UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Supplementary Examination - 2018

Roll No. ....

Subject: Mathematics A Course-II
PAPER: (Linear Algebra and Differential Equations)

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

Note: Attempt SIX Questions by selecting TWO Questions from Section-I, ONE Question from Section-II, ONE Question from Section-III and TWO Questions from Section-IV.

Section - I

Q.1. (a) Prove that the product of matrices $A=\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B=\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is the zero matrix when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$. (9, 8)

(b) For 2 × 2 matrices A and B the equation $\det(A + B)^2 = [\det(A + B)]^2$ hold?

Q.2. (a) Solve the system of equations by Gauss-Jordan elimination method: (9, 8)

$3x_1 + 2x_2 + 4x_3 = 7$; $2x_1 + x_2 + x_3 = 4$; $x_1 + 3x_2 + 5x_3 = 3$

(b) Prove that $\begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} = (x-a)^3(x+3a)$

Q.3. (a) Write the vector V= (1, -2, 3) ∈ R^3 as a linear combination of the vector: $V_1 = (1, 1, 1)$, $V_2 = (1, 2, 3)$ and $V_3 = (2, -1, 1)$ (9, 8)

(b) Let U and W be 2-dimentional subspaces of R^3. Show that $U \cap W \neq \{0\}$.

Q.4. (a) Find the rank of the matrix $A=\begin{bmatrix} -3 & 5 & 1 & 2 \\ 7 & 2 & 0 & -4 \\ -8 & 3 & 1 & 6 \end{bmatrix}$. Also write an echelon matrix row equivalent to A. (9, 8)

(b) The matrix $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 5 & 6 \\ -2 & 3 & -1 \end{bmatrix}$ of a linear transformation $T: R^n \to R^m$.

Determine *m*, *n* and express T is terms of coordinates.

Section - II

Q.5. (a) Show that {(1, −1, 0), (2, −1, −2), (1, −1, −2)} is a basis of R^3. Find an orthonormal basis of R^3 using the Gram-Schmidt process. (8, 8)

(b) Find an orthogonal matrix whose first row is $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.

Q.6. (a) For matrix $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$, find the characteristic polynomial, all eigen values and a basis of eigenspace. (8, 8)

(b) For symmetric matrix $A=\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$, find an orthogonal matrix P for which P^TAP is diagonal.

(P.T.O.)

Section - III

Q.7. (a) Solve D.E. $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$

(b) Solve D.E. $\frac{dy}{dx} = -\frac{ax + hy}{hx + by}$ (8, 8)

Q.8. (a) Solve the initial value problem: $e^x[y - 3(e^x + 1)^2]dx + (e^x + 1)dy = 0;\ y(0) = 4$

(b) Find the orthogonal trajectories of the curve of the family $r^n = a^n \cos n\theta$ (8, 8)

Section - IV

Q.9. (a) Solve D.E. $(D^3 + D^2 - 4D - 4)y = e^{2x}\cos 3x.$ (9, 8)

(b) Solve D.E. $x^2y'' + 2xy' - 6y = 10x^2$; $y(1) = 1$, $y'(1) = -6$

Q.10. (a) Find the particular solution of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = e^{-2x}\sec x$

(b) Solve D.E. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{1}{(1 + e^x)^2}$ (9, 8)

Q.11. (a) Evaluate (i) $\angle\{t^3 e^{-t}\}$ (ii) $\angle\{\cos(at + b)\}$ (4, 4)

(b) Evaluate (i) $\angle^{-1}\left\{\frac{s - 2}{s^2 - 2}\right\}$ (ii) $\angle^{-1}\left\{arc\tan\frac{a}{s}\right\}$ (4, 5)

Q.12. (a) Use the Laplace transform method to solve D.E.

$\frac{d^2y}{dt^2} + y = \cos t,\ y(0) = 0,\ y'(0) = -1$ (9, 8)

(b) Apply the power series method to solve D.E. $y' + y - 1 = 0$

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Supplementary Examination - 2018

Roll No. ....

Subject: Mathematics B Course-II
PAPER: (Mathematical Methods, Group Theory & Matrix Space)

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

Note: Attempt SIX question in all by selecting TWO questions from Section-I, TWO questions from Section-II, ONE question from Section-III and ONE questions from Section-IV.

SECTION I

Q.1 (a) Separate into real and imaginary part$(\alpha + i\beta)^{(P+iq)}$ 9,8

(b) Prove that $cos4\theta = 8cos^4\theta - cos^2\theta + 1$

Q.2 (a) Prove that $tan^{-1}z = \frac{1}{2i}log\left(\frac{1+iz}{1-iz}\right)$ 9,8

(b) Examine $f(x,y) = 2x^2 - 4x + xy^2 - 1$ for relative extrema.

Q.3 (a) A rectangular plate expands in such a way that its length changes from 10 to 10.03 and its breadth changes from 8 to 8.02 .Find approximate value for the change in its area. 9,8

(b) Let $f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2} & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$ Examine the continuity at (0,0). Do $f_x\,(0,0)\ and\ f_y(0,0)$ exist.

Q.4 (a) Find the area bounded by the parabola $y = x^2$and the straight line $y = 2x + 3$ 9,8

(b) Use the cylindrical coordinate to evaluate $I = \iiint z\sqrt{x^2+y^2}\ dv$, S is the hemisphere

$$x^2 + y^2 + z^2 \leq 4, z \geq 0$$

SECTION II

Q5(a)Test the series for absolute convergence ,conditional convergence or divergence$\sum_1^\infty \frac{sin\sqrt{n}}{\sqrt{n^3+1}}$.

(b)Test the series$\sum_1^\infty \frac{1}{(2n-1)^{1/3}}$ 9,8

Q6(a) If a,b ∈ Z,where a,b are not both zero and (a,b)=d then show that$\left(\frac{a}{d},\frac{b}{d}\right) = 1$ 9,8

(b)Find the solution set of the equation $23x - 49y = 179$.

Q7(a) Find the remainder when7^{23} is divided by 8. 9,8

(b)Prove that if$\sum_1^\infty a_n$converges then$\lim_{n\to\infty} a_n = 0$.

(P.T.O.)

Q8(a) Using Integral Test show that Harmonic series $\sum_1^\infty \frac{1}{n}$ is divergent. 9,8

(b) Define a Prime Divisor and prove that every Integer n>1 has a prime divisor.

SECTION III

Q9(a)Let G be a group and H is a subgroup of G.Then the set $aHa^{-1} = \{aha^{-1}: h \in H\}$ is a subgroup of G.

(b)Determine whether the Permutation is even or odd$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 2 & 1 & 6 & 4 \end{pmatrix}$. 8,8

Q10(a) State and prove Lagrange Theorem. 8,8

(b)Let H.K be subgroups of an Abelian group G. Then show that the set $HK = \{hk! h \in H , k \in K\}$ Is a subgroup of G.

SECTION IV

Q11(a) Let X be a metric space and Let $\{x_0\}$ be a single ton subset of X. Then show that X- $\{x_o\}$ is open.

(b)Show that $d: R \times R \to R$ defined by d(x,y)=$\left[\frac{1}{x} - \frac{1}{y}\right]$ for all x,y $\in R - \{0\}$ is metric. 8,8

Q12(a) If A and B are two subsets of a metric space X.Then A$\subseteq B$ Implies that $A^d \subseteq B^d$. 8,8

(b) If A and B are two subsets of a metric space X. Then $\overline{AUB} = \bar{A}U\bar{B}$

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Supplementary Examination - 2018

Roll No. ....

Subject: Mathematics **TIME ALLOWED: 3 Hrs.**
PAPER: Optional **MAX. MARKS: 100**

Note: Attempt any *FIVE* Questions in all, selecting at least *TWO* questions from each section.

SECTION-I

Q.1 (a) Evaluate $\lim_{x \to 2} \frac{\sqrt{x^2+12}-4}{x-2}$ **(10)**

(b) Discuss the continuity at x=2, $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 4, & x = 2 \end{cases}$ **(10)**

Q.2 (a) Find $\frac{dy}{dx}$ when $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$ **(10)**

(b) If $y = e^{ax}\sin bx$ then show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y = 0$ **(10)**

Q.3 (a) Find y_1, y_2 when $x^3 + y^3 = a^3$ **(10)**

(b) If $z = f(x,y) = e^{-x}\cos y$ then show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ **(10)**

Q.4 (a) Solve $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ **(10)**

(b) Solve $\int e^x\left(\frac{1+x\ln x}{x}\right)dx$ **(10)**

SECTION-II

Q.5 (a) Determine whether or not the set of vectors $\{(2,4,-3),(0,1,1),(0,1,-1)\}$ is a basis for R^3? **(10)**

(b) Determine whether the vectors are linearly independent or not? **(10)**
$v_1 = (1,1,1), v_2 = (0,1,1), v_3 = (0,1,-1)$.

Q.6 (a) Solve the system of linear equations **(10)**

$2x + z = 1$, $2x + 4y - z = -2$, $x - 8y - 3z = 2$

(b) If A is any non-singular matrix, then show that $(A^{-1})^{-1} = A$ **(10)**

(P.T.O.)

Q.7 (a) Show that $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2y^2$ **(10)**

(b) Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$ **(10)**

Q.8 (a) Evaluate the determinant of the matrix $A = \begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$ **(10)**

(b) Find the reduced echelon form of the matrix **(10)**

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

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B.A. / B.Sc. Part – II

Supplementary Examination - 2018

Roll No. ....

Subject: Mathematic General-II

PAPER: (Mathematical Methods (Geomt, Series, Compl. No. LA, DE)

TIME ALLOWED: 3 Hrs.

MAX. MARKS: 100

Note: Attempt six questions by selecting TWO Questions from Section – I. TWO Questions from Section – II. One Question from Section – III and ONE Question from Section – IV.

SECTION-I

Q.1 (a) Evaluate $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^6$. 9

(b) Prove that $\left(\frac{1+\sin x + i\cos x}{1+\sin x - i\cos x}\right)^n = \cos n\left(\frac{\pi}{2}-x\right) + i\sin n\left(\frac{\pi}{2}-x\right)$. 8

Q.2 (a) Show that $\log(1+\cos\theta + i\sin\theta) = \ln\left(2\cos\frac{\theta}{2}\right) + \frac{i\theta}{2}$. 9

(b) Find the sum of the infinite series, $\cos\theta - \frac{1}{2}\cos 2\theta + \frac{1}{3}\cos 3\theta - \frac{1}{4}\cos 4\theta + \ldots\ldots$ 8

Q.3 (a) Using ratio test determine whether the series $\sum_{n=1}^{\infty}\frac{(n+2)!}{4!\,n!\,2^n}$ converges or diverges. 9

(b) Apply appropriate test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{(a+1)(2a+1)\ldots(na+1)}{(b+1)(2b+1)\ldots(nb+1)}$, $a>0,\ b>0$. 8

Q.4 (a) Test the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^2}{(n+2)!}$ for (i) absolute convergence (ii) conditional convergence 9

(b) Find radius of convergence and interval of convergence for the series $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{n^2}$. 8

SECTION-II

Q.5 (a) Find volume of tetrahedron bounded by the coordinate planes and the plane: $15x+10y+2z-30=0$ 9

(b) Show that the line through (3, –4, –2) parallel to the vector [9, 6, 2] has equation $\frac{1}{9}(x-3) = \frac{1}{6}(y+4) = \frac{1}{2}(z-2)$. Also find points on the line distant 22 from A. 8

Q.6 (a) If $\bar{a} = 2xi - 3yzj + x^2zk$ and $\phi = 2z - x^3y$, then find $\bar{a}\times\nabla\phi$ at (1, –1, 1). 9

(b) Using vectors prove that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$. 8

Q.7 (a) Find equation of the straight line through the point A (5, –4, –4) and intersecting the line $\frac{x}{-1} = \frac{y-1}{1} = \frac{z}{-2}$ at right angle. 9

(b) Find distance of the point A (3, –1, 2) to the plane $2x+y-z-4=0$ 8

Q.8 (a) Find equation of the sphere with centre at C (4, 1, –6) and tangent to the plane $2x-3y+2z-10=0$. 9

(b) Find the direction of Qibla at Karachi with latitude $24^\circ\ 51\cdot 5'$ N and longitude $67^\circ\ 2'$E. 8

P.T.O.

SECTION-III

Q.9 (a) Find Rank of the Matrix $A = \begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{bmatrix}$ by converting it to echelon form. 8

(b) Find Inverse of Matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 3 & 2 & -2 \end{bmatrix}$. 8

Q.10 (a) Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2(a+b+c)^3abc.$ 8

(b) Suppose that u, v and w are linearly independent vectors. Prove that $u + v - 2w$, $u - v - w$ and $u + w$ are linearly independent. 8

SECTION-IV

Q.11 (a) Solve $(x-1)^3 \frac{dy}{dx} + 4(x-1)^2 y = x + 1$. 8

(b) Find an equation of orthogonal trajectories of the family of curves $y^2 = x^2 + cx$. 8

Q.12 (a) Find general solution of $(D^2 + 4)\, y = 4 \sin^2 x$. 8

(b) Solve $(x^2D^2 - 3xD + 5)\, y = x^2 \sin x$. 8