UNIVERSITY OF THE PUNJAB

Roll No. ....

B.A. / B.Sc. Part – I Annual Exam – 2019

Subject: Mathematics A Course-I **Paper: Calculus and Analytical Geometry** **Time: 3 Hrs. Marks: 100**

NOTE: ***Attempt SIX questions by selecting TWO questions from Section-I, TWO questions from Section-II, ONE questions from Section-III and ONE question from Section-IV.***

SECTION – I

Q.1: **(9,8)**

(a) Solve the inequality $\left|\frac{8+x}{12}\right| < \frac{x-1}{10}$

(b) Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$, discuss the continuity of f at x = 0.

Q.2: **(9,8)**

(a) Differentiate with respect to x, $\arctan\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)$

(b) Find $y^{(n)}(0)$ if $y = \ln\left(x+\sqrt{1+x^2}\right)$.

Q.3: **(9,8)**

(a) Differentiate with respect to x, $(\arcsin x)^{x^{1/2}}$

(b) Use differentials to approximate $Cos\ 61^\circ$

Q.4: **(9,8)**

(a) If x > 0, prove that $x - \ln(1+x) > \frac{x^2}{2(1+x)}$.

(b) Evaluate $\lim_{x\to 0}\left(\frac{1}{x\arcsin x} - \frac{1}{x^2}\right)$

SECTION – II

Q.5: **(9,8)**

(a) Find a reduction formula for $\int x^n \sin ax\, dx$ .where n > 1 is an integer. Hence evaluate $\int x^4 \sin 4x\, dx$.

(b) Integrate $\frac{1}{(x^2+4x+5)\sqrt{x+2}}$

P.T.O.

Q.6: **(8,9)**

(a) Show that $\int_0^{\pi} \frac{x\,dx}{1+\sin x} = \pi$

(b) If I_n denotes $\int_0^1 x^p(1-x^q)^n\,dx$,, where p, q and n are positive, prove that (qn + p + 1) $I_n = qn\, I_{n-1}$. Evaluate I_n when n is a positive integer.

Q.7: **(9,8)**

(a) Show that the normal at any point of the curve $x = a\cos\theta + a\theta\sin\theta,\ y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

(b) Find the pedal equation of the given curve $\frac{l}{r} = 1 + e\cos\theta$

Q.8: **(9,8)**

(a) If $p = x\cos\theta + y\sin\theta$ touches the curve $\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$. Prove that $p^n = (a\cos\theta)^n + (b\sin\theta)^n$

(b) Show that at any point of the lemniscate $r^2 = a^2\cos 2\theta$, $0 \le \theta \le \pi/4$, the measure of the angle between the radius vector and the outward – pointed normal is 2θ.

SECTION – III

Q.9: **(8,8)**

(a) Find point at which $r = 1 + \cos\theta$ has horizontal and vertical tangents.

(b) Find relation maximum and minimum of r if $r = 1 - \cos\theta$.

Q.10: **(8,8)**

(a) Prove that the intrinsic equation of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $s = 4a\sin\alpha$.

(b) If ρ_1, ρ_2 are the radius of curvature at the extremities of any chord of the cardioid $r = a(1 + \cos\theta)$ which passes through the pole, then prove that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$.

SECTION – IV

Q.11: **(8,8)**

(a) The direction cosines l, m, n, of two straight lines are given by the equations $l+m+n = 0$ and $l^2 + m^2 - n^2 = 0$. Find measure of the angle between them.

(b) Find a equation of the plane through the point (1,0,1) and (2,2,1) and perpendicular to the plane $x - y - z + 4 = 0$.

Q.12: **(8,8)**

(a) Find an equation of sphere through the circle $x^2 + y^2 + z^2 = 1,\ 2x + 4y + 5z - 6 = 0$ and touching the plane $z = 0$.

(b) Find the direction of Qibla of Karachi, latitude $\varphi = 24°51.5'$N and longitude $\lambda = 67°2'$E.

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I Annual Exam – 2019

Roll No. ....

Subject: Mathematics A Course-I **Paper: Calculus and Analytical Geometry** **Time: 3 Hrs.** **Marks: 100**

NOTE: Attempt SIX questions by selecting TWO questions from Section-I, TWO questions from Section-II, ONE questions from Section-III and ONE question from Section-IV.

SECTION – I

Q.1: **(9,8)**

(a) Discuss the continuity of f at x = a, where $f(x)=\begin{cases}(x-a)\,Sin\left(\dfrac{1}{x-a}\right) & if\ x\neq a\\ 0 & if\ x=a\end{cases}$

(b) Solve the inequality $\dfrac{x^2-2}{1-2x}>1$

Q.2: **(9,8)**

(a) Use differentials to find approximate value of $\cos 29^0$.

(b) If y = sin (arc + sin^2 x), prove that $(1-x^2)y^{n+2}=(2n+1)y^{n+1}-(n^2-a^2)y^n$.

Q.3: **(9,8)**

(a) Use differentials to find approximate value of $\sin 44^o$.

(b) If y = arc tan x, show that $(1+x2)\,y''+2xy'=0$, differentiate this equation n times and find the value of y^n at x = 0.

Q.4: **(9,8)**

(a) Prove that $\dfrac{x}{x+1}<\ell n(x+1)$ for all x > 0.

(b) Use L' hospital's rule to prove that $\lim\limits_{x\to\infty}\left(\dfrac{a^{1/x}+b^{1/x}}{2}\right)^x=\sqrt{ab}$

SECTION – II

Q.5: **(9,8)**

(a) Find reduction formula for $\int x^n e^{ax}\,dx$ and apply it to evaluate $\int x^3 e^{ax}\,dx$

(b) Integrate $\dfrac{1}{(1+x)\sqrt{x^2-1}}$

Q.6: **(9,8)**

(a) Prove that $\int_0^1 \dfrac{\ln(1+x)}{1+x^2}dx=\dfrac{\pi}{8}\ln 2$

(b) Show that $\int \sec^{2n+1}x\,dx=\dfrac{\sec^{2n-1}x\tan x}{2n}+\left(1-\dfrac{1}{2n}\right)\int \sec^{2n-1}x\,dx$

P.T.O.

Q.7: **(9,8)**

(a) Find the condition that the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ should intersect orthogonally.

(b) Show that the pedal equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{1}{p}=\frac{1}{a^2}+\frac{1}{b^2}-\frac{r^2}{a^2b^2}$

Q.8: **(9,8)**

(a) If $p = x\cos\theta + y\sin\theta$ touches the curve $\left(\frac{x}{a}\right)^{\frac{n}{n-1}}+\left(\frac{y}{b}\right)^{\frac{n}{n-1}}=1$.

Prove that $p^n = (a\cos\theta)^n + (b\sin\theta)^n$.

(b) Prove that an equation of the normal to the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ can be written in the form $x\sin t - y\cos t + a\cos 2t = 0, t$ being parameter.

SECTION – III

Q.9: **(8,8)**

(a) Find the dimension of the rectangle of maximum area that can be inscribed in a circle of radius r.

(b) Find the area of the region benumbed by the cardioid $r = a(1-\cos\theta)$

Q.10: **(8,8)**

(a) Show that the intrinsic equation of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $s=\frac{3a}{2}\sin^2\alpha$

(b) Prove that the radius of curvature at the point (2a, 2a) on the curve $x^2y = a(x^2+y^2)$ is $2a$.

SECTION – IV

Q.11: **(8,8.)**

(a) The direction cosines l, m, n of two straight lines are given by the equations $l + m + n = 0$ and $2lm + 2ln - mn = 0$. Find measure of the angle between them.

(b) Find equations of the planes bisecting the angles between the planes $3x + 2y - 6z + 1 = 0$ and $2x + y + 2z - 5 = 0$.

Q.12: **(8,8)**

(a) Find an equation of the sphere which passes through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$, and the point (1,2,3).

(b) Find the direction of Qibla of Peshawar Latitude $\varphi = 34°1'$ N and longitude $\lambda = 71°40'$E.

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I Annual Exam – 2019

Roll No. ....

Subject: Mathematics B Course-I **Paper: Vector Analysis and Mechanics** **Time: 3 Hrs. Marks: 100**

NOTE: ***Attempt SIX questions in all. selecting ONE question from Section-I, TWO questions from Section-II, TWO questions from Section-III and ONE question from Section-IV.***

SECTION – I

1. a. If $\bar{a}$ and $\bar{b}$ are unit vectors and θ is the angle between them, show that: $\sin\frac{\theta}{2} = \frac{1}{2}|\bar{b} - \bar{a}|$. **8**

b. Prove that: $(\bar{b} \times \bar{c}) \times (\bar{c} \times \bar{a}) = [\bar{a}\ \bar{b}\ \bar{c}]\bar{c}$ **8**

2. a. If $\vec{f}(t)$ is a vector function of t, then differentiate $\frac{\vec{f}}{|\vec{f}|}$ with respect to t. **8**

b. If $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$, then show that: $div\,(grad\, r^m) = m(m+1)r^{m-2}$. **8**

SECTION – II

3. a. Forces $P_1, P_2, P_3, P_4, P_5, P_6$ act along the sides of a regular hexagon taken in order. Show that they will be in equilibrium if $\sum P = 0$ and $P_1 - P_4 = P_3 - P_6 = P_5 - P_2$. **9**

b. A couple of moment G acts on a square board ABCD of side a. Reduce the couple by forces acting along AB, BD and CA. **8**

4. a. Find the force necessary just to support a heavy particle on an inclined plane of inclination α $(\alpha > \lambda)$. **9**

b. A hemi – spherical shell rests on a rough inclined plane whose angle of friction is λ. Show that the inclination of the plane base to the horizontal cannot be greater than $sin^{-1}(2\sin\lambda)$. **8**

5. a. State and prove PRINCIPLE OF VIRTUAL WORK for a single particle. **9**

b. Four uniform rods are freely joined at their extremities and form a parallelogram ABCD. Which is suspended from A and is kept in shape by an inextensible string AC. Prove that the tension of the string is equal to half the whole weight. **8**

6. a. Find the centre of gravity of uniform lamina forming a quadrant of an ellipse bounded by its semi – axes. **9**

b. A rod of length $5a$ is bent so as to form five sides of a regular hexagon. Show that the distance of the centre of gravity from either end of the rod is $\frac{a}{10}\sqrt{133}$. **8**

SECTION – III

7. a. Find radial and transverse components of velocity and acceleration of a particle. **9**

b. Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with uniform speed v when particle is at (0, b). **8**

8. a. Define the projectile and derive the equation of the trajectory of a projectile by neglecting air resistance. **9**

b. A shell burst on contact with the ground and pieces from it fly in all directions with all speeds upto 80 feet per second. Prove that a man 100 feet away is in danger for $\frac{5}{\sqrt{2}}$ seconds. **8**

9. a. Prove that in a conservative field of force the total energy remains constant. **9**

b. Prove that the field of force $\vec{F}$ determined by $\vec{F} = (y^2 - 2xyz^3)\hat{i} + (3 + 2xy - x^2z^3)\hat{j} + (6z^3 - 3x^2yz^2)\hat{k}$ is conservative and find its potential. **8**

10. a. Discuss the motion of a particle moving in a straight line with an acceleration x^3 where x is the distance of the particle from a fixed point O on the line, if it starts at $t = 0$ from a point $x = c$ with velocity $\frac{c^2}{\sqrt{2}}$ **9**

b. A particle is projected upwards. After a time t, another particle is sent up from the same point with the same velocity and meets the first at height h during the downward flight of the first. Find the velocity of projection. **8**

SECTION – IV

11. a. Define APSE, APSIDAL distance, APSIDAL line and APSIDAL angle; prove that the radius vector and tangent are perpendicular to each other. **8**

b. A particle describes curve $r^n = a^n \cos n\theta$ under central attractive force $\vec{F}$. Find the law of force. **8**

12. a. A heavy elastic ball is dropped upon a horizontal floor from a height of 20ft. and after rebounding twice, it is observed to attain a height of 10ft. Find the co-efficient of restitution. **8**

b. Prove that when two smooth sphere impinge obliquely, the kinetic energy is always lost by impact, unless the elasticity is perfect. **8**

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I Annual Exam – 2019

Roll No. ....

Subject: Mathematics B Course-I **Paper: Vector Analysis and Mechanics** *Time: 3 Hrs.* *Marks: 100*

NOTE: Attempt **SIX** questions in all. Selecting **ONE** question from **SECTION – I**, **TWO** questions from **SECTION – II**, **TWO** questions from **SECTION – III** and **ONE** question from **SECTION – IV**.

SECTION – I

1. a. Find two unit vectors which make angle of $\frac{\pi}{3}$ with both of the vectors $[1,\ -1,\ 0]$, $[1,\ 0,\ -1]$. **8**

b. Prove that: $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}) = [\vec{a}\vec{b}\vec{c}]\vec{b}$ **8**

2. a. If $\vec{r}(t)$ is a vector function of t, show that

$$\frac{d}{dt}[\vec{r}\times(\vec{r}'\times\vec{r}'')] = \vec{r}'\times(\vec{r}'\times\vec{r}'') + \vec{r}\times(\vec{r}'\times\vec{r}''').$$ **8**

b. If $\vec{F} = grad(x^3+y^3+z^3-3xyz)$, then find $div\,\vec{F}$ and $curl\,\vec{F}$. **8**

SECTION – II

3. a. P is any point in the plane of a triangle ABC and D, E, F are the middle points of its sides. Prove that the forces $\overrightarrow{AP}, \overrightarrow{BP}, \overrightarrow{CP}$, $\overrightarrow{PD}, \overrightarrow{PE}, \overrightarrow{PF}$ are in equilibrium. **9**

b. Forces P, 2P, 3P, 6P, 5P and 4P act respectively along the sides AB, CB, CD, ED, EF and AF of a regular hexagon of side a, the sense of the forces being indicated by the order of letters. Prove that the six forces are equivalent to a couple. **8**

4. a. The least force which will move a weight up an inclined plane is of magnitude P. Show that the least force acting parallel to the plane which will move the weight upward is $P\sqrt{1+\mu^2}$. **9**

b. A light ladder is supported on a rough floor and leans against a smooth wall. How far up the ladder can a man climb without slipping taking place? **8**

5. a. A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that, in the position of equilibrium, the beam is inclined to the wall at an angle $sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{2}}$. **9**

b. Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagon. The rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string. Prove that its tension is 3W. **8**

6. a. Find the centre of gravity of the area of astroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ between two consecutive cusps. **9**

b. Find the centre of gravity of a semi – circular lamina of radius r when its density varies as the cube of the distance from the centre. **8**

SECTION – III

7. a. A particle is moving with uniform speed v along the curve $x^2y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$. Show that its acceleration has maximum value $\frac{10\,v^2}{9\,a}$. 9

b. Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with uniform speed v when particle is at (0, b). 8

8. a. Obtain equation of parabola of safety. 9

b. A projectile having a range R, reaches a maximum height H. Prove that it must have been launched with (i) an initial speed equal to $\sqrt{\frac{g(R^2+16H^2)}{8H}}$

(ii) at an angle with the horizontal given by $\sin^{-1}\left(\frac{4H}{\sqrt{R^2+16H^2}}\right)$. 8

9. a. Determine whether the field of force $\vec{F}$ determined by $\vec{F} = \log x\,\hat{\imath} + \frac{1}{1+y^2}\hat{\jmath}$ is conservative. 9

b. Under the influence of a force field $\vec{F}$, a particle of mass m moves along the ellipse $\vec{r} = a\cos\omega t\,\hat{\imath} + b\sin\omega t\,\hat{\jmath}$, if $\vec{P}$ is the momentum, show that (i) $\vec{r} \times \vec{P} = ma\,b\,\omega\,\hat{k}$

(ii) $\vec{r} \cdot \vec{P} = \frac{1}{2}m(b^2 - a^2)\sin 2\omega t$. 8

10.a. A particle moving along a straight line from rest and is accelerated uniformly till it attains a velocity v. The motion is then restarted and the particle comes to rest after traversing a total distance x. If the acceleration is *f*, find the retardation and the total time taken by the particle from rest to rest. 9

b. A particle describing simple harmonic motion has velocities 5m/sec and 4m/sec, when its distance from the centre are 12m and 13m respectively. Find the time period of the motion. 8

SECTION – IV

11.a. State KEPLER'S LAWS of Planetary motion and derive NEWTON'S LAW of Gravitation from KEPLER'S LAWS. 8

b. A particle describes curve $a = r\,e^{n\theta}$ under central attractive force $\vec{F}$. Find the law of force. 8

12.a. Three perfectly elastic balls of masses m, 2m and 3m are placed in a straight line. The first impinge directly on the second with a velocity u and then the second impinges on the third. Find the velocity of the third ball after impact. 8

b. Two elastic spheres, each of mass m collide directly. Show that the energy lost during the impact is $\frac{1}{4}m(U^2 - V^2)$, where U and V are the relative velocities before and after impact. 8

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I Annual Exam – 2019

Roll No. ....

Subject: Mathematics General-I Paper: Calculus (Differential and Integral Calculus) Time: 3 Hrs. Marks: 100

NOTE: ***Attempt SIX questions by selecting TWO questions from Section-I, TWO questions from Section-II, ONE questions from Section-III and ONE question from Section-IV.***

Section-I

Q. 1. (a)

i. Prove that $||a| - |b|| \leq |a - b|$ for every $a, b \in R$. 4+4

ii. Evaluate $\lim_{y \to x} \frac{y^{2/3} - x^{2/3}}{y - x}$

(b) 4+5

i. Evaluate $\lim_{x \to \pm\infty} \left[\frac{x^2}{x+1} - \frac{x^2}{x+3}\right]$

ii. Let $f(x) = x^2$ and

$$g(x) = \begin{cases} -4 & if\ x \leq 0 \\ |x - 4| & if\ x > 0 \end{cases}$$

Determine whether $f_o g$ and $g_o f$ are continuous at $x = 0$.

Q. 2. (a) Find the values of a and b so that the function f is continuous and differentiable at $x = 1$, where 8+9

$$f(x) = \begin{cases} x^3 & if\ x < 1 \\ ax + b & if\ x \geq 1 \end{cases}$$

(b) Differentiate with respect to x : $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$

Q. 3. (a) Find $y^{(n)}(0)$ if $y = \left(x + \sqrt{1 + x^2}\right)^m$ 8+9

(b) Use Mean Value Theorem to show that $|\tan x + \tan y| \geq |x + y|$ for all real numbers x and y in the interval $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

Q. 4. (a) Evaluate the given limits 4+4

i. $\lim_{x \to 0} \left(\frac{1}{x \arcsin x} - \frac{1}{x^2}\right)$ ii. $\lim_{x \to 0} (\tan x)^{\sin 2x}$

(b) Show that, under certain conditions to be stated, 9
$f(a + h) = f(a) + hf'(a + \theta h)$ where $0 < \theta < 1$. Prove also that the limiting value of θ, when h decreases indefinitely, is $\frac{1}{2}$.

Section-II

Q. 5. Evaluate the following integrals: 8+9

a. $\int \frac{\cos x}{2 - \cos x} dx$ b. $\int \frac{x^2 + 2x + 3}{\sqrt{x^2 + x + 1}} dx$

P.T.O.

Q. 6. (a) Evaluate $\int_0^{\pi/2} \tan x \ln(\sin x)\, dx$ 8+9

(b) Evaluate $\int_0^3 \frac{dx}{x^2+2x-3}$

Q. 7. (a) Show that $16xy - 6x + 8y - 3 = 0$ represents a pair of straight lines. Also prove that this together with the coordinate axes form a rectangle and find the area enclosed by the rectangle. 8+9

(b) If $PF_2Q,\ PF_1R$ be two chords of an ellipse through the foci F_2, F_1, show that $\frac{|PF_2|}{|F_2Q|} + \frac{|PF_1|}{|F_1R|}$ is independent of the position of P.

Q. 8. (a) Find the pedal equation of the curve $r = a + b\cos\theta$ 8+9

(b) Find the equations of tangent and normal at $\theta = \frac{\pi}{2}$ to the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

Section-III

Q. 9 (a) Find equations of the asymptotes of the curve 8+8

$$(x - y + 1)(x - y - 2)(x + y) = 8x - 1$$

(b) Locate the points of relative extrema of the curve $f(x) = \sin x\, cos^2 x$

Q. 10 (a) Find the area of the region bounded by the curve $xy^2 = 4(2 - x)$ and the y – axis. 8+8

(b) Find the centre of curvature for the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Show that its evolute is $(ax)^{2/3} - (by)^{2/3} = (a^2 + b^2)^{2/3}$

Section-IV

Q. 11. (a) If $f(x, y) = x^y + y^x$, verify that $f_{xy} = f_{yx}$. 8+8

(b) Use differentials to approximate $\tan 29^\circ$.

Q. 12. (a) Examine $f(x, y) = \frac{1}{x} + xy - \frac{8}{y}$ for relative extrema. 8+8

(b) Evaluate $\int_0^2 \int_0^{\sqrt{4-z^2}} \int_{y^2+z^2-4}^{4-y^2-z^2} dx\, dy\, dz$.

UNIVERSITY OF THE PUNJAB

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Subject: Mathematics General-I Paper: Calculus (Differential and Integral Calculus) Time: 3 Hrs. Marks: 100

Attempt any **SIX** questions by selecting **TWO** questions from **Section-I**, **TWO** questions from **Section-II**, **ONE** question from **Section-III** and **ONE** question from **Section-IV**.

Section-I

Q. 1. (a) 4+4

i. Express $3 < x < 7$ in modulus notation.

ii. Evaluate $\lim_{x \to 0} \frac{\csc x - \cot x}{x}$

(b) 4+5

i. Find constants a and b such that the function

$$f(x) = \begin{cases} x^3 & if\ x < -1 \\ ax + b & if\ -1 \le x < 1 \\ x^2 + 2 & if\ x \ge 1 \end{cases}$$

is continuous for all x.

ii. If $x, y \in R$, then prove that $|x + y| \le |x| + |y|$, $\forall x, y\ R$

Q. 2. (a) Find $y^{(n)}(0)$ if 8+9

$$y = \ln(x + \sqrt{1 + x^2})$$

(b) Show that $f(x) = |x| + |x - 1|$ is continuous for every real value of x but is not differentiable at $x = 0$ and $x = 1$

Q. 3. (a) Differentiate with respect to x: 8+9

$$y = (tanx)^{cotx} + (cotx)^{tanx}$$

(b) Use the Mean Value Theorem to show that $\frac{1}{6} < \sqrt{27} - 5 < \frac{1}{5}$.

Also approximate $\sqrt{168}$ by using the Mean Value Theorem.

Q. 4. (a) Evaluate the given limits: 4+4

i. $\lim_{x \to 0} \left[\frac{x\cos x - \ln(1+x)}{x^2}\right]$

ii. $\lim_{x \to 0} x\tan(\frac{\pi}{2} - x)$

(b) Find by Maclaurin's Formula, the first four terms of the expansion of $f(x) = e^{ax} cosbx$ and write the remainder after n terms. 4+5

Section-II

Q. 5. (a) Find a reduction formula for $\int x^n \sin ax\, dx$, where $n > 1$ is an integer. Hence evaluate $\int x^4 \sin 4x\, dx$. 8+9

P.T.O.

(b) Evaluate $\int \frac{x^4}{x^4+2x^2+1} dx$

Q. 6. (a) Show that $\int_0^{\pi} \frac{x \sin x}{1+\sin x} dx = \frac{\pi^2}{2} - \pi$ 8+9

(b) Evaluate $\int_0^{\pi} \frac{\sin^4 x}{(1+\cos x)^2} dx$

Q. 7. (a) If a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with centre C, meets the major and minor axes in T and t. Prove that $\frac{a^2}{CT^2} + \frac{b^2}{Ct^2} = 1$ 8+9

(b) Analyze the conic represented by the equation $5x^2 - 2xy + 5y^2 - 12 = 0$

Q. 8. (a) If $x = a \cos g(t)$, $y = b \sin g(t)$, prove that 8+9

$$xy^2 \frac{d^2y}{dx^2} = b^2 \frac{dy}{dx}$$

(b) Find the pedal equation of the curve: $r = a + b \cos \theta$

Section-III

Q. 9. (a) Find the points of inflection of the curve 8+8

$$y = \frac{x^3 - x}{3x^2+1}$$

(b) Find equations of the asymptotes of the curve

$$r \sin 2\theta = a \cos 3\theta$$

Q. 10. (a) Prove that for the cardioid $r = a(1 + \cos \theta)$, $\frac{\rho^2}{r}$ is constant, where ρ is the radius of curvature 8+8

(b) Calculate the perimeter of the limacon : $r = a + b \cos \theta$, if $\frac{b}{a}$ is small.

Section-IV

Q. 11. (a) Verify that $f_{xy}(x, y) = f_{yx}(x, y)$ if $f(x, y) = x^y + y^x$ 8+8

(b) Use differentials to approximate $\sqrt[3]{123}$

Q. 12. (a) If $u = arc \sin\left(\frac{x^2+y^2}{x+y}\right)$, show that 8+8

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$$

(b) Find the area outside the circle $r = 3$ and inside the cardioid $r = 2(1 + \cos \theta)$.