q. 11. (a) Find the extrema of $f(x,y) = 2x^4+y^2-x^2-2y$. 8+8

(b) Use differentials to approximate $\sqrt{299^2+399^2}$.

Q. 12. (a) Find the first order partial derivatives of $f(x,y)=\ln\left[\frac{\sqrt{x^2+y^2}-x}{\sqrt{x^2+y^2}+x}\right]$ 8+8

(b) Find the area outside the circle r=3 and inside the Cardioid $r=2(1+\cos\theta)$

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Roll No. ....

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PAPER: Calculus (Differential and Integral Calculus) **MAX. MARKS: 100**

Attempt any **SIX** questions by selecting **TWO** questions from **Section-I, TWO** questions from **Section-II, ONE** question from **Section-III** and **ONE** question from **Section-IV**.

Section-I

Q. 1. (a)

i. Solve the inequality 4+4

$$\frac{2x}{x+2} \geq \frac{x}{x+2}$$

ii. Evaluate

$$\lim_{x \to -1} \frac{x^{\frac{1}{3}}+1}{x+1}$$

(b) 4+5

i. Examine the continuity of

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} & if\ x \neq 0 \\ 1 & if\ x = 0 \end{cases} \quad \text{at } x = 0$$

ii. If $a \geq 0$, then $|x| \leq a$ if and only if $-a \leq x \leq a$.

Q. 2. (a) Find the values of 'a' and 'b' so that the function f is continuous and differentiable at $x = 1$, where 8+9

$$f(x) = \begin{cases} x^3 & if\ x < 1 \\ ax + b & if\ x \geq 1 \end{cases}$$

(b) If $f(x) = \frac{1}{\sqrt{b^2-a^2}} \ln \left[\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{x}{2}}\right]$, then show that $f'(x) = \frac{1}{a+b\cos x}$

Q. 3. (a) Differentiate arc tan $\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to arc $\cos x^2$. 8+9

(b) Find first four terms of the Maclaurin Series of

$f(x) = \sec x$

Q. 4. (a) Evaluate the given Limits: 4+4

i. $\lim_{x \to \infty} (e^x + e^{-x})^{\frac{2}{x}}$

ii. $\lim_{x \to 0} \left(\frac{1}{x}\right)^{\tan x}$

(b) If $x > 0$, prove that: $x - \ln(1+x) > \frac{x^2}{2(1+x)}$ 9

PTO

Section-II

8+9

Q. 5. (a) Evaluate $\int \arcsin\left(\sqrt{\frac{x}{x+a}}\right) dx$.

(b) Evaluate $\int \frac{x}{(x^2-2x+2)\sqrt{x-1}} dx$.

Q. 6. (a) Show that $\int_0^{\pi} \frac{x\tan x}{\sec x+\cos x} dx = \frac{\pi^2}{4}$. 8+9

(b) Prove that $\int_0^{\frac{\pi}{2}} \cos^m x \sin nx \, dx = \frac{1}{m+n} + \frac{m}{m+n} \int_0^{\frac{\pi}{2}} \cos^{m-1} x \sin(n-1)x \, dx$.

Q. 7. (a) Find the locus of the middle points of a system of parallel chords of the parabola $y^2=4ax$. 8+9

(b) If PF_2Q, PF_1R be two chords of an ellipse through the foci F_2, F_1, show that $\frac{|PF_2|}{|F_2Q|} + \frac{|PF_1|}{|F_1R|}$ is independent of the position of P.

Q. 8. (a) Show that the pedal equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}-\frac{r^2}{a^2b^2}$ 8+9

(b) Find the points at which $r = 1+\cos\theta$ has horizontal and vertical tangents.

Section-III

Q. 9. (a) Find the intrinsic equation of the cardioids: $r = a(1-\cos\theta)$ 8+8

(b) Find the area of the region bounded by the curve $xy^2=4(2-x)$ and the y-axis.

Q. 10. (a) Find an equation of the osculating circle to the hyperbola $\frac{x^2}{4}-\frac{y^2}{9}=1$ at the point (-2,0) 8+8

(b) Find the length of the arc of the parabola $y^2=4ax$ cut off by the straight line $3y=8x$

Section-IV

Q. 11. (a) Verify that $f_{xy}(x,y) = f_{yx}(x,y)$ if $f(x,y)=\ln\left(\frac{x^2+y^2}{xy}\right)$. 8+8

(b) The side of a building is in the shape of a square surmounted by an equilateral triangle. If the length of the base is 15m with an error of 1%, find the percentage error in the area of the side.

Q. 12. (a) Find the point on $x^2 - 2y^2 - 4z^2=16$ at which the tangent plane is parallel to the plane $4x - 2y + 4z = 5$. 8+8

(b) Find the volume of the solid bounded by the graphs of $x^2 + y^2 = 4$, $z = \sqrt{16 - x^2 - y^2}$, $z=0$.

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I
Annual Examination - 2017

Roll No. ....

Subject: Mathematics A Course-I | **TIME ALLOWED: 3 hrs.**
PAPER: Calculus and Analytical Geometry | **MAX. MARKS: 100**

NOTE: **Attempt SIX questions by selecting TWO questions from Section-I, TWO questions from Section-II, ONE questions from Section-III and ONE question from Section-IV.**

SECTION – I

Q.1: **(9,8)**

(a) Solve the inequality $\dfrac{x^2-2}{1-2x} > 1$

(b) Let $f(x) = \begin{cases} x\sin\dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, discuss the continuity of f at x = 0.

Q.2: **(9,8)**

(a) Find $\dfrac{dy}{dx}$, *if* $y = arc\,\tan\left(\dfrac{x\sin\alpha}{1-x\cos\alpha}\right)$

(b) Use differentials to find approximate value of $\cos 61^\circ$.

Q.3: **(9,8)**

(a) If $y = (arc\,\sin x)^2$, show that $(1-x^2)y'' - xy' - 2 = 0$, differentiate this equation n times and find the value of $y^{(n)}$ at x = 0.

(b) If $x > 0$. Prove that $\dfrac{x}{x+1} < \ell n(x+1) < x$.

Q.4: **(9,8)**

(a) Find by Maclaurin formula, the first four terms of expansion of $f(x) = e^{ax}\cos bx$ and remainder after n terms.

(b) Evaluate $\lim\limits_{x\to 0}(\tan x)^{\sin 2x}$

SECTION – II

Q.5: **(5,5,7)**

Integrate the following.

(*i*) $\displaystyle\int \frac{\sin x}{\sin 3x}dx$

(*ii*) $\displaystyle\int \frac{dx}{(1+x)\sqrt{x^2-1}}$

(*iii*) $\displaystyle\int \frac{dx}{ax^n + bx}$

PTO

Q.6: **(8,9)**

(a) Prove that $\int_0^{\pi} \frac{x\,dx}{1+\sin x} = \pi$

(b) Obtain a reduction formula for $\int \frac{x^n}{\sqrt{1-x^2}}\,dx$ and hence evaluate $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

Q.7: **(9,8)**

(a) Show that the pedal equation of the astroid $x = a\cos^3\theta,\ y = a\sin^3$ *is* $r2 = a^2 - 3p^2$

(b) Show that tangents to cardioids r = a (1 + cosθ) at points $\theta = \frac{\pi}{3}$ and $\theta = \frac{2\pi}{3}$ are respectively parallel and perpendicular to initial line.

Q.8: **(9,8)**

(a) Show that the curves $r^m = a^m \cos m\theta \ and \ r^m = a^m \sin m\theta$ cut each other orthogonally.

(b) Prove that the area enclosed by parallelogram formed by the tangents at end of conjugate diameters of ellipse is constant.

SECTION – III

Q.9: **(8,8)**

(a) Find the asymptotes of the curve $x^2y + xy^2 + xy + y^2 + 3x = 0$

(b) Find the relative maxima and minima of $r = 1 - \cos\theta$

Q.10: **(8,8)**

(a) Show that the intrinsic equation of the parabola $x^2 = 4ay$ is
$S = a\tan\alpha\sec\alpha + a\ln(\tan\alpha + \sec\alpha)$

(b) Prove that radius of curvature at point (2a,2a) on the curve $x^2y = a(x^2 + y^2)$ *is* $2a$

SECTION – IV

Q.11: **(5,5,6)**

(a) A straight line makes angle of measure $\alpha, \beta, \gamma, \delta$ with the diagonals of a cube. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{3}{4}$

(b) Find an equation of the plane passing through the point (2,–3,1) and containing the lines x – 3 = 2y = 3z – 1.

Q.12: **(8,8)**

(c) Find equation of sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y - 4z - 8 = 0$ is a great circle.

(d) Find the direction of Qibla of Badshai Mosque, Lahore, latitude = 31°35.4′N and longitude = 74°18.4′E.

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I
Annual Examination - 2017

Roll No. ....

Subject: Mathematics B Course-I
PAPER: Vector ANALYSIS AND MECHANICS

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

NOTE: Attempt SIX questions in all, selecting one question from section-I, Two questions from section-II, Two questions from section-III and one question from section-IV

SECTION-I

1. (a) Prove that (i) $\int\left[\bar{a}.\bar{f}(t)\right]dt=\bar{a}.\int\bar{f}(t)dt$ (ii) $\int\left[\bar{a}\times\bar{f}(t)\right]dt=\bar{a}\times\int\bar{f}(t)dt$ 8

   (b) Prove that $\left(\bar{a}\times\bar{b}\right).\left(\bar{c}\times\bar{d}\right)=\begin{vmatrix}\bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{d}\\ \bar{b}\cdot\bar{c} & \bar{b}\cdot\bar{d}\end{vmatrix}$. 8

2. (a) If $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then evaluate grad r^n. 8

   (b) If $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then prove that $curl\left(\bar{a}\times\bar{r}\right)=2\bar{a}$, where $\bar{a}$ is a constant vector. 8

SECTION-II

3. (a) Forces P, Q, R act along the sides BC, CA, AB of a triangle ABC. Find the condition that their resultant is parallel to BC and determine its magnitude. 9

   (b) A couple of moment G acts on a square board ABCD of side a. Replace the couple by the forces acting along AB, BD and CA. 8

4. (a) Two equal smooth spheres, each of weight W and radius r, are placed inside a hollow cylinder open at both ends which rests on a horizontal plane; if a(<2r) be the radius of the cylinder, show that the least weight it can have so as not to be upset is $2W\left(1-\frac{r}{a}\right)$. 9

   (b) Find the c.g. of a semi-circular lamina of radius r when the density varies as the cube of the distance from the centre. 8

5. (a) A uniform ladder rests in limiting equilibrium with one en on a rough floor whose coefficient of friction is μ and with the other against a smooth vertical wall. Show that its inclination to the vertical is $\text{Tan}^{-1}(2\mu)$. 9

   (b) Find the centroid of the region bounded by the coordinate axes and the circle $x^2+y^2=a^2$ which lies in the first quadrant. 8

6. (a) Find the least force necessary to support a heavy particle on an inclined plane. 9

   (b) A uniform rod of length 2a rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that, in the position of equilibrium the beam is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$. 8

SECTION-III

7. (a) A particle is moving with uniform speed V along the curve $x^2y = a(x^2 + a^2/\sqrt{5})$. Show that its acceleration has the maximum value $10V^2/9a$. 9

   (b) Discuss the motion of a particle executing S.H.M in a straight line. 8

P.T.O.

8.(a) A particle is projected vertically upward with a velocity $\sqrt{2gh}$ and another is let fall from a height h at the same time. Find the height of the point where they meet each other. 9

(b) A shell of mass M is moving with speed V. An internal explosion generates an amount of energy E and breaks the shell into two portions whose masses are in the ratio $m_1 : m_2$. The fragments continue to move in the original line of motion of the shell. Show that their speeds are $V+\sqrt{\frac{2m_2E}{m_1M}}$ and $V-\sqrt{\frac{2m_1E}{m_2M}}$. 8

9. (a) A particle is projected at time t = 0 in a fixed vertical plane from a given point O with a speed $\sqrt{2ga}$ of which the vertical component is V. Show that at time t=2a/V, the particle is on a fixed parabola (parabola of safety), that its path touches the parabola, and that its direction of motion is then perpendicular to its direction of projection. 9

(b) A particle is launched at an angle α from a cliff of height H above sea level. If it falls into the sea at a distance D from the base of the cliff, prove that the maximum height above the sea level is $H+D^2\tan^2\alpha/4(H+D\tan\alpha)$. 8

10.(a) A particle of mass m moves along the curve defined by $\vec{r}=a(\cos\omega t)\hat{i}+b(\sin\omega t)\hat{j}$. Find the torque and the angular momentum about the origin. 9

(b) A particle of mass m moves along x-axis under the influence of a conservative field of force having potential V(x). If the particle is located at positions x_1 and x_2 at respective times t_1 and t_2. Prove that if E is the total energy then $t_2-t_1=\sqrt{\frac{m}{2}}\int_{x_1}^{x_2}\frac{dx}{\sqrt{E-V(x)}}$. 8

SECTION-IV

11.(a) A particle describes the curve $\frac{a}{r}=\cosh n\theta$ under force F to the pole, show that the force is stated as $F\propto\frac{1}{r^3}$. 8

(b) A particle of elasticity e is dropped from a vertical height a upon the highest point of plane which is of length b and is inclined at an angle α to the horizontal and descend to the bottom into three jumps. Prove that b = 4ae (1+e)(1+ e + e^2)(1 + e^2)sin α. 8

12. (a) A rubber ball is dropped from a height h after rebounding twice from the ground. It reaches to height h/2. Find the coefficient of restitution. What would be the coefficient of restitution had the ball reached at height h/2 after rebounding three times. 8

(b) Two equal smooth and perfectly elastic spheres moving at right angle to one another impinging obliquely. Show that after impact, they will still move at right angle to each other. 8