UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – I
Supplementary Examination - 2017

Roll No. ....

Subject: Mathematics A Course-I **TIME ALLOWED: 3 hrs.**
PAPER: Calculus and Analytical Geometry **MAX. MARKS: 100**

NOTE: . **Attempt SIX questions by selecting TWO questions from Section-I, TWO questions from Section-II, ONE questions from Section-III and ONE question from Section-IV.**

SECTION-I

Q.1. (a): Evaluate $\lim_{x\to 0}\frac{a^x-1}{x}$ (a> 0 >>, $a \neq 0$), 9

(b): Solve the inequality $\left|\frac{x+8}{12}\right| < \frac{x-1}{10}$ 8

Q.2. (a): Find $\frac{dy}{dx}$ if y = $(tanx)^{cotx} + (cotx)^{tanx}$ 9

(b): Let f(x)=$\begin{cases} x\,\cos\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = o \end{cases}$

Discuss the continuity of f at x=0 8

Q.3 (a): Use differentials to find approximate value of sin 44^0 9

(b): If y = arc tanx , show that $(1+x^2)\, y'' + 2\, xy' = 0$, 8
Differentiate this equation n times and find the value of y^n at x=0

Q.4...(a): If f(x) = $\sin^2$x on [0,π]. Discuss the validity of Roll's theorem. 9
Find c if possible such that f' (c) = 0

(b): Find $\lim_{x\to 1}\left[\tan\left(\frac{\pi x}{4}\right)\right]^{tan\left(\frac{\pi x}{2}\right)}$ 8

SECTION-II

Q.5. (a) Integrate $\int\frac{1}{(x^2-2x+2)\sqrt{x-1}}dx$ 8

(b) Show that $\int x^n \arctan x dx = \frac{x^{n+1}}{n+1}\arctan x - \frac{1}{n+1}\int\frac{x^{n+1}}{1+x^2}dx$

Hence evaluate $\int x^3 \arctan x dx$ 9

Q.6. (a): Prove that $\int_0^{\pi}\frac{xsinx}{1+sinx}dx = \frac{\pi^2}{2} - \pi$ 9

(b) Evaluate $\int\frac{\sin^2 x}{\cos^5 x}dx$ 8

PTO

Q.7 (a): Analyze and graph the conic represented by $xy + x - 2y + 3 = 0$ 9

(b): Show that the tangent at the vertex of a diameter of a parabola is parallel to the chords bisected by the diameter. 8

Q.8 (a): Show that the pedal equation of the 9

$x = ae^{\theta}(\sin\theta - \cos\theta), y = ae^{\theta}(\sin\theta = \cos\theta)$ is $r = \sqrt{2}p$.

(b): Find the measure of the angle of intersection of the curves $r = a\theta$ and $r\theta = a$ 8

SECTION-III

Q.9 (a): Find the dimensions of the rectangle of maximum area that can be inscribed in a circle of radius r 9

(b): Find the area of the region benumbed by the cardioid $r = a(1 + \cos\theta)$ 8

Q.10 (a): Show that the intrinsic equation of the cardioid $r = a(1 + \cos\theta)$ is $S = 4a\sin\alpha/3$ 9

(b): Find the radius of curvature at any point of the curve $x = a(\cos t + t\sin t), y = a(\sin t - t\cos t), a > 0$ 8

SECTION-IV

Q.11 (a): Find equations of the straight line passing through the point P(2, 0, -2) and perpendicular to each of the straight lines. 8

$$\frac{x-3}{2} = \frac{y}{2} = \frac{z+1}{2} \text{ and } \frac{x}{3} = \frac{y+1}{-1} = \frac{z+2}{2}$$

(b): Find an equation of the plane which passes through the point (3, 4, 5) has an x-intercept equal to -5 and is perpendicular to the plane $2x + 3y - z = 8$ 8

Q.12 (a): Find an equation of the sphere passing through the Points (0,-2,-4), (2,-1,-1) and having its center on the st-line $2x - 3y = 0 = 5y + 2z$. 8

(b): Find the direction of Qibla of shahi Mosque, Islamabad, 8

Latitude = $33^0 40'$N and longitude = $73^0 8'$E

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B.A. / B.Sc. Part – I
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Roll No. ....

Subject: Mathematics B Course-I TIME ALLOWED: 3 hrs.
PAPER: Vector Analysis and Mechanics MAX. MARKS: 100

NOTE: Attempt SIX questions in all, selecting one question from section-I, Two questions from section-II, Two questions from section-III and one question from section-IV

SECTION-I

1. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then evaluate grad r^n. 8

   (b) Show that $(\vec{b}\times\vec{c}).(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b}) = (\vec{a}.\vec{b}\times\vec{c})^2$. 8

2. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then prove that $curl(\vec{a}\times\vec{r}) = 2\vec{a}$, where $\vec{a}$ is a constant vector. 8

   (b) Prove that $\vec{\nabla}\times(\vec{\nabla}\times\vec{f}) = \vec{\nabla}(\vec{\nabla}.\vec{f}) - \nabla^2\vec{f}$. 8

SECTION-II

3. (a) Three forces P, Q and R act along the sides BC, CA and AB respectively of a triangle ABC. Prove that if P sec A + Q sec B + R sec C = 0, then the line of the action of the resultant passes through the orthocentre of the triangle. 9

   (b) The greatest resultant that two forces can have is of magnitude P and the least is of magnitude Q. Show that, when they act at an angle α, their resultant is of magnitude $\sqrt{P^2\cos^2 \alpha/2 + Q^2 \sin^2 \alpha/2}$. 8

4. (a) Two equal smooth spheres, each of weight W and radius r, are placed inside a hollow cylinder open at both ends which rests on a horizontal plane; if a(<2r) be the radius of the cylinder, show that the least weight it can have so as not to be upset is $2W\left(1-\frac{r}{a}\right)$. 9

   (b) From a semi-circular lamina of radius 2a, a circular lamina of radius a is removed. Prove that the c.m. of the remainder is at a distance $\frac{16a}{3\pi} - a$ from the diameter. 8

5. (a) Find the c.g. of a semi-circular lamina of radius r when the density varies as the cube of the distance from the centre. 9

   (b) A rod 4 feet long, rests on a rough floor against the smooth edge of a table of height 3 feet. If the rod is on the point of slipping when inclined at an angle of 60° to the horizontal, find the coefficient of friction. 8

6. (a) A uniform rod of weight W is placed with its lower end on a rough horizontal floor and its upper end against equally rough vertical wall. The rod makes an angle α with the wall and is just prevented from slipping down by a horizontal force P applied at its middle point. Prove that $P = W\tan(\alpha - 2\lambda)$, Where λ is the angle of friction and $\lambda < \frac{1}{2}\alpha$. 9

   (b) Six equal uniform rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a regular hexagon. The rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string. Show that its tension is 3W. 8

(P.T.O.)

SECTION-III

7.(a) Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with uniform speed V when the particle is at (0 , b). 9

(b) A particle is projected vertically upwards. After a time t, another particle is sent up from the same point with the same velocity and meets the first at height h during the downward flight of the first. Find the velocity of projection. 8

8. (a) A particle describes simple harmonic motion with frequency N. If the greatest velocity is v, find the amplitude and maximum value of the acceleration of the particle. Also show that the velocity v at a distance x from the centre of motion is given by $v=2\pi N\sqrt{a^2-x^2}$, where a is the amplitude. 9

(b) The angular velocity of a particle about a point in its plane of motion is constant. Prove that the transverse component of its acceleration is proportional to the radial component of its velocity. 8

9.(a) Find the range of a rifle bullet when α is the elevation of projection and V_0 the speed . Show that, if the rifle is fired with the same elevation and speed from a car travelling with speed V towards the target, the range will be increased by $\frac{2v_0 v\sin\alpha}{g}$. 9

(b) A shell bursts on contact with the ground and pieces from it fly in all directions with all speeds up to 80 ft/sec. Prove that a man 100 ft. away is in danger for $5/\sqrt{2}$ sec. 8

10.(a) A shell of mass M is moving with speed V. An internal explosion generates an amount of energy E and breaks the shell into two portions whose masses are in the ratio $m_1 : m_2$ .The fragments continue to move in the original line of motion of the shell. Show that their speeds are $V+\sqrt{\frac{2m_2E}{m_1M}}$ and $V-\sqrt{\frac{2m_1E}{m_2M}}$. 9

(b) A particle of mass m moves along x-axis under the influence of a conservative field of force having potential V(x).If the particle is located at positions x_1 and x_2 at respective times t_1 and t_2 . Prove that if E is the total energy then $t_2-t_1=\sqrt{\frac{m}{2}}\int_{x_1}^{x_2}\frac{dx}{\sqrt{E-V(x)}}$. 8

SECTION-IV

11.(a) A particle describes the curve $r^n\cos n\theta=a^n$ under force F to the pole, show that the force is stated as $F\propto r^{2n-3}$. 8

(b) Two elastic spheres of mass m and m' moving with velocity u and u' impinges directly. If e being the coefficient of restitution. Find their velocities after impact. 8

12.(a) From a point on a smooth horizontal plane, a ball is projected with velocity u at an angle α to the horizon. Show that it will keep rebounding from the plane for a time $\frac{2u\sin\alpha}{g(1-e)}$ and will have a range $\frac{u^2\sin 2\alpha}{g(1-e)}$, e being the coefficient of restitution. 8

(b) Two elastic spheres each of mass m collide directly. Show that the energy lost during the impact is $\frac{1}{4}m\left(U^2-V^2\right)$, where U and V are the relative velocities before and after impact. 8

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B.A. / B.Sc. Part – I
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Roll No. ....

Subject: Mathematics General-I TIME ALLOWED: 3 hrs.
PAPER: Calculus (Differential and Integral Calculus) MAX. MARKS: 100

Attempt any **SIX** questions by selecting **TWO** questions from **Section-I**, **TWO** questions from **Section-II**, **ONE** question from **Section-III** and **ONE** question from **Section-IV**.

Section-I

Q. 1. (a)

i. Solve the inequality 4+4

$$x^2 - 5x + 6 < 0$$

ii. Evaluate

$$\lim_{y\to x}\frac{y^{\frac{2}{3}} - x^{\frac{2}{3}}}{y - x}$$

(b) 4+5

i. Let $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{1+e^{\frac{1}{x}}} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$ Examine the continuity of $f(x)$ at $x = 0$.

ii. Find 'c' such that the function

$$f(x) = \begin{cases} \frac{1-\sqrt{x}}{x-1} & if\ 0 \leq x < 1 \\ c & if\ x = 1 \end{cases}$$

is continuous for all $x \epsilon [0,1]$

Q. 2. (a) If $y = a\cos(lnx) + b\sin(lnx)$, prove that (17)

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

(b) If $f(x) = (sinx)^{cosx} + (cosx)^{sinx}$, find $f'(x)$.

Q. 3. (a) A dome is in the shape of a hemisphere with radius 60 feet. The dome is to be painted with a layer of 0.01 inch thickness. Use differentials to estimate the amount of paint required. 8+9

(b) If $f(x) = x(x-1)(x-2), a = 0, b = \frac{1}{2}$; find 'c' of the Mean Value Theorem.

Q. 4. (a) Evaluate the given Limits: 4+4

i. $\lim_{x\to 0}\frac{ln(1-x^2)}{\ln(cosx)}$ ii. $\lim_{x\to\frac{\pi}{2}}(secx)^{cotx}$

PTO

(b) Show that the number θ which occurs in the Taylor's theorem with Lagrange's form of remainder after n terms approaches the limit $\frac{1}{1+n}$ as $h \to 0$ provided that $f^{(n+1)}(x)$ is continuous and different from zero at $x = a$

(9)

Section-II

Q. 5. (a) Evaluate $\int \frac{2-\cos x}{2+\cos x} dx$ 8+9

(b) Evaluate $\int \frac{x^2+2x+3}{(x+2)\sqrt{x^2+1}} dx.$

Q. 6. (a) Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$ 8+9

(b) Find a reduction formula for $\int \frac{x^n}{\sqrt{ax^2+2bx+c}} dx.$

Q. 7. (a) Find the condition that the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ should intersect orthogonally. 8+9

(b) Find the angle of intersection of the curves $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$

Q. 8. (a) Analyze the conic represented by the eqation $xy + x - 2y + 3 = 0$ 8+9

(b) Find equations of the tangent and normal at $\theta = \frac{\pi}{2}$ to the cycloid $x = a(\theta - \sin\theta)\,,\, y = a(1 - \cos\theta)$

Section-III

Q. 9. (a) Find equations of the tangents at the multiple points of the curve $(y-2)^2 = x(x-1)^2$ 8+8

(b) Find the radius of curvature at any point of the curve $r(1 + \cos\theta) = a$

Q. 10. (a) Find the envelope of the family $y = mx + \sqrt{a^2m^2 + b^2}$, m being the parameter.

(b) Show that the intrinsic equation of $3ay^2 = 2x^3$ is $9s = 4a(\sec^3\alpha - 1)$. 8+8

Section-IV

Q. 11. (a) Verify that $f_{xy}(x,y) = f_{yx}(x,y)$ if $f(x,y) = \frac{xy}{\sqrt{1+x^2+y^2}}$. 8+8

(b) The Lateral surface of a cone is computed from the formula $= \pi r\sqrt{r^2 + h^2}$, r is the radius of the base and h is the height. If r is calculated as 6 with an accuracy of 1% and h as 8 with an accuracy of 0.25% , with what % accuracy will be the area S.

Q. 12. (a) Find $\frac{d^2y}{dx^2}$ if $x^3 + y^3 = 3axy$ 8+8

(b) Find the volume of the solid in the first octant bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, a, b and c being positive.