UNIVERSITY OF THE PUNJAB

NOTIFICATION

It is hereby notified that the Syndicate at its meeting held on 28-03-2025 has approved the recommendations of the Academic Council made at its meeting dated 27-01-2025 regarding approval of the Revised Curriculum/Scheme of Studies/Syllabi and Courses of Reading for M.Phil/Ph.D. in Mathematics Programs w.e.f the Academic Session, Fall, 2025 to onward *subject to segregation of core courses and elective courses per HEC policy/guidelines.*

The Revised Curriculum/Scheme of Studies/Syllabi and Courses of Reading for M.Phil/Ph.D. in Mathematics Programs is attached herewith as Annexure-A.

Sd/-
Dr. Ahmad Islam
Registrar

Admin. Block,
Quaid-i-Azam Campus,
Lahore.

No. D/ 3173 Acad. **Dated: 18 -04 /2025.**

Copy of the above is forwarded to the following for information and further necessary action: -

1. Dean, Faculty of Sciences
2. Director, Institute of Mathematics
3. Controller of Examinations.
4. Director, IT Centre *(for uploading on website)*.
5. Secretary to the Vice-Chancellor
6. Secretary to Registrar
7. Assistant Registrar (Statutes)
8. A.O. Syllabus (W.F)

Assistant Registrar (Academic)
for Registrar

Revised Syllabi (Annexure - S3)
for
M.Phil./Ph.D. Mathematics

Department of Mathematics
University of the Punjab
Lahore

SYLLABI FOR M.PHIL. MATHEMATICS
(SEMESTER SYSTEM PROGRAMME)

To be offered in Department of Mathematics, University of the Punjab, Lahore with effect from Admissions 2025 to onwards.

Programme	M.Phil. Mathematics
Duration	2 Years
Semesters	4
Credit Hours	30
Department	Mathematics
Faculty	Science

Introduction

The Department of Mathematics at the University of the Punjab is one of the university's oldest academic units, founded in 1921. Over the years, it has produced numerous distinguished graduates, many of whom have gained prominence in Mathematics. For a considerable period, the department operated on an inter-collegiate basis, with faculty members from local institutions such as Government College Lahore, Forman Christian College, Dyal Singh College, Islamia College Civil Lines, and M.A.O. College, Lahore, conducting M.Sc. classes at the university. In 1956, the department became an independent institution with the appointment of two full-time faculty members: one reader and one senior lecturer. Since then, the department has steadily grown and now offers a range of programs, including BS (4 years), BS (5 semesters), M.Phil., and Ph.D.
In 1982, the department established a computer center to enhance the computational capabilities of university students, faculty, and staff. Furthermore, the Department of Mathematics publishes the Punjab University Journal of Mathematics, with its inaugural issue released in 1967 under the editorship of Prof. Dr. Syed Manzur Hussain.
The department is home to four robust research groups: Computational Mathematics, Fuzzy Mathematics, Gravitation & Cosmology, and Pure Mathematics. These groups contribute to the academic strength of the department, fostering innovation and advancing knowledge in their respective fields.

Vision

The Department of Mathematics aims to be recognized as an internationally top-ranking center of excellence in both teaching and research.

Mission

In pursuit of our vision, the Department of Mathematics strives to provide quality education at both undergraduate and postgraduate levels, aiming to produce high-calibre graduates who will excel in their chosen careers in industry, the professions, and academia. Our students are selected based on intellectual merit, without discrimination based on gender, race, or physical disabilities. We are committed to fostering a diverse and well-balanced portfolio of research of the highest quality, encompassing a wide range of interests.

Objectives

The following objectives are designed to guide the Department of Mathematics toward achieving its vision of becoming an internationally recognized center of excellence in teaching and research.

- Expand and diversify the faculty to encompass all disciplines of Mathematics.
- Strengthen all existing academic programs, with particular emphasis on the MPhil/PhD programs to facilitate world-class research.
- Develop an industry-based Mathematics curriculum, fostering close collaboration between mathematics and industry.
- Encourage faculty engagement in research projects, paper presentations at international conferences, organizing conferences, international research collaborations, and postdoctoral training.
- Promote the "Punjab University Journal of Mathematics" as a premier research journal of international repute.
- Provide strong support for both individual researchers and research groups.
- Enhance the abilities and character of students, nurturing them into well-rounded individuals.
- Recognize and reward the department's staff, acknowledging them as its greatest asset.

Admission Eligibility Criteria

- BS(4 years)/ M.Sc. (2 years) or equivalent in the relevant subject from a HEC recognized University (At least 16 years of education after F.Sc or Equivalent; 120 credit hours).
- No third division in the whole career.
- Qualifying marks in the admission test conducted by the University shall be 50%.
- Qualifying marks in the interview shall be 50%

Requirement for the Award of M. Phil. Degree

The following requirement shall be fulfilled before the award of M. Phil. degree:

- Complete 30 credit hours that include 24 credit hours (8 courses) of course work and 6 credit hours of thesis.

Coding Scheme of Courses

The course code consists of two parts: letter code and numeric code. All courses use the alphabetical code "MATH" which contains 4 characters and the numeric code contains 3 digits.

SEMESTER-WISE WORKLOAD

A Master of Philosophy degree in Mathematics is structured to be comprised of Four regular semesters over a period of Two years. The semester-wise workload is as under:

Semester I

Sr. No.	Course Title	Credit Hours
1	Core Course I	3
2	Core Course II	3
3	Elective Course I	3
4	Elective Course II	3
Total Semester Credits		**12**

Semester II

Sr. No.	Course Title	Credit Hours
1	Core Course I	3
2	Core Course II	3
3	Elective Course I	3
4	Elective Course II	3
Total Semester Credits		**12**

Semester III & IV

Course Code	Course Title	Credit Hours
MATH-690	M.Phil. Thesis	6
Total Semester Credits		**6**

List of Core Courses

Details of the core courses are given in the Table 1 .

Table 1: List of Core Courses

Course Code	Core Courses	Credit Hours
MATH-501	Applied Linear Algebra	3
MATH-502	Applied Graph Theory	3
MATH-503	Partial Differential Equations	3
MATH-504	Mathematical Methods for Physics	3
MATH-505	Numerical Approximation Theory	3
MATH-506	Integral Equations	3
MATH-507	Riemannian Geometry	3
MATH-508	Theory of Groups	3
MATH-509	Module Theory	3
MATH-510	Measure Theory	3
MATH-511	Lie Algebras	3

List of Elective Courses

Table 2: List of Elective Courses

Course Code	Course Title	Credit Hours
MATH-512	Lie Symmetries of Differential Equations	3
MATH-513	Linear Groups and Group Representations	3
MATH-514	Field Extensions & Galois Theory	3
MATH-515	Representation Theory of Finite Groups	3
MATH-516	Theory of Manifolds	3
MATH-517	Number Theory and Cryptography	3
MATH-518	Applied Combinatorics	3
MATH-519	Algebraic Geometry	3
MATH-520	Lie Groups	3
MATH-521	Category Theory	3
MATH-522	Fixed Point Theory	3
MATH-523	Knot Theory	3
MATH-524	Operator Theory	3
MATH-525	Banach Algebras	3
MATH-526	Functional Analysis	3
MATH-527	Harmonic Analysis	3
MATH-528	Advanced Topology	3
MATH-529	Algebraic Topology	3
MATH-530	Topological Groups	3

Table 3: List of Elective Courses

Course Code	Course Title	Credit Hours
MATH-531	Homological Algebra	3
MATH-532	Commutative Algebra	3
MATH-533	Algebraic Graph Theory	3
MATH-534	Introduction to Fuzzy Systems	3
MATH-535	Fuzzy Decision Making Methods	3
MATH-536	Fuzzy Graph Theory	3
MATH-537	Fuzzy Fractional Differential Equations	3
MATH-538	Introduction to Biomathematics	3
MATH-539	Mathematical Cosmology	3
MATH-540	Stellar Dynamics	3
MATH-541	Elastodynamics	3
MATH-542	General Relativity	3
MATH-543	Mathematical Plasma Dynamics	3
MATH-544	Magnetohydrodynamics	3
MATH-545	Fluid Dynamics	3
MATH-546	Biofluid Mechanics	3
MATH-547	Classical Field Theory	3
MATH-548	Quantum Field Theory	3
MATH-549	Classical Electrodynamics	3
MATH-550	Non-Newtonian Fluid Mechanics	3
MATH-551	Relativistic Theory of Black Holes	3
MATH-552	Fractional Differential Equations	3

Table 4: List of Elective Courses

Course Code	Course Title	Credit Hours
MATH-553	Quantum Information Theory	3
MATH-554	Quantum Computing	3
MATH-555	Optimization Techniques	3
MATH-556	Theory of Spline Functions	3
MATH-557	Minimal Surfaces	3
MATH-558	Scientific Computing	3
MATH-559	Subdivision Methods for Geometric Design	3
MATH-560	Spline Solutions of Boundary Value Problems	3
MATH-561	Numerical Methods for Partial Differential Equations	3
MATH-562	Numerical Methods for Ordinary Differential Equations	3
MATH-563	Sobolev Gradients and Differential Equations	3
MATH-564	Theory of Differential Equations	3
MATH-565	Mathematical Modeling in Epidemiology	3
MATH-566	Artificial Intelligence with Python	3
MATH-567	Data Science Fundamentals	3
MATH-568	Fundamentals of Cryptography	3
MATH-569	Modular Forms and Elliptic Curves	3
MATH-570	Mathematical Statistics	3
MATH-571	Financial Mathematics	3

Course Title: Applied Linear Algebra
Course Code: MATH- 501
Prerequisites: Linear Algebra
Credit Hours: 3
Course Overview:
Linear algebra is a cornerstone of modern mathematics and serves as the foundation for many scientific and computational disciplines. Applied linear algebra, in particular, provides essential tools for solving complex problems in fields such as physics, engineering, computer science, data analysis, and economics. This course offers a comprehensive introduction to the fundamental concepts of applied linear algebra, bridging the gap between theory, algorithm development, and practical implementation.
Course Contents:
Preliminaries: Matrix multiplication, Orthogonal vectors, Orthogonal matrices, Vector and matrix norms, Condition number of a matrix, Convergent matrices, Projection matrices.
Linear Equations: Existence of a solution, Gaussian elimination, LU decomposition, Perturbation analysis, Sensitivity of linear systems, Uncoupled linear Systems. Applications: Electrical networks, Network flows.
Orthogonality: Orthogonal subspaces, Fundamental subspaces of a matrix, Orthogonal projection, Gram-Schmidt orthogonalization, QR-decomposition, Least squares approximation. Application: Fitting models to data.
Eigenvalues: Diagonalization, Spectral theorem, Singular value decomposition (SVD), Pseudo inverse, SVD expansion, Power iterative method, QR-iterative method. Applications: Image deblurring, Page Rank. Complex vector spaces.

Recommended Books:

1. Golub, G. H, *Matrix Computations*, Johns Hopkins University Press, 2013.
2. Nicholson, W. K, *Linear Algebra with Applications*, University of Calgary, 2018.
3. Strang, G, *Linear Algebra and Learning from Data*, Wellesley-Cambridge Press,U.S, 2019.
4. Strang, G, *Linear Algebra and Its Applications*, 4th edition, Cengage Learning, 2006.
5. Leon, S.J., *Linear Algebra with Applications*, Maxwell Macmillan International Edition, 1990.

Course Title: Applied Graph Theory
Course Code: MATH- 502
Prerequisites: N/A
Credit Hours: 3
Course Overview:
Graph Theory is a fundamental area of discrete mathematics that explores the study of graphs, which are mathematical structures used to model pairwise relations between objects. This course provides a deep understanding of the theoretical foundations of graph theory. This course focuses on the mathematical theory of graphs, a few applications, and algorithms.
Course Outline:
Preliminaries: Undirected graph, Directed graphs, Degree sequence, Handshaking lemma, Simple graphs, Representations of graph, Associated matrices of graph, Graph isomorphism.
Connectivity: Vertex and edge connectivity, Cut-vertex, Bridge, Hamilton cycles and Euler circuits.
Trees: Rooted tree, Spanning tree, Minimum spanning tree, Depth-first search, Breadth-first search.
Graph Algorithms: Algorithmic complexity, Greedy algorithm, Shortest path algorithm, Minimal spanning tree algorithm, Chinese postman algorithm.
Planar Graphs: Properties of planar graphs, Euler formula, Dual graph, Crossing number, Thickness of graph, Graph on surfaces.
Coloring of Graphs: Vertex coloring, Chromatic number, Edge coloring, Chromatic index, Chromatic polynomial.
Network Flow Problems: Flows in networks, Max-flow /Min-cut theorem, Algorithm to find maximum flow in a network, Menger's theorem.
Matching Theory: Matching, Perfect matching, Matching in bipartite graph.
Extremal Graph Theory: Ramsey number, Topological indices, Connectivity index, Graph energy.

Recommended Books:

1. Aldous, J. M. and Wilson, R. J., *Graphs and Applications*, Springer, 2000.
2. Bondy J. A. and Murty, U. S. R., *Graph Theory with Applications*, Elsevier Science, 1984.
3. Chartrand, G. and Oellermann, O. R., *Applied and Algorithmic Graph Theory*, McGraw-Hill College, 1992.
4. Diestel, R., *Graph Theory*, Springer, 2017.
5. Wilson, R. J., *Introduction to Graph Theory*, Pearson, 2010.

Course Title: Partial Differential Equations
Course Code: MATH- 503
Prerequisites: Ordinary Differential Equations
Credit Hours: 3
Course Overview:
This course provides an overview of basic concepts of partial differential equations (PDEs) and includes some important methods to find the analytical solutions of first and second order partial differential equations. The Laplace equation, wave equation and heat equation are particularly focused as important examples of elliptic equations, hyperbolic equations and parabolic equations, respectively.
Course Outline:
First Order PDEs: Classification, Solutions of linear and quasilinear equations, Characteristic curves.
Second Order PDEs: Linear second order PDEs in n independent variables, Functionally invariant pairs, Exponential type solutions of linear PDEs, Classification of linear and almost linear second order PDEs in two and more variables, Normal forms for hyperbolic, Parabolic and elliptic equations, Cauchys problems for linear second order equations in n independent variables, Adjoint operator and self adjoint operator, Lagranges identity, Greens formula for self adjoint operator, Laplace equation, Heat equation.
The Wave Equation: D'Alembert's solution of the initial-value problem, Mathematical model for vibrating string and its solution by separation of variables, Spherical waves, Cylindrical waves.

Recommended Books:

1. Dennemyer, R., *Introduction to Partial Differential Equations and Boundary Value Problems*, McGraw Hill Book Company, 1968.

2. McOwen, R., *Partial Differential Equations*, Prentice Hall, 1996.

3. Pinsky, M. A. *Partial Differential Equations and Boundary-Value Problems with Applications*, American Mathematical Society, 2011.

4. Strauss, W. A., *Partial Differential Equations*, Wiley, 2008.

5. Zauderer, E. *Partial Differential Equations of Applied Mathematics*, Wiley-Interscience, 2006.

Course Title: Mathematical Methods for Physics
Course Code: MATH- 504
Prerequisites: Ordinary Differential Equations, Partial Differential Equations
Credit Hours: 3
Course Overview:
This course covers a wide range of mathematical techniques necessary to solve advanced problems in physics and engineering. This course will focus on the application of mathematical methods to physics problems. This course covers some important mathematical methods and solutions to some well-known ordinary and partial differential equations that appear frequently in physics. This course also provides a brief introduction to variational methods and matrix algebra.
Course Contents:
Nonlinear ordinary differential equations: Bernoulli's equation, Riccati equation, Lane Emden equation, Nonlinear pendulum, Duffing's equation, Pinne's equation.
Linear partial differential equations: Classification, Initial and boundary values problems, Fourier analysis, Heat equations, Wave equations, Laplace equation.
Integral equations: Classification, Integral transform separable kernels, Singular integral equations, Wiener-Hopf equations, Fredholm theory, Series solutions.
Variational methods: Euler-Lagrange equations, Solutions to some famous problems, Sturm-Liouville problem and variational principles, Rayleigh-Ritz methods for partial differential equations.
Matrix algebra: Method of Faddeev, Caley-Hamilton' theorem function of matrices, Functions of matrices, Kronecker and tensor product, Special matrices.

Recommended Books:

1. Arfken, G. B. and Weber, H. J., *Mathematical Methods for Physicists*, Academic Press, 2013.
2. Riley, K. F., Hobson, P. M. and Bence, S. J., *Mathematical Methods for Physics and Engineering*, Cambridge University Press, 2006.
3. Stephenson, G. and Radmore, P. M., *Advanced Mathematical Methods for Engineering and Science Students*, Cambridge University Press, 1990.
4. Stone, M. and Goldbart, P., *Mathematics for Physics*, Cambridge University Press, 2009.
5. Tang, K. T., *Mathematical Methods for Engineers and Scientists*, Springer, 2007.

Course Title: Numerical Approximation Theory
Course Code: MATH- 505
Prerequisites: Numerical Analysis
Credit Hours: 3
Course Overview:

Numerical Approximation Theory is the study of algorithms with the use of numerical approximations, in many disciplines of mathematics such as linear algebra and differential equations. Approximation theory has become so vast that it intersects with every other branch of analysis and plays an increasingly important role in applications in the applied sciences and engineering. Numerical approximation theory presents a systematic, in-depth treatment of some basic topics in approximation theory designed to emphasize the rich connections of the subject with other areas of study. The course includes existence and uniqueness theorems for approximation problems, polynomial and rational approximations.

Course Contents:

Introduction: Approximation problems, Generalized Fourier series, Error analysis, Optimal polynomials
Polynomial Approximations: Existence of best approximations, Uniqueness of best approximations, Approximation operators and some approximating functions, Polynomial interpolation, Uniform convergence of polynomial approximations, Minimax approximation theory, Function spaces and related theorems, Existence and uniqueness theorems for linear approximation problems, Degree of approximation by trigonometric and algebraic polynomials.
Non-Polynomial Approximations: Approximation by rational functions. Chebyshev approximation, Padé approximation, Least squares approximation, Gram-Schmidt orthonormalization process, Orthogonal functions, Remez algorithm, Approximation to periodic functions, Uniform boundedness theorem.

Recommended Books:

1. Lorentz, G. G., *Approximation of Functions*, AMS Chelsea publishing, 2005.
2. Mhaskar, H. N. and Pai, D. V., *Fundamentals of Approximation Theory*, CRC Press, 2000.
3. Powell, M.J.D., *Approximation Theory and Methods*, Cambridge University Press, 2001.
4. Rivlin, T. J., *An Introduction to the Approximation of Functions*, Dover Publications, 1981.
5. Trefethen, L. N., *Approximation Theory and Approximation Practice*, Society for Industrial and Applied Mathematics, 2013.

Course Title: Integral Equations
Course Code: MATH- 506
Prerequisites: Ordinary Differential Equations and Real Analysis
Credit Hours: 3
Course Overview:
This course emphasizes concepts and techniques for solving integral equations from an applied mathematics perspective. Many physical problems that are usually solved by differential equation methods can be solved more effectively by integral equation methods. This course will help students gain insight into the application of advanced mathematics and guide them through derivation of appropriate integral equations governing the behavior of several standard physical problems.
Course Contents:
Introductory concepts: Series solutions of ODEs, Leibniz's rule for differentiating integrals, Reducing multiple to single integrals, Laplace transforms for ODEs, Convolution theorem, Infinite geometric series. Properties of linearity and homogeneity, singular and integro-differential equations, Special kinds of kernels.
Linear integral equations: Linear integral equations of the first and second kind, Relationship between differential equations and integral equations (IVP and BVP), Neumann series, Fredholm integral equation of the second kind with separable kernels, The Adomian and modified decomposition method, Variational iteration and successive approximation method for the Volterra and Fredholm integral equations, Volterra-Fredholm integro-differential equations, Systems of Volterra and Fredholm integral equations, Eigenvalues and eigenvectors, Iterated functions.
Homogeneous and singular integral equations: Homogeneous integral equations of the second kind, Fredholm integral equations of the first kind, Fredholm integral equations of the second kind, Singular integral equations, Abel's integral equations. *Hilbert-Schmidt theory:* Integral equations with symmetric kernels, Hilbert-Schmidt theory of integral equations. *Nonlinear integral equations:* Nonlinear Fredholm integral equations, Nonlinear Volterra integral equations. *Integro-differential equations:* Fredholm integro-differential equations, Volterra integro-differential equations. *Applications of integral equations:* Applications in physics, engineering, and mathematical modeling.

Recommended Books:

1. Lovitt, W.V., *Linear Integral Equations*, Dover Publications, 2005.
2. Rahman, M., Integral Equations and their Applications, WITPRESS, 2007.
3. Squire, W., *Integration for Engineers and Scientists*, American Elsevier, 1970.
4. Wazwaz, A.M., *Linear and Nonlinear Integral Equations*, Springer, 2011.
5. Zemyan, S. M., *The Classical Theory of Integral Equations*, Birkhäuser Boston, 2012.

Course Title: Riemannian Geometry
Course Code: MATH- 507
Prerequisites: Differential Geometry
Credit Hours: 3
Course Overview:
This course provides an in-depth introduction to Riemannian geometry, a branch of differential geometry that focuses on smooth manifolds equipped with a Riemannian metric. This framework includes the measurement of geometric quantities such as angles, curve lengths, surface areas, and volumes, along with tangent spaces, vector fields, symmetries, geodesics, and curvature. Originating from Bernhard Riemann's pioneering work, this field has driven major breakthroughs in Einstein's general relativity, quantum gravity, cosmology, and gauge theory. It has applications in various scientific fields and provides students with a comprehensive understanding of the subject.
Course Outline:
Theory of curves and surfaces: Regular parameterized curves in R^3, Moving triad, Associated planes, curvature, Torsion, Serret-Frenet apparatus, spherical indicatrices. *Regular parameterized surfaces in* R^3: Tangent planes, Normal line, Singular points, Coordinate curves on a surface, Fundamental forms, Normal curvature, Principal curvature, Gaussian and mean curvature, The Shape operator, Theorem Egregium, Gauss-Weingarten and Mainardi-Codazzi equations, The fundamental theorem of surface theory, Geodesic curvature, Beltrami's formula, Geodesic deviation, Geodesics in higher dimensions, and the Gauss-Bonnet theorem and its applications to surfaces.
Tensor and manifold theory: Definition and examples of smooth manifolds, Coordinatization, atlas, Submanifolds and their topological properties, Tangent vectors, Immersions, Curves, Vector fields on manifolds, Derivation and dual derivation, Cotangent and tangent bundles, Parallel transport, Tensor fields and tensor algebra, Covariant, Contravariant and mixed tensors, Christoffel symbols, Riemannian and semi-Riemannian metrics (positive definite and indefinite signature), Covariant derivatives and geodesics, Levi-Civita connection, Riemann curvature, Ricci curvature, Ricci scalar and sectional curvature. Isometries and symmetries in both Riemannian and semi-Riemannian settings, Riemann and Weyl tensors.

Recommended Books:

1. Do Carmo, M. P., *Riemannian Geometry*, Birkhauser Boston, 1992.
2. Jost, Jürgen, *Riemannian Geometry and Geometric Analysis*, Universitext, Springer, 2017.
3. Lee, John M., *Introduction to Smooth Manifolds*, Graduate Texts in Mathematics, Springer, 2012.
4. O'Neill, Barrett, *Semi-Riemannian Geometry with Applications to Relativity* , Academic Press, 1983.
5. Spivak, Michael, *A Comprehensive Introduction to Differential Geometry,* Publish or Perish, 1999.

Course Title: Theory of Groups
Course Code: MATH- 508
Prerequisites: Elementary Group Theory
Credit Hours: 3
Course Overview:
Group theory plays a pivotal role in various areas of mathematics and its applications. A group provides a structured framework to study symmetry, transformations, and operations, making it essential for understanding more complex algebraic systems such as rings, fields, and vector spaces. Special emphasis is placed on the classification of finite abelian groups and the implications of the Sylow theorems in the study of finite groups. This course aims to equip students with a deep comprehension of group theory powerful tools and their relevance to both pure and applied mathematics
Course Contents:
Preliminaries: Abelian groups, Cyclic groups, Dihedral groups, Quaternion groups, Matrix groups, Permutation groups, Normal subgroups, Automorphisms, Group actions, Orbit-stabilizer theorem, Burnside's lemma and its applications to combinatorics.
Sylow Theory: p-groups and their structures, Cauchy's theorem, Sylow's theorems, Applications of Sylow's theorems, Classification of finite Abelian groups.
Series in Groups: Subnormal and normal series, Zassenhaus lemma, Schreier's theorem, Jordan-Hölder theorem.
Solvable Groups: Definition of Solvable groups, Examples of solvable groups, Structure of solvable groups.
Nilpotent Groups: Definition of nilpotent groups, Examples of nilpotent groups, Upper and lower central series, Characterization of finite nilpotent groups.
Free Groups: Free groups, Free product of groups, Wreath product of groups.

Recommended Books:

1. Gallian, C. J., *Contemporary Abstract Algebra*, Chapman and Hall/CRC, 2020.
2. Rotman, J., *An Introduction to the Theory of Groups*, Springer, 1995.
3. Rose, H.E., *A Course on Finite Groups*, Springer, 2009.
4. Smith, G. C. and Tabachnikova, O. M., *Topics in Group Theory*, Springer, 2000.
5. Wehrfritz, B., *Finite Groups*, World Scientific Publishing Company, 1999.

Course Title: Module Theory
Course Code: MATH- 509
Prerequisites: Ring Theory
Credit Hours: 3
Course Overview:
This course provides a comprehensive introduction to module theory that generalizes both Abelian groups and vector spaces. Students will explore foundational topics along with advanced concepts like injective and projective modules, and modules over a principal ideal domain (PID). The course emphasizes the structural theorems for finitely generated modules, including the Jordan canonical form and Smith normal form, providing essential tools for understanding algebraic structures.
Course Contents:
Introduction to Module Theory: Definition of module with examples, Submodules, Quotient modules, Module homomorphisms, Isomorphism, Noether theorems, Correspondence Theorem, Generation of modules, Direct sums, Free modules, Tensor product of modules.
Injective and Projective Modules: Exact sequences, The short five lemma, Projective modules, Schanuel's Lemma, Injective modules, Baer's criterion, Flat modules.
Modules over a PID: Noetherian R-modules, Rank of an R-module, Fundamnental Theorem for finitely generated modules, Elementary Divisor Form, The Primary decomposition Theorem, The fundamental Theorem of finitely generated Abelian groups, The rational canonical form, Smith normal form, The Jordan cononical form, Application to canonical form of matrices.

Recommended Books:

1. Adamson, I, T., *Rings Modules and Algebras*, Oliver & Boyd, 1971.
2. Adhikari, M. R. and Adhikari, A., *Basic Modern Algebra with Applications*, Springer, 2014.
3. Blyth, T. S., *Module Theory*, Oxford University Press, 1990.
4. Dauns, J., *Modules and Rings*, Cambridge University Press, 1994.
5. Dummit, D. S., Foote, R. M., *Abstract Algebra*, John Wiley & Sons, 2003.

Course Title: Measure Theory
Course Code: MATH- 510
Prerequisites: Real Analysis, Topology
Credit Hours: 3
Course Overview:
This course provides a rigorous introduction to measure theory and Lebesgue integration, foundational tools in modern analysis, probability theory, and various applied fields. Topics include σ-algebras, measures, measurable functions, integration, convergence theorems, product measures, and differentiation. The course emphasizes both theoretical understanding and practical applications.
Course Contents:
Measurable sets: σ-algebras and measurable spaces, Outer measure, Lebesgue measure, Lebesgue measurable sets, Outer measures and Caratheodory's criterion, Borel sets, Non measurable sets.
Measurable functions: Lebesgue measurable functions, Simple functions, Characteristic functions, Borel measurable function, Littlewood three principle.
The Lebsegue integration: Review of the Riemann integral and Riemann-Stieltjes integral, Properties of the Lebesgue integral, Lebsegue integral of bounded measurable functions and non-negative functions, Integrable functions, Monotone convergence theorem, Fatou's lemma, Dominated convergence theorem, Convergence in measure, Convergence of measurable functions. Pointwise convergence, Uniform convergence, Almost everywhere convergence.
Classical Banach spaces: L^P-spaces, Convergence and completeness in L^P-spaces, Bounded linear functional on the L^P-spaces, General convergence theorem, Singed measures, Absolutely continuous functions, Differentiation under the integral sign.

Recommended Books:

1. Axler, S., *Measure, Integration & Real Analysis*, Springer, 2020.

2. Halmos, P. R., *Measure Theory*, Springer, 1950.

3. Khanfer, A., *Measure theory and integration*, Springer, 2023.

4. Royden, H. L. and Fitzpatrick, P. M., *Real Analysis*, Pearson Education, 2010.

5. Ross, S. M. and Pekoz, E. A., *Measure theory and laws of large numbers. In A second course in probability*, Cambridge University Press, 2023.

Course Title: Lie Algebras
Course Code: MATH- 511
Prerequisites: Linear Algebra
Credit Hours: 3
Course Overview:
This is an introductory course on Lie algebras, used in both Mathematics and Physics. The course will deal with finite-dimensional Lie algebras, that is, with anticommutative algebras satisfying the Jacobi identity. These algebras have various applications in representation theory, mathematical physics, geometry, engineering and computer graphics. Lie theory is currently a very active area of research and provides many interesting examples and patterns to other branches of mathematics.
Course Contents:
Introduction: Lie algebras with examples, Ideals and homomorphisms, Lie algebra of derivations, Witt Algebra, Lie algebra representation.
Classification of Lie Algebras: Classification of complex Lie algebras of dimension 1, 2,3, Nilpotent and solvable Lie algebras, Simple, semisimple, General linear Lie algebras and its subalgebras, Nilpotent maps, Weights and invariance lemma.
Engel and Lie Theorems: Engel and Lie theorems, Reductive Lie algebras and their properties.
Basic representation theory: Examples of representations, Modules, Reducible and irreducible representation, Faithful representation.
Cartan's Criteria and Killing Form: Jordan Decomposition, Killing forms, Criteria for semisimplicity, Solvability and nilpotency.

Recommended Books:

1. Erdmann, K. and Wildon, M. J., *Introduction to Lie Algebras*, Springer, 2006.
2. Gilmore, R., *Lie Groups, Lie Algebras and Some of Their Applications*, Dover Publication, 2006.
3. Humphreys, J.E. *Introduction to Lie Algebras and Representation Theory*, Springer, 1972.
4. Iachello, F., *Lie Algebras and Applications*, Springer, 2006.
5. Jacobson, N., *Lie Algebras*, Dover Publication, 1979.

Course Title: Lie Symmetries of Differential Equations
Course Code: MATH- 512
Prerequisites: Differential Geometry, Differential Equations
Credit Hours: 3
Course Overview:
Symmetry principles play an important role in the laws of nature. Invariance principles provide a structure and coherence to the laws of nature, just as the laws of nature provide a structure and coherence to the set of events. In fact, it is hard to imagine that progress could have been made in deducing the laws of nature without the existence of certain symmetries. This course will help to determine a useful, systematic, computational method that will explicitly determine the symmetry group of differential equations or system of differential equations and solve the corresponding differential equation. Lie symmetry analysis of differential equations proves to be a powerful tool to solve or at least reduce the order and nonlinearity of the equation. This course covers the fundamental symmetries of differential equations.
Course Contents:
Differential Geometry of Manifolds: Manifolds, Change of coordinates, Maps between manifolds, The maximal rank condition, Submanifolds, Regular submanifolds, Implicit submanifolds, Curves and surfaces, Connectedness, Lie groups, Vector fields, Lie algebras, Lie Subgroup, Local Lie subgroup, Local transformation group, Orbits, Flows, Action on functions and differentials, Lie brackets, Tangent spaces, Vectors fields on submanifolds, Frobenius' Theorem, One-parameter subgroups subalgebras, The exponential map, Lie algebras of local Lie groups, Structure constants. Commutator tables, Infinitesimal group actions, Basis of differential forms, Pull-back and change of coordinates, Interior products. The differential, Lie derivatives, Integration and Stokes' Theorem.
Symmetry Group of Differential Equations: Invariant subsets, Invariant functions, Infinitesimal invariance, Local invariance, Invariants and functional dependence, Methods for constructing invariants.
Prolongation: Systems of differential equations, Prolongation of group actions, Invariance of differential equations, Prolongation of vector fields, Infinitesimal invariance, The prolongation formula, Total derivatives, The general prolongation formula, Properties of prolonged vector fields, Characteristics of symmetries, Calculation of symmetry groups.
Integration of Ordinary Differential Equations: First order equations, Higher order equations, Differential invariants, Multi-parameter symmetry, Groups, Solvable groups, Systems of ordinary differential equations.
Nondegeneracy Conditions for Differential Equations: Local solvability, Invariance criteria, The Cauchy-Kovalevskaya theorem, Characteristics, Normal systems, Prolongation of differential equations.

Recommended Books:

1. Arrigo, D.J., *Symmetry Analysis of Differential Equations*, Wiley, 1st edition, 2015.

2. Bluman, A. Cheviakov, S. Anco, *Applications of symmetry methods to partial differential equations*, Springer, 2010.

3. Hans, S., *Differential Equations: Their Solution Using Symmetries*, Cambridge University Press, 1989.

4. Olver, P.J., *Applications of Lie groups to differential equations*, Oxford University, 1980.

5. Peter, E. H., *Symmetry Methods for Differential Equations: A Beginner's Guide*, Cambridge University Press, 2000.

6. Stillwell J., *Naive Lie Theory*, Springer, 2008.

Course Title: Linear Groups and Group Representations
Course Code: MATH-513
Prerequisites: Group Theory, Linear Algebra.
Credit Hours: 3
Course Overview:
This course offers a comprehensive exploration of group theory, focusing on key topics such as Sylow theorems and group extensions, with main emphasis on general linear groups. It delves into the properties of general, special, and projective linear groups.
Course Contents:
Direct product of groups, Pull-back and Push-out in nonabelian groups, Classification of finite Abelian Groups, p-groups and Sylow Theorems, Semi direct products, Regular and permutationwreath products. Holomorph of a group, Generalized Dihedral groups, Extension of a Group, Section and sectional factor set, Central and cyclic extensions, General linear groups, Order and center of general linear groups, Decomposition of general linear groups, The projective special linear groups, Transvections, Generators of linear groups, Simple linear groups, Classification of groups with at most 31 elements.

Recommended Books:

1. Alperin, J.L. and Bell, R.B., *Groups and Representations*, Springer, 1995.
2. Dixon, J.D., *The Structure of Linear Groups*, Van Nostrand Reinhold Company, 1971.
3. Grove, L.C., *Classical Groups and Geometric Algebra*, American Mathematical Society, 2002.
4. Humphreys, J. F., *A Course in Group Theory*, Oxford University Press, USA, 1996.
5. Rotman, J. J., *An Introduction to the Theory of Groups*, Springer, 1994.
6. Wilson, R. A., *The Finite Simple Groups*, Springer, 2009.

Course Title: Field extensions and Galois Theory
Course Code: MATH- 514
Prerequisites: Algebra
Credit Hours: 3
Course Overview:
The course on Field Extensions and Galois Theory provides a thorough exploration of Field Theory, covering essential topics like algebraic and transcendental extensions, Galois correspondences, and finite fields. This course equips students with the tools needed to tackle complex problems in algebra.
Course Outline:
Preliminaries: Rings, Ideals, Fields, Field of fractions, Characteristics, Prime fields. *Field Extensions*: Extension of fields, Simple extensions, Irreducible polynomials, Algebraic element, Algebraic and transcendental extensions, Classical ruler and compasses constructions, Splitting fields, Normal extensions, Algebraically closed fields, Normal extensions, Fundamental theorem of algebra, Separable and inseparable polynomials, Frobenius automorphisms, Separable extensions, Purely inseparable extensions. *Galois Theory:* Automorphisms of fields, The general Galois correspondence, Galois extensions, Finite Galois theory, Roots of unity, Primitive elements, Separable and inseparable degrees, Norms and traces, Cyclic extensions, Solvability by radicals, Finite fields.

Recommended Books:

1. Bastida, J. R, *Field Extensions and Galois Theory*, Cambridge University Press, 1984.
2. Borceux, F. and Janelidze, G., *Galois Theories*, Cambridge University Press, 2001.
3. Edwards, H. M, *Galois Theory*, Springer, 1984.
4. Dummit. D and Foote. R., *Abstract Algebra*, John Wiley & sons, 2003.
5. Stewart, I., *Galois Theory*, Chapman and Hall, 2015.
6. Weintraub, S. H., *Galois Theory*, Universitext, Springer, 2009.

Course Title: Representation Theory of Finite Groups
Course Code: MATH-515
Prerequisites: Theory of Groups, Linear Algebra
Credit Hours: 3
Course Overview:
This course provides a comprehensive study of group representation, covering fundamental concepts like algebras, group modules, and reducibility. It delves into representation theory for abelian and permutation groups, incorporating pivotal theorems such as Maschke's, Clifford's, and Burnside's. Group characters are explored with an emphasis on character tables, solvability criteria, and key theorems by Frobenius, Schur, and Brauer. The curriculum also includes applications of these concepts to linear groups, the lifting process, and advanced induction theorems.

Course Contents:
Representation of groups: Algebras and group modules, reducibility and decomposition of a representation, Linear representation. Representation of abelian groups. Permutation representation, Maschke's theorem and its application, Chifford's theorem. *G*-homomorphisms, Schur's lemma. A module and regular representation. Semi simple algebras. Theorems of Burnside. Frobenius. Schur and wedderburn. Representation of direct and central product.
Group Characters: Characters as class function. Character ring. Tensor products. Basic concepts and application. Character table. Burnside's criterion for solvable groups. The Frobenius Wieland theorem on existence of normal subgroups in a group. Theorems of Jordan, Burnside and Schur on linear groups. The lifting process. Brauer's theorem on induced characters and its application. The generalization induction theorems.
Recommended Books:

1. Curtis, C.W. and Reiner, I., *Representations Theory of Finite Groups and Associative Algebras*, Interscience, 1962.

2. Huppert, B., *Character Theory of Finite Groups*, De Gruyter, 1998.

3. Isaac, I. M., *Character Theory of Finite Groups*, Academic Press, 1976.

4. James, G. and Liebeck, M., *Representations and Characters of Group*, Cambridge University Press, 2001.

5. Serre, J. P., *Linear Representations of Finite Groups*, Springer, 1971.

Course Title: Theory of Manifolds
Course Code: MATH- 516
Prerequisites: Differential Geometry
Credit Hours: 3
Course Overview:
A manifold is a generalization of curves and surfaces to higher dimensions. It is locally Euclidean in that every point has a neighborhood, called a chart, homeomorphic to an open subset of $\mathbb{R}^n$. The coordinates on a chart allow one to carry out computations as though in a Euclidean space, so that many concepts from $\mathbb{R}^n$, such as differentiability, point-derivations, tangent spaces, and differential forms, carry over to a manifold. This course covers the fundamental concepts of manifolds.
Course Outline:
Manifolds and Differentiable Structures: Basic definition and examples, Topological manifold, Smooth manifold, Atlas and charts, Smooth structures, Transition maps, Partial derivatives, Critical points, Immersion theorem, Submersion, Examples of manifolds, Integral curve and flows.
Tangent and Cotangent Bundle on a Manifold: Tangent space of $\mathbb{R}^n$, Tangent space of an imbedded manifold, Vector bundle, Tangent and cotangent bundle of a manifold, Derivations and vector fields on a manifold.
Tensors Fields on the Manifolds: Basic definition and examples tensors and tensor bundles, Metric tensor, Tensor and exterior algebras, Differential forms, Exterior product, Lie derivative, Interior product.
Fibre Bundles: Principal bundle, Connections in bundle, Linear connection, Torsion and curvature of the connection. *Riemannian Manifolds*: Reimannian manifolds and connection, Geodesics, Curvature and Ricci tensors, Affine killing conformal jacobi and harmonic vector fields.

Recommended Books:

1. Gadea, P. M., Masque, J.M., *Analysis and Algebra on Differentiable Manifolds*, Springer, 2009.

2. Lee, J. M., *Introduction to Smooth Manifolds*, Springer, 2000.

3. Spivak, M., *A comprehensive introduction to Differential Geometry*,VOL 1-5 Publish or Perish Inc., 1999.

4. Tu, L. W., *Introduction to Manifolds*, 2nd edition, Springer, 2008.

Course Title: Number Theory and Cryptography
Course Code: MATH-517
Prerequisites: Elementary knowledge of integers
Credit Hours: 3
Course Overview:
This course covers divisibility, modular arithmetic, and arithmetic functions. Students will explore primitive roots, indices, and their applications, along with solving linear Diophantine equations and Congruences. In addition, the course covers cryptography, ranging from classical cryptosystems to advanced methods like RSA and public key cryptography.
Course Contents:
Preliminaries: Modular arirhmetic, Congruences, The Chinese remainder theorem, Sigma and Tao functions, Mobius function and its properties, Mobius inversion formula, Bracket functions.
Primitive roots and indices: Integers belonging to a given exponent, Composite moduli, primitive roots modulo a prime, Determination of integers having primitive roots, Indices, Properties of indices, Applications of indices.
Fermat's last theorem and quadratic reciprocity law: Proof of Fermat's last theorem for n=3, 4), The Jacobi and Legender symbols with applications, Gauss's quadratic reciprocity law.
Algebraic number theory: The ring $Z[\sqrt{-5}]$, Finite fields, Primitive polynomials. Irreducibility. Gauss lemma and Eisenstein criterion of irreducibility, Number fields and integral dependence, Integers in number fields, Cyclotomic polynomials, Cyclotomic fields and class groups.
Cryptography: Caesar cipher cryptosystem, Subsitution in cryptosystem, Applications of elementary number theory to cryptography, Symmetric ciphers, Public key cryptography, Discrete log problems and RSA cryptosystems.

Recommended Books:

1. Adler, A. and Cloury, J. E., *The Theory of Numbers*, Jones & Bartlett Pub; 1st edition, 1995.

2. Burton, D. M., *Elementary Number Theory*, McGraw Hill Company, 2007.

3. Koblitz, N., *A Course in Number Theory and Cryptography*, Springer, 1994.

4. Schneier, B., *Applied Cryptography*, John Wiley & Sons Inc, 1996.

5. Stallings, W., *Cryptography and Network Security*, Pearson, 2016.

Course Title: Applied Combinatorics
Course Code: MATH- 518
Prerequisites: Discrete Mathematics
Credit Hours: 3
Course Overview:
Combinatorics is a broad subject that mainly deals with the counting, construction, and optimization of discrete objects. Such problems are near the intersection of mathematics and computer science. This course is designed to provide students with a rigorous introduction to combinatorial theory, with a focus on both classical and modern techniques.
Course Contents:
Counting techniques: Addition rule, Multiplication rule, Combinations, Permutations, Pigeonhole principle, Principle of inclusion and exclusion, Binomial coefficient, Bell numbers, Catalan numbers, Stirling numbers of the first and second kind, Linear homogeneous recurrence relations, Algebraic solutions of linear recurrence relations and constant functions, The method of generating functions, Non-linear recurrence relations.
Discrete probability: Axioms of probability, Addition and multiplication rules of probability, Conditional probability, Bayes's rule.
Graph theory: Undirected graphs, Directed graphs, Eulerian graphs, Hamiltonian graphs, Trees, Graph coloring, Matching in bipartite graphs.
Ramsey Theory: Ramseys theorem, Bounds for Ramsey numbers and applications.
Polya Theory: Necklace problem, Group actions and symmetry in counting problems, Burnsides lemma, cycle index of a permutation group, Polya's enumeration theorem and its applications to coloring and combinatorial configurations.

Recommended Books:

1. Bryant, V., *Aspects of Combinatorics*, Cambridge University Press, 1993.
2. Keller, M. T. and Trotter, W. T., *Applied Combinatorics*, Open Textbook Library, 2017
3. Godsil, C., *Algebraic combinatorics*, Routledge, 2017
4. Richard, A.B., *Introductory Combinatorics*, Prentice Hall, 1999.
5. Roberts, F.S. and Tesman, B., *Applied combinatorics*, CRC Press, 2024.
6. Tucker, A., *Applied Combinatorics*, Wiley, 2012.

Course Title: Algebraic Geometry
Course Code: MATH- 519
Prerequisites: Commutative Algebra
Credit Hours: 3
Course Title:
Algebraic geometry is a branch of mathematics that uses abstract algebraic techniques derived primarily from commutative algebra to solve geometric problems. Traditionally, it studies the zeros of multivariate polynomials; modern methods generalize this in several different ways.
Course Contents:
Introduction: Noetherian rings, Radical of an ideal, Hilbert's Nullstellensatz, Primary decomposition of ideals in noetherian rings, Affine algebraic sets, The Hilbert basis theorem, The Zariski topology, The correspondence between algebraic sets and radical ideals, Decomposition of an algebraic set into irreducible algebraic sets, Regular functions, Regular maps, Noether normalization theorem, Dimension theory.
Affine Algebraic Varieties: Sheaves, Ringed space, Subvarieties, Category of affine algebraic varieties.
Localization: Tangent spaces to plane curves, Tangent cones to plane curves, Localization and examples, The local ring at a point on a curve, Tangent spaces to algebraic subsets of $\mathbb{A}^m$, The differential of a regular map, Tangent spaces to affine algebraic varieties, Singular locus.
The prime Spectrum of a Ring: Affine scheme, Morphism of schemes, Examples.
Projective Varieties: Projective space $\mathbb{P}^n$ and subsets, Zariski topology on $P\mathbb{P}^n$, The homogeneous co-ordinate ring of a projective variety, Regular function on a projective space, Some classical maps on projective varieties, Grassmann varieties, Rigidity theorem and Abelian varieties.

Recommended Books:

1. Dolgachev I. V., *Classical Algebraic Geometry. A modern view*, Cambridge University Press, 2012.
2. Eisenbud, D., *Commutative Algebra with a View Toward Algebraic Geometry*, Springer, 1995.
3. Milne, J. S., *Algebraic Geometry*, Allied Publishers, 2012.
4. Harstone, R., *Algebraic Geometry*, Springer, 1977.
5. Harris, J., *Algebraic Geometry,* Springer, 1995.

Course Title: Lie Groups
Course Code: MATH- 520
Prerequisites: Group Theory, Topology, Geometry
Credit Hours: 3
Course Overview:
Lie groups are continuous groups of symmetries, like the group of rotations of n-dimensional space or the group of invertible n-by-n matrices. In studying such groups we can use tools from calculus to linearise our problems, which leads us to the notion of a Lie algebra: a vector space with an antisymmetric product associated to any Lie group, which remembers everything about its algebraic structure. Here, we also focus on the basic examples of Lie group representations as well.
Course Outline:
Lie Groups : Review of groups and manifold, Smooth manifolds, Atlas and smooth structures, Smooth group operations, Examples of Lie groups, n-Spheres($\mathbb{S}^n$), Lie subgroups, Lie group homomorphism.
Matrix Lie Groups: Matrix Lie groups, Closed subgroups, Examples of general linear group and its subgroups, Quotient groups, Orthogonal and unitary groups, Triangular groups, One parameter Lie subgroups, Generalized orthogonal and Lorentzian groups, Symplectic groups, Euclidean and Poincare groups, The Heisenberg group.
Topological Properties of Lie Groups: Connected Lie groups, Path connected Lie groups, Simply connected Lie groups, Classification of connected Lie groups, Compact Lie groups, Classification of compact Lie groups, Classification of connected compact Lie groups.
Lie Groups and Lie Algebras: Exponential map, Exponential of a matrix, Matrix logarithm, The polar decomposition, The Lie algebra of a matrix Lie group, The commutator, Adjoint action and Jacobi identity, Subalgebras, Ideals, and center, Lie algebra of vector fields, Stabilizers and the center, Campbell -Hausdorff formula, Fundamental theorems of Lie theory, Complex and real forms, The Lie algebras $so(3,R)$, $su(2)$, and $sl(2,C)$, Lie group and Lie algebra homomorphisms.
Representation of Lie Groups: Representations and examples, Faithful representation, Linear action, Examples of representations.

Recommended Books:

1. Hall, B. C., *Lie Groups, Lie Algebras and Representations*, Springer, 2015.
2. Humphreys, J.E. *Introduction to Lie Algebras and Representation Theory*, Springer, 1972.
3. Knapp A. W., *Lie groups beyond an Introduction*, Springer, 1996.
4. Kirillov, A. Jr., *An introduction to Lie Groups and Lie Algebras*, Cambridge University Press, 2008.
5. Stillwell J., *Naive Lie Theory*, Springer, 2008.

Course Title: Category Theory
Course Code: MATH- 521
Prerequisites: Topology, Algebra
Credit Hours: 3
Course Overview:
Category theory serves as a unifying framework for various domains within pure mathematics, while also maintaining significant connections to logic and computer science. It is founded on the premise that numerous mathematical concepts can be elegantly unified and simplified through the use of diagrams composed of arrows, wherein these arrows signify functions of appropriate types. Furthermore, many constructions within pure mathematics can be articulated through the lens of "universal properties" associated with such diagrams, enhancing our understanding and facilitating the exploration of mathematical structures.
Course Contents:
Categories, Functors and natural transformations, Adjoints, Adjunctions via units and counits, Adjunctions via initial objects, Representables, The Yoneda lemma and its consequences. Limits, Colimits, Interactions between functors and limits, Limits in terms of representables and adjoints, Limits and colimits of presheaves, Interactions between adjoint functors and limits.

Recommended Books:

1. Agore, A., *A First Course in Category Theory*, Springer 2023.
2. Awodey, S., *Category Theory*, Oxford University Press, 2010.
3. Lawvere, F. M. and Schanuel, S. H., *Conceptual Mathematics: A First Introduction to Categories*, Cambridge University Press, 2009.
4. Leinster, T., *Basic Category Theory*, Cambridge Studies in Advanced Mathematics, 2014.
5. Pierce, B. C., *Basic Category Theory for Computer Scientists*, MIT, 1991.

CHAIRMAN
Department of Mathematics
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Lahore-Pakistan

Course Title: Fixed Point Theory
Course Code: MATH- 522
Prerequisites: Topology, Functional Analysis
Credit Hours: 3
Course Overview:

Fixed point theory is a fascinating subject, with an enormous number of applications in various fields of mathematics. This course is intended as a brief introduction to the subject with a focus on Fixed Point Theorems and its application to nonlinear differential equations, nonlinear integral equations, real and complex implicit functions and system of nonlinear equations.

Course Contents:

Preliminaries: Metric spaces, Complete metric spaces, Normed spaces, Banach spaces.
Banach Fixed Point Theorem: Applications of Banach's theorem to linear equations, differential equations and integral equations. Generalized contractions, Nonexpansive mappings, Sequential approximation techniques for nonexpansive mappings, Properties of fixed points set and minimal sets.
Multi-valued mappings: Brouwer's fixed point theorem. Best approximation theorems.

Recommended Books:

1. Agarwal, R., Meehan, M. and D. O'Regan, *Fixed Point Theory and Applications*, Cambridge University Press, 2009.

2. Bollobas, B., Fulton, W., Katok, A., at al., *Fixed Point Theory and Applications*, Cambridge University Press, 2001.

3. Dugundji, J. and Granas, A., *Fixed Point Theory*, Polish Scientific Publishers, 1982

4. Kirk, W.A. and Sims, B., *Handbook of Metric Fixed Point Theory*, Klawer Academic Publishers, 2001.

5. Pata, V., *Fixed Point Theorems and Applications*, Springer Cham, 2019.

Course Title: Knot Theory
Course Code: MATH- 523
Prerequisites: Topology, Abstract Algebra and Linear Algebra
Credit Hours: 3
Course Overview:

Knot theory is a classical subject concerning knotting and linking in 3-space. This course provides an in-depth study of knot theory and braid groups, focusing on the analysis of knots, links, and their algebraic and geometric properties. Students will explore both classical results and recent developments in the field, with a particular focus on knot invariants and their applications.

Course Contents:

Knots and Links: Basic ideas of Knots, Oriented knots, Concepts of isotopy, Reidemeister moves, Tricolorability and projections of knots, Amphicheiral and invertible knots, and the relationship between knots and braids via Alexander and Markov theorems.

Braids and Braid Groups; Braids, Closure of braids, Geometric braids, Algebraic presentations of braid groups, Positive braids, Normal forms, Word and conjugacy problems in braid groups, Braid moniod and Hilbert series.

Knot invariants: Fundamental group of knot complements, Wirtinger presentation of knot groups, Alexander matrix, elementary ideals, knot polynomials.

Polynomial related to Knots: Alexander Polynomial, Jones polynomials of knots, Related other polynomial in graph theory.

Khavanov homology: HOMFLY polynomial and introduction to Khovanov homology, Computing Khavanov homology of knots and braids.

Recommended Books:

1. Adams, C. C., *The Knot Book: An elementary introduction to the mathematical theory of knots*, American Mathematical Society, 3rd Edition, 2004.
2. Birman, J.S., *Braids, Links, and Mapping Class Groups*, Princeton University Press and University of Tokyo Press.
3. Birman, J.S. and Brendle, T., *Braids: A Survey. In Handbook of Knot Theory*, Elsevier, 2005.
4. Crowell, R.H. and Fox, R.H., *Introduction to Knot Theory*, Ginn and Company, 1963.
5. Manturov, V., *Knot Theory*, Chapman & Hall/CRC, 2004.

Course Title: Operator Theory
Course Code: MATH- 524
Prerequisites: Linear Algebra, Real and Complex Analysis, Topology
Credit Hours: 3
Course Overview: This course focuses on operator theory, emphasizing applications in mathematical physics, engineering, and functional analysis. Topics include bounded and unbounded operators, spectral theory, and special classes of operators (compact, self-adjoint, and normal). The course emphasizes both theoretical understanding and practical applications.
Course Contents:
Normed Spaces: Review of normed spaces, Banach spaces and Hilbert spaces, Definitions and examples of bounded linear operators in normed Spaces, Linear functionals in normed spaces, Spaces of bounded linear operators (functionals) and their completeness properties, Bounded linear operators on Hilbert spaces.
Dual spaces and Weak Topology: Definition of weak topology, The coarsest topology such that all continuous linear functionals remain continuous, Weak convergence, Convergence with respect to the weak topology, A sequence converges weakly if it converges pointwise under linear functionals, Dual spaces, The role of the dual space in defining weak topology, Banach-Alaoglu theorem, Compactness of the unit ball in the weak* topology of the dual space.
Operator Theory: Bounded linear operators, Continuous linear operators, Norm of a bounded linear operator, Linear transformations, Matrices of linear transformations, Relationship of operators and matrices, Self-adjoint, Normal, and unitary operators, Symmetric and self-adjoint operators, Functional calculus for self-adjoint operators, Unbounded operators, Introduction to unbounded operators, Closed operators and graph norms, Extensions of symmetric operators, The Friedrichs extension, Adjoint of an operator, Operator norm and examples, Compact linear operators on normed spaces, Compact operators and their properties, Further properties of compact linear operators, Spectral properties, Operations of bounded self adjoint linear operators, Positive operators, Square roots of positive operators. Projection operators, Spectral theorems for different classes of operators, polar-decomposition, Schmidt-decomposition, Monotone convergence theorem for self-adjoint operators.

Recommended Books:

1. Dunford, N., and Schwartz, J. T., *Linear Operators*, Wiley-Interscience, 1988.
2. Eidelman, Y., Milman, V., and Tsolomitis, A., *Functional Analysis*, American Mathematical Society, 2004.
3. Kreyszig, E., *Introductory Functional Analysis with Applications*, Wiley, 1989.
4. Putnam, C. R., *An Introduction to Operators on Hilbert Space*, Springer, 1967.
5. Rudin, W., *Functional Analysis*, McGraw-Hill, 1991.

Course Title: Banach Algebras
Course Code: MATH- 525
Prerequisites: Algebra, Real and Complex analysis, Topology
Credit Hours: 3
Course Overview:
This course introduces students to the theory of Banach algebras, focusing on their structure, properties, and applications in various branches of analysis and operator theory. The course covers topics such as the definition of Banach algebras, spectral theory, topological aspects of normed rings, Gelfand representation, and applications to commutative Banach algebras, C^*-algebras and harmonic analysis. The course emphasizes both theoretical understanding and practical applications.
Course Contents:
Banach Algebras: Review of algebraic structures, Definition of Banach algebras and examples, Normed algebras and Banach algebras, Matrix algebras, Function algebras, and operator algebras, Spectrum of an element, Definition of the spectrum and resolvent set, Spectral radius and basic properties, Spectral theorem for Banach algebras, Applications to linear operators and matrices.
Topological Rings: Definition and examples of rings, Rings with identity, Center and Ideals, The radical, Homomorphism and isomorphism of rings, Regular representations of rings, Topological rings, Topological adjunction of the identity, Rings with continuous inverse, Resolvents in a ring with continuous inverse, Topological division rings with continuous inverse, Rings with continuous quasi-inverse.
Normed Rings: Definition of a normed ring, Q-ring (or Q-algebra), Adjunction of the identity, Banach rings with identity, Continuous homomorphisms of normed rings, Regular representations of a normed ring, Basic examples C^n, $C(X)$ (continuous functions on a compact space), Algebraic properties, Closure, Associativity, Distributivity. Norm properties, Positivity, Scalability, Triangle inequality.
Topological Aspects of Normed Rings: Topology induced by norms, Convergence and continuity in the context of normed rings, Completeness and Banach rings, The relationship between normed rings and Banach algebras, Homomorphisms and ideals in normed rings, Ideals and quotient rings, Construction of quotient rings and their properties.

Recommended Books:

1. Dales, H. G. and Ülger, A., *Banach Function Algebras, Arens Regularity, and BSE Norms*, Springer, 2024.
2. Dales, H. G., *Banach Algebras and Automatic Continuity*, Oxford University Press, 2023.
3. Douglas, R. G., *Banach Algebra Techniques in Operator Theory*, Springer, 1998.
4. Naimark, M. A., *Normed Algebras*, Springer Science & Business Media, 2012.
5. Upmeier, H., *Introduction to Banach Algebras, Operators, and Harmonic Analysis*, Springer, 2020.

Course Title: Functional Analysis
Course Code: MATH-526
Prerequisites: Set Theory, Real analysis, Topology
Credit Hours: 3
Course Overview:
Functional analysis is mainly concerned with the study of vector spaces and operators acting upon them. It provides powerful tools in handling several problems in applied mathematics and theoretical physics. It is also basic for the understanding and development of very many other mathematical theories like the Theory of Partial Differential Equations and the Theory of Operators. The course covers fundamental theorems for normed and Banach spaces such as the Hahn-Banach theorem for complex vector spaces and normed spaces. Additionally, the course emphasizes the application of these concepts and techniques to various branches of mathematics, including partial differential equations, quantum mechanics, and optimization problems. The course emphasizes both theoretical understanding and practical applications.
Course Contents:
Metric Space: Continuity of metric spaces, Convergence in metric spaces, Cauchy sequences, Complete metric spaces, Baire category theorem.
Normed Spaces: Normed linear spaces, Banach spaces, Equivalent norms, Convex sets, Quotient spaces, Linear operator, Finite dimensional normed spaces, Continuous and bounded linear operators, Linear functionals, Dual Spaces.
Inner Product Spaces: Orthonormal sets and bases, Projections, Linear functionals on Hilbert spaces, Reflexivity of Hilbert spaces, The Riesz representation theorem, Annihilators and orthogonal complements, Direct decomposition.
Fundamental Theorems of Functional Analysis: Hahn-Banach theorem (for real spaces, complex spaces, and Banach spaces), the Banach-Steinhaus theorem, Stone-Weierstrass theorem, and the spectral theorem, Open mapping theorem, Closed graph theorem, Uniform boundedness principle. Consequences of Hahn-Banach theorems.
Spectral Theory: Eigenvalues and eigenvectors in infinite-dimensional spaces, Spectral theorem for bounded self-adjoint operators on Hilbert spaces, Compact operators and their spectra.
Applications: Banach fixed point theorem, Classical Banach spaces, Distance measures, Classical Banach spaces, L^p-spaces.

Recommended Books:

1. Axler, S., *Measure, Integration & Real Analysis*, Graduate Texts in Mathematics, Springer, 2020.
2. Balakrishnan, A. V., *Applied Functional Analysis*, Springer, 1981.
3. Conway, J. B., *A Course in Functional Analysis*, Springer, 2007.
4. Kreyszig, E., *Introduction to Functional Analysis with Applications*, Wiley, 1991.
5. Yosida, K., *Functional Analysis*, Springer, 1995.

Course Title: Harmonic Analysis
Course Code: MATH-527
Prerequisites: Linear Algebra, Topology, Functional Analysis
Credit Hours: 3
Course Overview:
Harmonic analysis, as a subfield of analysis, is study the quantitative properties of functions and how these change when various operators are applied. Over the past two centuries, it has become a broad discipline with applications in fields as diverse as signal processing, quantum mechanics, and neuroscience. The main ideas of the basic harmonic analysis course include Fourier transform analysis and interpolation analysis.

Course Contents:
Fourier Series and Transform on Group $\mathbb{T} = \frac{\mathbb{R}}{2\pi\mathbb{Z}}$ *and Line*: Review of some Function spaces, Fourier coefficients, Summablity in norm and homogenous Banach spaces on $\mathbb{T}$, Order of the magnitude of Fourier coefficients, Fourier series of square summable functions, Fourier coefficients of linear functionals, Convergence of Fourier series in norm, convergence and divergence of fourier series at a point, Fourier transform on L^p, and L^1, fourier transform on the line, tempered distributions and pseudo-measures, Almost periodic functions on the line, The weak-star spectrum of bounded functions, the paley-Weiner and kronecker theorems.
Fourier Analysis on Locally Compact Abelian Groups: Locally Compact Abelian Groups, The Haar measure, characters and the dual group, fourier transforms, Almost periodic functions and Bohr compactification.

Recommended Books:

1. Deitmar, A., *A First Course in Harmonic Analysis*, Springer, 2005.
2. Grafakos, L., *Modern Fourier Analysis*, Springer, 2009.
3. Lehmann, F. J., *Harmonic Analysis*, BiblioBazaa, 2008.
4. Katznelson, Y., *An Introduction to Harmonic Analysis*, Cambridge University Press, 2004.
5. Stein, E. M., and Shakarchi, R., *Fourier Analysis*, Princeton University Press, 2003.

Course Title: Advanced Topology
Course Code: MATH-528
Prerequisites: Set Theory, Real analysis
Credit Hours: 3
Course Overview:
This course introduces the concepts of topology, focusing on topological properties and their applications in various branches of analysis and geometry. Understand the basic concepts of topological spaces, continuity, and convergence. Study important topological properties such as compactness, connectivity, and separation axioms. Develop skills in applying general topology to various branches of mathematics, including analysis and algebra. The course emphasizes theoretical understanding.

Course Contents:
Topological Spaces: Review of continuous functions and homeomorphism, Topological properties and classification of topological spaces, lower limit and K- topologies on Real Line, box topology, ordered topology, Quotient topology, Metric topology, Metrizable topological spaces.
Bases and Subbases: Base and subbases, Neighborhood bases, First and second axioms of countability, Separable spaces, Lindelöf spaces.
Separation Axioms: T_0, T_1, T_2 (Hausdorff) spaces. Regular and normal spaces, Urysohn's lemma and the Tietze extension theorem,
Connectedness: Connected and disconnected spaces, Connectedness in metric and general spaces, connected subspaces of $\mathbb{R}^n$, disconnected spaces and components, Path-connected spaces, Locally connected spaces, totally disconnected spaces.
Compact Spaces: Compactness, Definition of compactness, Compact sets in metric spaces, Compactness in general topological spaces, Compact subspaces of R^n. Tychonoffs Theorem, Local compactness, Sequentially compact and paracompact spaces.
Convergence and Filters: Convergence of sequences and nets, Introduction to filters and their convergence properties. Ultrafilters and their role in topology, Function spaces with the compact-open topology,
Topological Manifolds: Basic idea of topological manifold and examples, Construction of new examples using connected sum and surgery.

Recommended Books:

1. Kelley, J. L., *General Topology*, Springer, 1955.
2. Morris, S. A., *Topology Without Tears*, University of New England, 2018.
3. Munkres, J. R., *Topology*, Prentice Hall, 2000.
4. Willard, S., *General Topology*, Addison-Wesley Publishing Co., London, 1970.
5. Simmons, G. F., *Introduction to Topology and Modern Analysis*, McGraw-Hill, 1963.

Course Title: Algebraic Topology
Course Code: MATH-529
Prerequisites: Algebra, Topology
Credit Hours: 3
Course Overview:
Algebraic topology is a fundamental part of modern mathematics, and some knowledge of this area is essential for any advanced work related to geometry, including topology itself, differential geometry, algebraic geometry, and Lie groups. It actually uses the tools of abstract algebra to study topological spaces. The basic goal is to find algebraic invariants that classify topological spaces as homeomorphisms, although in general most classifications are Homotopy equivalences.
Course Contents:
Classification of Surfaces: Orientable and non-orientable surfaces, Genus, Connected compact surfaces in 3D-space, Euler characteristics, Classification theorem for surfaces and examples, Connected sum of surfaces.
Homotopy and Fundamental Group: Paths and path connected spaces, Homotopy of continuous mappings, Homotopy of paths, Linear homotopy, Null homotopic, Homotopy classes, The fundamental group of a topological space, Fundamental group of a circle.
Applications of Fundamental Groups: Brouwer fixed-point theorem, Borsuk'Ulam theorem, Fundamental theorem of algebra, Lusternik Schnirelmann theorem, Higher homotopy groups.
Covering Spaces: Induced maps and properties of induced maps, Examples of covering spaces, Coverings of circles, Covering spaces of wedge sums of circles, Universal covering spaces and examples, Universal covering spaces and fundamental groups.
Introduction to Homology: Basic idea of boundary maps, Simplicial complex, Homology group and betti numbers, Poincare polynomial.

Recommended Books

1. Hatcher, A., *Algebraic Topology*, Cambridge University Press, 2002.
2. Kosniowski, C., *A First Course in Algebraic Topology*, Cambridge University Press, 1980.
3. Munkres, K.R., *Topology*, Prentice Hall of India, 1983.
4. Peter M. J., *A Concise Course in Algebraic Topology*, University of Chicago Press, 1999.
5. Spanier, E.H., *Algebraic Topology*, Springer, 1966.

Course Title: Topological Groups
Course Code: MATH- 530
Prerequisites: Group Theory, Topology
Credit Hours: 3
Course Overview:
This course on topological groups explores the intersection of topology and group theory, covering fundamentals, continuous operations, and examples. It examines separation axioms, metrization, and the properties of subgroups within matrix groups. Overall, it provides a deep understanding of the structures and applications of topological groups.
Course Outline:
Topological Groups: Fundamentals of topology and groups, Continuous group operations, Topological groups and examples, Subgroups and quotient groups of topological groups, Function spaces as topological groups, Transformation groups, Direct products of topological groups.
Separation Axioms and Metrizability in Topological Groups: Separation axioms for topological spaces, Hausdorff reflection of a topological group, Complete topological groups, Metrizable complete topological groups.
Subgroups of $\mathbb{R}^n$ *and Matrix Groups*: Closed subgroups, Quotient groups, Dense subgroups, General linear groups, Orthogonal and unitary groups, Triangular groups, One parameter subgroups.
Connectedness of Topological Groups: Connected topological spaces, Connected matrix groups, Totally disconnected topological groups.
Compactness and local Compactness in Topological Groups: Invariant metrics, Compact Abelian groups, Matrix representations, Specific properties of (local) compactness, The open mapping theorem, Compactness vs connectedness, compact product spaces.

Recommended Books:

1. Chandrasekharan, K., *A Course on Topology Groups*, Amer Mathematical Society , 2011.
2. Gamkrelidze, R.V., Topological Groups, Routledge, 1987.
3. Higgins, P.J. *An Introduction to Topology Groups*, Cambridge University Press, 12013.
4. Nelson G. M. *Topological Groups*, Wiley, 2010.
5. Pontriagin, L.S., *Topological Groups*, Princeton University Press, 1958.

Course Title: Homological Algebra
Course Code: MATH-531
Prerequisites: Module theory, Linear Algebra
Credit Hours: 3
Course Overview:
This course provides a rigorous introduction to Homological Algebra, focusing on key concepts such as categories, functors, and exact sequences. It explores the homology and cohomology of modules and groups, as well as the foundational tools like Ext and Tor functors. The course emphasizes projective and injective resolutions, derived functors, and their applications in advanced algebraic structures. Students will also study cohomology in the context of finite cyclic groups and Galois cohomology. By bridging theory with practical methods, this course equips students with a solid foundation in modern algebraic techniques.
Course Contents:
Preliminaries: Categories, Functors, Natural transformation, Yoneda lemma, Modules, Free modules, Tensor products, Five lemma, Adjoint isomorphism.
Exact Sequences and Homology: Exact sequences, Short exact sequences, Chain complexes, Homology groups, Spliting map and split exact sequence, Chain homotopies and null homotopic Simplicial maps.
Projective and Injective Objects: Left and right exact functors, Exact functors, Projective modules, Injective modules, Projective and injective resolutions, Flat modules.
Ext and Tor Functors: Derived functors, Hom, Ext and Tor, Double and total complexes, Yoneda product, Universal coefficient theorem.
Cohomology of Groups and Modules: Coboundary homomorphisms, Cohomology of finite cyclic groups, Dimension shifting, Shapiro lemma, Galois cohomology, Crossed homomorphism and bar complex.

Recommended Books:

1. Hilton, P.J. and Stammbach, U., *A Course in Homological Algebra*, Springer,1997.
2. Northcott, D.G, *A First Course of Homological Algebra*, Cambridge University Press, 1980.
3. Osborne, M.S, *Basic Homological Algebra*, Springer, 2000.
4. Rotman, J.J., An Introduction to Homological Algebra, Springer, 2009.
5. Weibel, C.A., *An Introduction to Homological Algebra*, Cambridge University Press, 1995.

Course Title: Commutative Algebra
Course Code: MATH-532
Prerequisites: Algebra
Credit Hours: 3
Course Overview:
Commutative algebra, formerly known as ideal theory, is the branch of algebra that studies commutative rings, their ideals, and modules over such rings. Both algebraic geometry and algebraic number theory build on commutative algebra. Commutative algebra is the main technical tool of algebraic geometry, and many results and concepts of commutative algebra are strongly related with geometrical concepts. This course gives a modern introduction to commutative algebra for students who had a first course in abstract algebra and who are familiar with the most basic notions of topology.
Course Contents:
Introduction: Basics of rings, Polynomial rings, Operations on ideals (Addition, Multiplication, Quotients, Intersection, Radical), Nill and Jacobson radicals, Extension and contraction, Hilbert's Nullstellensatz.
Ring of Fractions: Localization and local ring, Local properties, Extension and contraction of ideals in local rings, Regular local ring, Formal power series rings.
Primary Decomposition: Primary ideal, Primary decomposition, Noether normalization lemma, Going up and going down.
Finiteness Properties of Commutative Rings: Ascending chain condition and descending chain condition, Noetherian ring, Hilbert's basis theorem, Artinian ring.
Computational Algorithmic Aspects and Combinatorial Commutative Algebra: Monomial ideal, Monomial ordering, Elimination theory, Grobner bases, Buchberger's algorithm, Macualay bases theorem, Edge ideals, Cohen-Macaulay graphs, Constructions of Cohen-Macaulay graphs, Hilbert series of monomial ideals.

Recommended Books:

1. Atiyah, M. F. and Macdonald, I. G., *Introduction to Commutative Algebra*, Addison- Wesley, 1994.
2. Dummit, D. S., Foote, R. M., *Abstract Algebra*, Third Edition, John Wiley & Sons, 2003.
3. Eisenbud, D., *Commutative Algebra with a View Toward Algebraic Geometry*, Springer, 1995.
4. Kemper, G., *A Course in Commutative Algebra*, Springer, 2010.
5. Matsumura, H., *Commutative Algebra*, The Banjamin Publishing Company London, 2nd edition, 1980.

Course Title: Algebraic Graph Theory
Course Code: MATH-533
Prerequisites: Algebra
Credit Hours: 3
Course Overview:
This course provides an introduction to algebraic methods in graph theory. The goal is to explore the interplay between algebra and graph theory, using algebraic techniques to solve problems in graph theory and vice versa. Topics include spectral graph theory, automorphism groups, and combinatorial structures described by graphs. Upon successful completion of this course, students will be able to understand how to compute the spectra of graphs and their relevance to graph properties and have a solid understanding of how algebraic methods can be used to solve complex problems in graph theory and related fields.
Course Contents:
Introduction: Undirected graphs, Directed graphs, Graph models, Paths, Cycles, Trees, Spanning trees, Connectedness, Graph isomorphisms, Regular graphs, Complements, Walks, Diameter.
Graph Representation: Adjacency matrix, Incidence matrix, Laplacian matrix, Distance matrix, Properties of these matrices and their relationship with graph properties.
Graph Spectra: Eigenvalues and eigenspectra of the maticos associated with graphs, Spectrum of different classes of graphs (Complete graphs, Complete bipartite graphs, Cycles, Paths, Line graphs), Spectral properties of regular and strongly regular graphs, Spectral radius, Energy of graph, The Perron-Frobenius theorem, Kirchhoff's matrix tree theorem, Co-spectral graphs.
Graph Automorphisms: Automorphism group of a graph, Symmetry in graphs, Vertex-transitive, Edge-transitive graphs, Cayley graphs and their applications.
Algebraic Structure Graphs: Zero divisor graph, Power graph, Iteration digraphs, Commuting graph.
Applications: Graph invariants, Distance based topological indices, Degree based topological indices, Application of these invariants to predict molecular properties.

Recommended Books:

1. Beineke, L.W. and Wilson R. J., *Topics in Algebraic Graph Theory*, Cambridge University Press, 2004.
2. Biggs, N., *Algebraic Graph Theory*, Cambridge University Press, 1993.
3. Cvetkovic, D, Rowlinson, P. and Simic, S., *An Introduction to the Theory of Graph Spectra*, Cambridge University Press, 2009.
4. Chartrand, G. and Oellermann, O. R., *Applied and Algorithmic Graph Theory*, McGraw-Hill, 1993.
5. Godsil C. and Royle G. F., *Algebraic Graph Theory*, Springer, 2001.

Course Title: Introduction to Fuzzy Systems
Course Code: MATH-534
Prerequisites: Set Theory
Credit Hours: 3
Course Overview:
Fuzzy set theory was introduced by Lotfi Zadeh in 1965, as a generalization of classical set theory, for representing imprecise and vague phenomena. Zadeh, in his theory of fuzzy sets, proposed using a membership function (with a range covering the interval [0,1]) operating on the domain of all possible values. He proposed new operations for the calculus of logic and showed that fuzzy logic was a generalization of classical and Boolean logic. In artificial intelligence systems, fuzzy logic is used to imitate human reasoning and cognition. Rather than strictly binary cases of truth, fuzzy logic includes 0 and 1 as extreme cases of truth but with various intermediate degrees of truth. This course covers basic concepts of fuzzy set theory and fuzzy logic with applications.
Course Contents:
Fuzzy Set Theory : Fuzzy sets (Membership function, Cardinality, normality), Fuzzy set operations (Union, Intersection, Complementation), Distances between fuzzy sets (Hamming distance, Normalized Hamming distance, Euclidean distance, Normalized Euclidean distance), Similarity measures. Fuzzy relation and composition, Fuzzy function. Fuzzy numbers (Triangular fuzzy number, Trapezoidal fuzzy number), Arithmetic operations of fuzzy numbers. *Fuzzy control Systems*: Fuzzy logic, Linguistic variable, Fuzzy truth qualifier, Fuzzy inference, Fuzzy rules and implication, Defuzzification, Fuzzy logic controller, Configuration of fuzzy logic controller, Fuzzy expert systems.
Intuitionistic Fuzzy Sets (IFSs): Intuitionistic fuzzy sets and examples, Set operations (Union, Intersection, Complementation), Geometrical interpretations of IFSs, Distances between IFSs (Hamming distance, Normalized Hamming distance, Euclidean distance, Normalized Euclidean distance), Level cut sets, Intuitionistic fuzzy relation and composition, Intuitionistic fuzzy numbers. Applications of IFSs in medical diagnosis and pattern recognition.

Recommended Books:

1. Atanassov, K.T., *On Intuitionistic Fuzzy Sets Theory*, Springer Berlin, Heidelberg, 2012.
2. Dubois, D. and Prade, H., *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, 1980.
3. Lee, K. H., *First Course on Fuzzy Theory and Applications*, Springer Berlin, Heidelberg, 2009.
4. Klir, G. J. and Yuan, B., *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Pearson College Div., 1995.
5. Zimmermann, H. -J. *Fuzzy Set Theory and Its Application*, Studies in Fuzziness and Soft Computing, Springer, 2001.

Course Title: Fuzzy Decision Making Methods
Course Code: MATH- 535
Prerequisites: Fuzzy Set Theory and Operations Research
Credit Hours: 3
Course Overview:
Multi-criteria decision making (MCDM) is a well-known branch of decision theory that aims at achieving multiple, and usually conflicting, objectives in decision problems. Usually, these problems are also defined under high uncertain contexts. Therefore, the wide application of MCDM approaches to real-world applications demanding the treatment of uncertainty has led to the development of fuzzy MCDM as a paradigm to deal with uncertainty in MCDM problems. This course covers the theoretical and practical development of fuzzy MCDM methods.
Course Outline:
Preliminaries: Basic concepts of decision-making, Problem structuring, MCDM categories, Constructing the decision model, Role and types of criteria in MCDM problems, Normalization method, Weight assignment methods, The matrix representation of the MCDM problem, Fuzzy sets, Fuzzy linguistic set, Hesitant fuzzy set, Multipolar fuzzy sets.
Fuzzy Decision Making Methods: Fuzzy analytic hierarchy process, Fuzzy TOPSIS method, Fuzzy VIKOR method, Fuzzy WASPAS method, Fuzzy COPRAS method, Fuzzy EDAS method, TOPSIS method with multipolar fuzzy linguistic sets.
Fuzzy ELECTRE Methodology: Introduction to ELECTRE, Types of criterion used with ELECTRE, Fuzzy ELECTRE I method, Fuzzy ELECTRE II method, Fuzzy ELECTRE III method, Fuzzy ELECTRE IV method, Fuzzy ELECTRE TRi method.
Multipolar fuzzy ELECTRE methodology: Multipolar fuzzy ELECTRE (I, II, III, IV) methods, ELECTRE-I methods with multipolar fuzzy linguistic sets, Multipolar fuzzy linguistic ELECTRE-I method, Hesitant multipolar fuzzy ELECTRE-I method.
Fuzzy PROMETHEE Methods: Fuzzy PROMETHEE I method, Fuzzy PROMETHEE II method, Fuzzy PROMETHEE III method, Fuzzy PROMETHEE IV method, Multipolar fuzzy PROMETHEE (I, II) methods.

Recommended Books:

1. Akram, M. and Adeel A., *Multiple Criteria Decision Making Methods with Multi-polar Fuzzy Information*, Springer, 2023.

2. Akram, M., Shumaiza and Alcantud, J.C.R., *Multi-Criteria Decision Making Methods with Bipolar Fuzzy Sets*, Springer, 2023.

3. Pedrycz, W., Ekel, P. and Parreiras, R., *Fuzzy Multicriteria Decision-Making*, Wiley, 2010.

4. Tzeng, G.-H. and Huang, J.-J., *Multiple Attribute Decision Making*, Chapman and Hall/CRC, 2011.

5. Voskoglou, M. G., *Fuzzy Methods for Assessment and Decision Making*, Morgan Kaufmann, 2024.

Course Title: Fuzzy Graph Theory
Course Code: MATH- 536
Prerequisites: Graph Theory
Credit Hours: 3
Course Overview:
Fuzzy set theory owes its origin to the work of Zadeh. After the significant introduction of fuzzy set theory, this remarkable approach has been applied to various domains. In 1975, Rosenfeld first discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann in 1973. Fuzzy graph theory has experienced an impressive growth in recent years. Fuzzy graph theory has been important in technological development. Fuzzy graph theory has paved the way for engineers to build many rule-based expert systems. This course covers fundamental topics of group theory with applications.
Course Contents:
Preliminaries : Fuzzy sets, Fuzzy set operations, Fuzzy relation and composition.
Fuzzy Graphs: Fuzzy sets and operations, Fuzzy relation on fuzzy set, Fuzzy graphs, Operations on fuzzy graphs (Cartesian product, Composition, Union and Join), Fuzzy paths and connectedness, Fuzzy bridges and fuzzy cuts, Fuzzy forests and fuzzy trees, Fuzzy cycles, Metric in fuzzy graphs, Fuzzy bipartite graphs, Regularity of fuzzy graphs, Domination in fuzzy graphs, Automorphism of fuzzy graphs, Fuzzy planar graphs, Fuzzy competition graphs.
Bipolar Fuzzy Graphs: Bipolar fuzzy sets, Bipolar fuzzy graphs, Isomorphism of bipolar fuzzy graphs, Applications of bipolar fuzzy digraphs to decision making.
Multipolar Fuzzy Graphs: Multipolar (m-polar) fuzzy sets, 2-polar fuzzy set $\neq$ bipolar fuzzy set, m-polar fuzzy graphs, Energy of mpolar fuzzy graphs, Certain types of m-polar fuzzy graphs, Certain metrics in m-polar fuzzy graphs, m-polar fuzzy labeling graphs, Certain dominations in m-polar fuzzy graphs. Applications of m-polar fuzzy graphs.

Recommended Books:

1. Akram, M., *m-Polar Fuzzy Graphs*, Studies in Fuzziness and Soft Computing, Springer, 2019.
2. Akram, M., Sarwar, M. and Dudek, W.A., *Graphs for the Analysis of Bipolar Fuzzy Information*, Studies in Fuzziness and Soft Computing, Springer, 2021.
3. Mathew, S., Mordeson, J. and Malik, D. S., *Fuzzy Graph Theory*, Studies in Fuzziness and Soft Computing, Physica Verlag, Heidelberg, 2018.
4. Mordeson, J.N., Nair, P.S., *Fuzzy Graphs and Fuzzy Hypergraphs*, Springer, 2001.
5. Zimmermann, H. -J. *Fuzzy Set Theory and Its Application*, Studies in Fuzziness and Soft Computing, Springer, 2001.

Course Title: Fuzzy Fractional Differential Equations
Course Code: MATH- 537
Prerequisites: Fractional Differential Equations
Credit Hours: 3
Course Overview:
Fuzzy fractional differential equations are extremely useful in the modeling of scientific and engineering problems, such as population models, evaluating weapon systems, civil engineering, and modeling electro-hydraulics. This course covers theoretical and numerical aspects of the fuzzy fractional differential equations.
Course Outline:
Preliminaries: Fuzzy sets, Fuzzy set operations, Fuzzy numbers, Parametric form of fuzzy number, Operators on fuzzy numbers, Fuzzy centre, Fuzzy arithmetic.
Fuzzy Operators: H-derivative, Seikkala-derivative, SGH-derivative, π-derivative, gH-derivative, g-derivative, $\hat{D}$-derivative, Interactive derivative, gS-derivative, gr-derivative.
Fuzzy Fractional Operators: Fuzzy Riemann-Liouville fractional derivative, Caputo-type fuzzy fractional derivative, Granular Riemann-Liouville fractional derivative, gH-fractional derivative, g-fractional derivative, Fuzzy Caputo fractional integral, Fuzzy RiemannLiouville fractional integral, Generalized fuzzy fractional integral, Modified fuzzy Riemann-Liouville derivative, Fuzzy q-fractional derivative, Granular differentiability, Fuzzy Riemann integral operator.
Fuzzy Initial Value Problems (FIVPs): Fuzzy differential equations with fuzzy derivatives, Fuzzy differential inclusions, Existence of the solution, Fuzzy differential inclusions, Fuzzy fractional differential equations-Laplace transforms, Fuzzy solutions of time-fractional problems, Fuzzy impulsive fractional differential equations, Concrete solution of fractional differential equations, n-term linear fuzzy fractional linear differential equations.
Numerical Solutions of Fractional (FIVPs): Fuzzy Riemann-Liouville derivative-Fuzzy RL derivative, Fuzzy fractional differential equations-Caputo-Katugampola derivative, Fuzzy generalized Taylors expansion, Fuzzy fractional Euler method, Solution of the fuzzy fractional heat equation, Fuzzy finite difference method for solving the fuzzy fractional Poisson's equation.

Recommended Books:

1. Allahviranloo, T., *Fuzzy Fractional Differential Operators and Equations*, Springer, 2021.
2. Allahviranloo, T. and Pedrycz, W., *Soft Numerical Computing in Uncertain Dynamic Systems*, Elsevier, 2020.
3. Chakraverty, S., *Fuzzy Differential Equations and Applications for Engineers and Scientists*, Taylor and Francis Group, 2017.
4. Chakraverty, S., Tapaswini, S. and Behera, D., *Fuzzy Arbitrary Order System: Fuzzy Fractional Differential Equations and Applications*, Wiley, 2016.
5. Gomes, L.T., de Barros, L.C. and Bede, B., *Fuzzy Differential Equations in Various Approaches*, Springer, 2015.

Course Title: Introduction to Biomathematics
Course Code: MATH- 538
Prerequisites: Calculus, Linear Algebra and Differential Equations
Credit Hours: 3
Course Overview:
This course provides fundamental knowledge in applying mathematical modeling techniques to understand and predict the dynamics of biological systems. Students will learn the development, analysis, and interpretation of biomathematical models based on discrete-time and continuous-time models. Applications may include examples in population biology, ecology, infectious diseases, microbiology, and genetics.
Course Contents:
Continuous and discrete population models for single species, Models for interacting populations, Age-structured populations, Stochastic population growth, Dynamics of infectious diseases, Historical aside on epidemics, Simple epidemic models and practical applications, Modeling venereal diseases, Multi-Group model for gonorrhea and its control, Modeling of acquired immunodeficiency syndrome(AIDS) and human immunodeficiency virus (HIV), Modeling the transmission, Dynamics of the HIV, Modeling combination. Drug therapy, Delay model for HIV infection with drug therapy, Modeling the population dynamics of acquired immunity to parasite infection, Age-dependent epidemic model and threshold criterion, Existence and stability of equilibrium, The disease-free equilibrium and its stability, The endemic equilibrium and its stability, Local and global stability.

Recommended Books:

1. Brauer, F. and Chavez, C.C., *Mathematical Models in Population Biology and Epidemiology*, Springer, 2000.

2. Farkas, M., *Dynamical Models in Biology*, Academic Press, 2001.

3. Murray, J. D., *Mathematical Biology I*, Springer, 2002.

4. Murray, J. D., *Mathematical Biology II*, Springer, 2002.

5. Shier, D.R. and Wallenius K.T., *Applied Mathematical Modeling*, CRC Press, 1999.

Course Title: Mathematical Cosmology
Course Code: MATH- 539
Prerequisites: Fundamentals of Mechanics
Credit Hours: 3
Course Overview:
This course provides a comprehensive exploration of the history and development of astronomy, astrophysics, and cosmology, covering key observational discoveries and the fundamental components of the universe. It examines the cosmological principles, including both strong and weak formulations, and delves into models like Einstein's, de Sitter's, and the steady-state theory. The course also introduces the basics of high-energy physics, detailing the universe's evolution, dark matter, and challenges faced by the standard cosmological model.
Course Contents:
Historical background, Astronomy, Astrophysics and Cosmology, Observational facts about the universe and its contents, The strong and weak forms of the cosmological principle, The Einstein and de Sitter models of the universe, The Hubble law and the Friedmann models, Steady state models, The hot big bang model, The microwave background, Discussion of the significance of the start of time, Fundamentals of high energy physics, The chronology and composition of the universe, Non-baryonic dark matter, Problems of the standard model of cosmology, Structures and structure formation in the universe, The cosmological constant, The inflationary solutions to the problems of the standard model, Later developments on the inflationary models, The Bianchi classification of homogeneous spacetimes, The Kasner (mixmaster) models of the universe and the generic approach to the initial singularity, BKL- oscillations.

Recommended Books:

1. Howland, R., *An Introduction to Mathematical Cosmology*, CreateSpace Independent Publishing Platform, 2018.
2. Islam, J.N., *An Introduction to Mathematical Cosmology*, Cambridge University Press, 2002.
3. Joshi, P.S., *Global Aspects in Gravitation and Cosmology*, Oxford University Press, 1993.
4. Landsberg, P.T., Evans, D.A., *Mathematical Cosmology*, Clarendon Press, 1977.
5. Peebles, P.J.E., *Physical Cosmology*, Princeton University Press, 2015.
6. Plebanski, J., Kransinski, A., *An Introduction to General Relativity and Cosmology*, Cambridge University Press, 2006.

Course Title: Stellar Dynamics
Course Code: MATH- 540
Prerequisites: Mechanics
Credit Hours: 3
Course Overview:
This course covers the fundamental principles of static stellar structure, including equilibrium conditions and the basics of stellar modeling. It explores the Hertzsprung-Russell diagram and the processes of stellar evolution, leading to topics like gravitational collapse and the formation of degenerate stars, such as white dwarfs, neutron stars, and black holes. The course also examines star systems, including irregular and globular clusters, as well as the structure and distribution of galaxies, superclusters, and cosmic filaments. Additionally, it addresses the role of dark matter in astrophysics, focusing on its presence in galactic halos. These concepts are crucial for understanding stellar evolution, galaxy formation, and the large-scale structure of the universe, with applications in astrophysics, cosmology, and the study of dark matter.
Course Contents:
Introduction to galaxies and other stellar systems, Static stellar structure and the equilibrium conditions, Encounters and mergers of stellar systems, Spiral structure, Introduction to stellar modeling, The Hertzprung-Russel diagram and stellar evolution, Gravitational collapse and degenerate stars, White dwarfs, neutron stars and black holes, Systems of stars, Irregular and globular clusters, Galaxies superclusters and filaments, Astrophysical dark matter and galactic haloes.

Recommended Books:

1. Chandrasekhar, S., *Principles of Stellar Dynamics*, Dover Publications, 2005.
2. Ciotti, L., *Introduction to Stellar Dynamics*, Cambridge University Press, 2021.
3. Jeans, J., *Problems of Cosmology and Stellar Dynamics*, Cambridge University Press, 2009.
4. Misner, C.W., Thorne, K.S., Wheeler, J.A., *Gravitation*, Freeman Company, 2017.
5. Ogorodnikov, K.F., *Dynamics of Stellar Systems*, Elsevier Science, 2016.

Course Title: Elastodynamics
Course Code: MATH- 541
Prerequisites: Mechanics
Credit Hours: 3
Course Overview:
This course explores key concepts in elasticity theory, focusing on the equation of wave motion within elastic media. It examines phenomena such as reflection and transmission of waves at plane interfaces, the behavior of surface waves, and dispersion effects. Students will study wave propagation in infinite media and analyze half-space problems, which involve boundaries that extend infinitely in one direction. The course also addresses diffraction and scattering caused by irregular structures, providing a comprehensive understanding of how waves interact with materials.
Course Contents:
Some topics in theory of elasticity, Uniqueness of Solution, Equation of wave motion in an elastic medium, Wave motion with polar symmetry, Reflection and transmission at a plane interface, Reflection of SH-waves, Reflection of P-waves, Reflection of SV-waves, Reflection and partition of energy at a free space, Plane harmonic waves, Surface waves, Dispersion, Waves in infinite media, Half-space problems, Propagation and reflection of plane waves in half-space, Waves at oblique incidence, Waves at grazing incidence, Diffraction and scattering due to irregular structures, Scattering of waves by cavities.

Recommended Books:

1. Achenbach, J.D., *Wave Propagation in Elastic Solids*, North-Holland Publishing Company, 2012.
2. Dieulesant, D., Royer, F., *Elastic Waves in Solid*, John Wiley & Sons, 1999.
3. Graff, K.F., *Wave Motion in Elastic Solids*, Clarendon Press, 2012.
4. Hudson, J.A., *The Excitation and Propagation of Elastic Waves*, Cambridge University Press, 1985.
5. Kausel, E. *Fundamental Solutions in Elastodynamics*, Cambridge University Press, 2011.

Course Title: General Relativity
Course Code: MATH- 542
Prerequisites: Tensor Calculus
Credit Hours: 3
Course Overview:
This course provides a thorough introduction to the principles of Special Relativity, starting with its original formulation and covering essential topics like velocity addition in three dimensions and the four-vector formalism. It includes an examination of the Poincaré group and the concept of the null cone, as well as a review of electromagnetism and the four-vector formulation of Maxwell's equations. The course transitions to General Relativity, detailing the Einstein field equations, the stress-energy momentum tensor, and key solutions such as the vacuum Einstein equations and the Schwarzschild solution. Students will explore Birkhoff's theorem, the Reissner-Nordström solution, and the Kerr and Kerr-Newman solutions, along with the Newtonian limit of relativity and relativistic equations of motion. The course also addresses classical tests of relativity and their current status, the Schwarzschild interior solution, and introduces concepts of linearized gravity and gravitational waves.

Course Contents:
Formulation of special relativity, Velocity addition in 3-d formulation, 4-Vector formalism, Poincaré group, The null cone, Review of electromagnetism, 4-Vector formulation of Maxwell's equations, Special relativity with small accelerations, The principles of general relativity, The Einstein field equations, The stress-energy momentum tensor, The vacuum Einstein equations and the Schwarzschild solution, Birkhoff's theorem, The Reissner-Nordström solution and the generalized Birkhoff's theorem, The Kerr and the Kerr-Newmann solution, The newtonian limit of relativity, The Schwarzschild exterior solution and relativistic equations of motion, The classical tests of Relativity and their current status, The Schwarzschild interior solution, Linearized gravity and gravitational waves, Foliations.

Recommended Books:

1. Misner, C.W., Thorne, K.S., Wheeler, J.A., *Gravitation*, W. H. Freeman & Company, 2017.

2. Plebanski, J., Krasinski, A., *An Introduction to General Relativity and Cosmology*, Cambridge University Press, 2024.

3. Qadir, A., Relativity: *An Introduction to the Special Theory*, World Scientific, 1989.

4. Qadir, A., *Einstein's General Theory of Relativity*, Cambridge Scholars Publishing, 2020.

5. Stephani, H., *General Relativity: An Introduction to the Theory of Gravitational Field,* Cambridge University Press, 1990.

6. Wald, R.M., *General Relativity*, The University of Chicago Press, 2010.

Course Title: Mathematical Plasma Dynamics
Course Code: MATH- 543
Prerequisites: Mechanics
Credit Hours: 3
Course Overview:
This course offers an introduction to plasma physics, highlighting its natural occurrences and applications. It covers essential concepts such as temperature, Debye shielding, and plasma criteria. The course explores single-particle motion, focusing on charged particles' behavior in electric and magnetic fields, including adiabatic invariants. It examines plasmas as fluids, linking plasma behavior to electromagnetic theory and fluid dynamics, including equations of motion and continuity. Students will learn about various types of waves in plasmas, including group velocity, plasma oscillations, and distinctions between ion and electron waves.
Course Contents:
Introduction to plasma physics, Occurrence of plasmas in nature, Concept of temperature, Debye shielding, Criteria for plasmas, Applications of plasma physics, Single particle motion, motion of charged particles in uniform E and B fields, Adiabatic invariants, Plasmas as fluids, Relation of plasma physics to ordinary electromagnetic, The fluid equation of motion, Equation of continuity, The complete set of fluid equations, Plasma approximations, Waves in plasmas, Representation of waves, Group velocity, Plasma oscillations, Electron plasma waves, Sound waves, Ion waves, Validity of plasma approximation, Comparison of ion wave and electron wave, Electrostatic electron oscillations perpendicular to B, Electrostatic ion waves perpendicular to B, The lower hybrid frequency, EM waves with $B_0 = 0$, EM waves perpendicular to B_0, Cutoffs and resonances, EM waves parallel to B_0, Hydromagnetic waves, Magnetosonic waves.

Recommended Books:

1. Bittoncourt, J.A., *Fundamentals of Plasma Physics*, Springer-Verlag, 2004.
2. Chen, F.F., *Introduction to Plasma Physics and Controlled Fusion*, Plenum Press, 2013.
3. Goldston, R.J., Rutherford, P.H., *Introduction to Plasma Physics*, IOP Publishing Limited, 2020.
4. Rionero, S., Toscani, G., Boffi. V., *Mathematical Aspects of Fluid and Plasma Dynamics*, Springer, 2014.
5. Sentis, R., *Mathematical Models and Methods for Plasma Physics*, Birkhuser Cham, 2014.

Course Title: Magnetohydrodynamics
Course Code: MATH- 544
Prerequisites: Electrodynamics
Credit Hours: 3
Course Overview:

This course explores the fundamental equations of electrodynamics, fluid dynamics, and magnetohydrodynamics (MHD), focusing on the behavior of electrically conducting fluids. It covers Ohm's law, the motion of viscous fluids with linear current flow, and steady-state motion along magnetic fields. The course delves into the characteristics of waves, such as their propagation, distortion, and discontinuities, including simple and shock waves. It also addresses the stability and structure of shock waves, particularly in relativistic MHD, and investigates scenarios like the piston problem and oblique shock waves.

Course Contents:

Equations of electrodynamics, Equations of Fluid Dynamics, Ohm's law, Equations of magnetohydrodynamics, Motion of a viscous electrically conducting fluid with linear current flow, Steady state motion along a magnetic field, Wave motion of an ideal fluid. Magneto-sonic waves, Alfvén's waves, Damping and excitation of MHD waves, Characteristics lines and surfaces, Kinds of simple waves, Distortion of the profile of a simple wave, Discontinuities, Simple and shock waves in relativistic magnetohydrodynamics, Stability and structure of shock waves, Discontinuities in various quantities, Piston problem, Oblique shock waves.

Recommended Books:

1. Anderson, J.E., *Magnetohydrodynamics, Shock Waves*, MIT Press, 2003.
2. Anile, A.M., *Relativistic Fluids and Magneto-Fluids with Applications in Astrophysics and Plasma Physics*, Cambridge University Press, 2005.
3. Cowling, T.G., *Magnetohydrodynamics*, Interscience Publishers, 1963.
4. Kulikowsky, A.G., Lyabinov, A.G., *Magnetohydrodynamics*, A.Weslev, 1965.
5. Punsley, B., *Black Hole Gravitohyrodynamics*, Springer-Verlag, 2013.

Course Title: Fluid Dynamics
Course Code: MATH- 545
Prerequisites: Vector Analysis and Mechanics
Credit Hours: 3
Course Overview:
This course emphasizes the mathematical theory of fluid dynamics, including related physical quantities, conservation laws, and various mathematical representations. Its purpose is to enhance students' understanding of fluid flow phenomena and help them to solve real-world engineering problems in multiple areas. It also introduces students to the mathematical description of fluid flow and its solutions.
Course Contents:
Review of basic concepts, Continuum hypothesis, Viscosity, Types of fluid and flow, Stress, Newtons's law of viscosity, Methods to describe the fluid motion (Lagrangian and Eulerian), Material derivative, Continuity equation, Streamlines, Velocity potential, Circulation, Euler's equation of motion, Navier-Stokes' equations, Parallel flow, Couette flows, Poiseuille flow, Flow between two porous plates, Flow over an inclined plate, Hagen-Poiseuille flow, Flows between two coaxial cylinders, Unsteady flow, Stokes' first and second problem, Dynamical similarity and Reynolds number, Boundary layer concept and governing equations, Turbulent flow, Time averaging and its principles, Reynold's equations of turbulent motion, Stability and equilibrium problems.

Recommended Books:

1. Batchelor, G. K., *An Introduction to Fluid Dynamics*, Cambridge University Press, 2000.
2. Curie, I. G., *Fundamentals of Mechanics of Fluids*, CRC Press, 2012.
3. Fox, R. W., McDonald, A. T. and Mitchell, J. W., *Fox and McDonald's Introduction to Fluid Mechanics*, Wiley, 2020.
4. Meyer, R. E., *Introduction to Mathematical Fluid Dynamics*, Dover Publication, 2010.
5. Schlichting, H. and Gersten, K., *Boundary-Layer Theory*, Springer, 2017.

Course Title: Biofluid Mechanics
Course Code: MATH- 546
Prerequisites: Fluid Dynamics
Credit Hours: 3
Course Overview:
Biofluid dynamics may be considered as the discipline of biological engineering or biomedical engineering in which the fundamental principles of fluid dynamics are used to explain the mechanisms of biological flows and their interrelationships with physiological processes, in health and in diseases/disorder. The need for engineers with integrated multidisciplinary knowledge is expected to grow along with the rapid advances in biomedical science and technology. This course elaborates on the application of fluid mechanics principles to major human organ systems. The focus of the course is on the integration of various fluid mechanics concepts to address relevant problems of the human bodys systems.
Course Outline:
Review of basic fluid mechanics: Biorheology, Constitutive equations, Non-Newtonian fluid models. *Circulatory biofluid mechanics*: Circulatory system physiology, Function of circulatory system, circulation in heart, blood and lymphatic vessels, Blood properties, Hemorheology.
Models for blood flow: Steady flow in tubes, Pulsatile flow in a rigid tube, Pulsatile flow in an elastic tube, Wave propagation in elastic tubes.
Applications in circulatory system: Blood flow dynamics in arteries and veins, Flow in specific vessels and arteries, Heart-valve hemodynamics, Diseases related to obstruction of blood flow, Artificial heart valves and stents.
Synovial fluid in joints: Synovial joints physiology, Function of synovial fluid, Diseases, Synovial fluid properties and rheology, Lubrication theory, Application for synovial fluid flow, Arthritis, Knee and Hip injury.
Respiratory biofluid mechanics: Respiratory system physiology, Alveolar ventilation, Air flow in the lungs, Mechanics of breathing, Gas exchange and transport, Flow and pressure measurement techniques in human body.

Recommended Books:

1. Goyal, M. R., Biofluid Dynamics of Human Body Systems, Apple Academic Press, 2013.
2. Kleinstreuer, C., Biofluid Dynamics, CRC Press, 2006.
3. Mazumdar, J., Biofluid Mechanics, World Scientific Publishing Company, 1992.
4. Ostadfar, A., Biofluid Mechanics, Academic Press, 2016.
5. Waite, L, Applied Biofluid Mechanics, McGraw Hill, 2007.

Course Title: Classical Field Theory
Course Code: MATH- 547
Prerequisites: Mechanics
Credit Hours: 3
Course Overview:
This course offers an extensive review of continuum mechanics, focusing on both solid and fluid media. It examines constitutive equations and conservation laws, introducing the concept of fields in a four-dimensional context, particularly through the stress-energy momentum tensor. Students will explore scalar fields, including linear scalar fields governed by the Klein-Gordon equation and non-linear scalar fields related to fluids. The course also covers vector fields, linear massless scalar fields, and the Maxwell equations, along with the electromagnetic energy-momentum tensor and the properties of electromagnetic waves, including diffraction. Advanced and retarded potentials, multipole expansions of radiation fields, and the properties of massive and massless tensor fields are discussed, culminating in the Einstein field equations and gravitational waves.

Course Contents:
Review of continuum mechanics, Solid and fluid media, Constitutive equations and conservation equations, The concept of a field, The four dimensional formulation of fields and the stress-energy momentum tensor, The scalar field, Linear scalar fields and the Klein- Gordon equation, Non-linear scalar fields and fluids, The vector field. Linear massless scalar fields and the Maxwell field equations, The electromagnetic energy-momentum tensor, Electromagnetic waves, Diffraction of waves, Advanced and retarded potentials, Multipole expansion of the radiation field, The massive vector field, The tensor field, The massless tensor field and the Einstein field equations, Gravitational waves, The massive tensor field, Coupled field equations.

Recommended Books:

1. Carroll, S.M., *An Introduction to General Relativity: Spacetime and Geometry*, Addison Wesley, 2019.

2. Jackson, J.D., *Classical Electrodynamics*, John Wiley & Sons, 1999.

3. Landau, L.D, Lifshitz, M., *The Classical Theory of Fields,* Pergamon Press, 1980.

4. Misner, C.W., Thorne, K.S., Wheeler, J.A., *Gravitation,* W.H. Freeman & Company, 2017.

5. Scipio, L.A., *Principles of Continua with Applications*, John Wiley, 1969.

Course Title: Quantum Field Theory
Course Code: MATH- 548
Prerequisites: Quantum Mechanics
Credit Hours: 3
Course Overview:
This course delves into classical and quantum field theory, beginning with Lagrangian mechanics and the variational principle. It covers vibrating strings, Lorentz transformations, the Lorentz group, and its representations, along with classical scalar fields, the Klein-Gordon equation, and complex scalar fields. Students will study the energy-momentum tensor, the electromagnetic field, Maxwell's equations, and the Dirac equation for spinor fields. The course also explores symmetries, conservation laws, and Noether's theorem, as well as field quantization methods, including canonical and scalar field quantization, and the particle interpretation in quantum field theory. It addresses interacting quantum fields, perturbation theory, the S-matrix, cross sections, particle decay, and higher-order interactions using Wick's theorem and Feynman diagrams. Renormalization techniques, including mass, coupling constant, and field renormalization, are also discussed.
Course Contents:
Classical field theory, Lagrangian mechanics, variational principle, Vibrating stings, Classical field theory, Lorentz transformations, Lorentz group, Representations of Lorentz group, Classical scalar fields, Klein-Gordon equation, Complex scalar fields, Energy-momentum tensor, Electromagnetic field, Maxwell's equations, Spinor field, Dirac equation, Symmetries and conservation laws, Noether's theorem, Translation invariance, Quantization of fields, Canonical quantization of fields, Quantization of scalar fields, Particle interpretation of quantum field theory, Normal ordering, Non-hermitian fields, Interacting quantum fields, Interacting fields, Perturbation theory, Time ordering, S-matrix, Cross section, Decay rate of an unstable particle, Higher order perturbation theory, Wick's theorem second order perturbation theory, Feynman rules and diagrams, Renormalization, Mass renormalization, Coupling constant renormalization, Field renormalization.

Recommended Books:

1. Kaku, M., *Quantum Field Theory*, Oxford University Press, 2012.
2. Mandl, F., Shaw, G., *Quantum Field Theory*, Wiley, 2010.
3. Ryder, L.H., *Quantum Field Theory*, Cambridge University Press, 1999.
4. Ticciati, R., *Quantum Field Theory for Mathematicians*, Cambridge University Press, 2009.
5. Weinberg, S., *The Quantum Theory of Fields*, Cambridge University Press, 2005.

Course Title: Classical Electrodynamics
Course Code: MATH- 549
Prerequisites: Mechanics
Credit Hours: 3
Course Overview:
This course provides a comprehensive study of electromagnetic theory, focusing on Maxwell's equations and the electromagnetic wave equation. It covers boundary conditions and explores wave behavior in both conducting and non-conducting media, including reflection and polarization. The course examines energy density, energy flux, and the Lorentz force. Students will learn about waveguides, resonators, and the behavior of spherical and cylindrical waves. It also introduces the inhomogeneous wave equation, retarded potentials, and Lenard-Wiechert potentials. The course further analyzes fields produced by uniformly moving charges, radiation from groups of moving charges, oscillating dipoles, and accelerated point charges.
Course Contents:
Maxwell's equations, Electromagnetic wave equation, Boundary conditions, Waves in conducting and non-conducting media, Reflection and polarization, Energy density and energy flux, Lorentz formula, Wave guides and cavity, Resonators, Spherical and cylindrical waves, Inhomogeneous wave equation, Retarded potentials, Lenard Wiechart potentials, Field of uniformly moving point charge, Radiation from a group of moving charges, Field of oscillating dipole, Field of an accelerated point charge.
Recommended Books:

1. Corson, D., Lorrain, P., *Introduction to Electromagnetic Fields and Waves*, W. H. Freeman & Company, 2013.
2. Ghosh, S.N., *Electromagnetic Theory and Wave Propagation*, Narosa Publishing House, 2002.
3. Jackson, J.D., *Classical Electrodynamics*, John Wiley & Sons, 1999.
4. Penofskey, W.K.H., Philips, M., *Classical Electricity and Magnetism*, Addison- Wesley, 2012.
5. Reitz, J.R., Milford, F.J., Christy, R.W., *Foundations of Electromagnetic Theory*, Addison-Wesley, 2009.

Course Title: Non-Newtonian Fluid Mechanics
Course Code: MATH- 550
Prerequisites: Vector Analysis and Mechanics
Credit Hours: 3
Course Overview:
This course provides an in-depth study of non-Newtonian fluids, including their classification and behavior under various conditions. It covers rheological models such as time-independent, thixotropic, and viscoelastic fluids, and examines variable viscosity and cross viscosity fluids. It explores fundamental equations of motion, focusing on linear viscoelastic liquids and specific flow scenarios like Couette, Poiseuille, and oscillatory flows. The course also introduces boundary layer theory, including similarity solutions, turbulent boundary layers, and stability analysis.
Course Contents:
Classification of non-Newtonian fluids, Rheological formulae (Time-independent fluids, thixotropic fluids and viscoelastic fluids), Variable viscosity fluids, Cross viscosity fluids, The deformation rate, Viscoelastic equation, Materials with short memories, Time dependent viscosity, The Rivlin-Ericksen fluid, Basic equations of motion in rheological models, The linear viscoelastic liquid, Couette flow, Poiseuille flows, The current semi-infinite field, Axial oscillatory tube flow, Angular oscillatory motion, Periodic transients, Basic equations in boundary layer theory, Orders of magnitude, Truncated solutions for viscoelastic flow, Similarity solutions, Turbulent boundary layers, Stability analysis.

Recommended Books:

1. Bird, R. B., Armstrong R. C., Hassager, O., *Dynamics of Polymeric Liquids*, John Wiley & Sons, 1987.

2. Bird, R. B., Stewart, W. E., Lightfoot, E.N., *Transport Phenomena*, John Wiley & Sons, Inc., 2002.

3. Harris, J., *Theology and Non-Newtonian Flow*, Longman Inc., 1977.

4. Hughes, W. F., Brighton, J. A., *Fluid Dynamics*, McGraw Hill, 2004.

5. Schowalter, W. R., *Mechanics of Non-Newtonian Fluids*, Pergamon Press, 1978.

Course Title: Relativistic Theory of Black Holes
Course Code: MATH- 551
Prerequisites: General Relativity
Credit Hours: 3
Course Overview:
This course offers an in-depth study of black holes, including their singularities, event horizons, and the use of coordinate systems that extend through these horizons. It explores advanced concepts like Kruskal and Carter-Penrose diagrams for Schwarzschild and Reissner-Nordström geometries, as well as the maximal extension and Einstein-Rosen bridges (wormholes). The course also covers theoretical topics such as the no-hair and cosmic censorship conjectures, gravitational forces around black holes, and black hole thermodynamics. It discusses observational evidence for black holes, central black holes in galaxies, and the Kaluza-Klein theory. Quantum gravity issues, quantization in curved spacetime, Hawking radiation, and related symmetries like isometries and homotheties are also explored, along with the study of gravastars.
Course Contents:
Black holes, Coordinate and essential singularities, Horizons, Coordinates passing through horizons, The Kruskal and the Carter-Penrose (CP) diagrams for the Schwarzschild geometry, The maximal extension, The Einstein-Rosen bridge, Wormholes, The CP diagram for the RN metric, The no-hair and cosmic censorship hypotheses, Gravitational forces about black holes, Black hole thermodynamics, Observational status and central black holes, Kaluza-Klein theory, Problems of quantum gravity, Quantization in curved space backgrounds and Hawking radiations, Isometries. Homotheties and their significance in relativity, Gravastars.

Recommended Books:

1. Carroll, S.M., *An Introduction to General Relativity: Spacetime and Geometry*, Addison Wesley, 2013.
2. Misner, C.W., Thorne, K.S., Wheeler, J.A., *Gravitation*, W. H. Freeman and Company, 2017.
3. Plebanski, J., Kransinski, A., *An Introduction to General Relativity and Cosmology*, Cambridge University Press, 2024.
4. Stephani, H., *General Relativity: An Introduction to the Theory of Gravitational Field,* Cambridge University Press, 1990.
5. Wald, R.M., *General Relativity*, The University of Chicago Press, 2010.

Course Title: Fractional Differential Equations
Course Code: MATH-552
Prerequisites: Ordinary Differential Equations, Partial Differential Equations
Credit Hours: 3
Course Overview:
Fractional differential equations are generalizations of ordinary differential equations to an arbitrary (noninteger) order. Fractional differential equations have attracted considerable interest because of their ability to model complex phenomena. These equations capture nonlocal relations in space and time with power-law memory kernels. Due to the extensive applications of fractional differential equations in engineering and science, research in this area has grown significantly all around the world. This course covers theoretical and numerical aspects of the fractional differential equations.
Course Contents:
Preliminaries: Gamma function and its properties, Beta function, Contour integral representation.
Fractional calculus: Fractional integral, Riemann-Liouville fractional derivative, Caputo fractional derivative, Mittag-Leffler function, Fractional order evolution equation, Laplace transform in fractional calculus.
Fractional ordinary differential equations: Series solutions, Existence and uniqueness theorems, Fractional partial differential equations, initial and boundary value problems, Nonlinear fractional differential equations, Adomian decomposition method, Homotopy analysis method, Fractional models in biology and epidemiology.
Numerical method for fractional ODEs: Backward Euler method, Second-order backward difference methods, Adams-Bashforth method, Convolution, quadrature, Error estimate.
Numerical methods for fractional PDEs: Finite difference method for solving fractional diffusion equation, Spectral methods for fractional differential equations, High-order approximation of Caputo fractional derivatives, Adaptive and high-order scheme for non-integer order differential equations, Stability and convergence in numerical methods, Applications of numerical methods to nonlinear fractional differential equations, Fractional order systems in control theory, Modelling and simulation tools for fractional systems, Simulink, Mathematica, Applications in control of dynamic systems.

Recommended Books:

1. Balachandran, K., An Introduction to Fractional Differential Equations, Springer Singapore, 2023.
2. Baleanu, D., Diethelm, K., Scalas, E., & Trujillo, J. J., Fractional Calculus, World Scientific Pub. Co, Inc., 2014.
3. Guo, B., Pu, X. and Huang, F., Fractional Partial Differential Equations and Their Numerical Solutions, World Scientific Pub. Co, Inc., 2015.
4. Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J., Theory and Applications of Fractional Differential Equations, Elsevier Science, 2006.
5. Podlubny, I., Fractional Differential Equations, Academic Press, 1998.

Course Title: Quantum Information Theory
Course Code: MATH-553
Prerequisites: Linear Algebra, Functional Analysis
Credit Hours: 3
Course Overview: This course focuses on the fundamentals of quantum information theory. It deals with how information is stored, manipulated, and communicated in a quantum system. More precisely, quantum information theory extends the core principles of classical information theory using quantum mechanics. It also illustrates the most recent developments in quantum information and the possible implementations of new protocols for transmission and manipulation of information.
Course Contents:
Review of quantum mechanics: Postulates of quantum mechanics, Quantum states and observables, Dirac notation, Projective measurements, Density operator representation, Pure and mixed states, Tensor products.
Fundamentals of quantum information: Generalized measurements (CP maps, POVMs), No-cloning theorem and it implications, Quantum state discrimination.
Quantification of classical information: Shannon entropy, Relative entropy, Conditional entropy, Mutual information, Classical algorithms for data compression, Classical error detection and correction techniques.
Quantification of quantum information: Von Neumann entropy, Relation between von Neumann entropy and quantum information, Quantum mutual information and its statistical interpretation, Holevo's bound.
Communication of quantum information: Quantum communication channel, Capacity of a quantum channel, Quantum data compression, Fidelity, Helstrom's measurement, Quantum communication protocols, Super-dense coding, Teleportation, Entanglement swapping, Instantaneous transfer of information, Quantum key distribution, Entanglement, Entanglement witnesses, Peres- Horodecki criterion, Various mathematical measures of entanglement, Noisy quantum channels, Phase damping, Amplitude damping and depolarization channels, Kraus operators representation for noisy quantum channels, Quantum error detection and correction methods for noisy quantum channels.

Recommended Books:

1. Liboff, R., *Introductory Quantum Mechanics*, 4th Edition, Pearson, 2011.
2. Nielson, M. A. and Chuang, I. L., *Quantum Computation and Quantum Information*, 10th Edition, Cambridge University Press, 2010.
3. Steeb, W. and Hardy, Y., *Problems and Solutions in Quantum Computing and Quantum Information*, 3rd Edition, World Scientific Publishing, 2011.
4. Vedral, V., *Introduction to Quantum Information Science*, 1st Edition, Oxford University Press, 2007.
5. Watrous, J., *The Theory of Quantum Information*, 1st Edition, Cambridge University Press, 2018.

Course Title: Quantum Computing
Course Code: MATH-554
Prerequisites: Linear Algebra, Functional Analysis, Elementary Statistics, Basic Programming
Credit Hours: 3
Course Overview:
This course provides an introduction to the theory and practice of quantum computing. The course deals with the designing of quantum algorithms and implementing them on quantum computers. These quantum algorithms demonstrate computational superiority over classical algorithms for several complex problems in physics, mathematics, optimization and machine learning.
Course Contents:
Introduction to classical computing: Boolean logic, classical logical gates and circuits, deterministic and probabilistic computing models.
Introduction to quantum mechanics: Basic principles and postulates of quantum mechanics, Hilbert space, Quantum states, Evolution of quantum state, Quantum measurement, Superposition, Operator function.
Fundamentals of quantum computing: Pure and mixed quantum states, Bloch sphere representation of pure and mixed quantum states, Density matrix representation, Quantum entanglement, EPR states and Bells inequality, Measures of quantum entanglement, No-cloning theorem, Positive operator valued measurements (POVM), Single and multi-qubit quantum operation, Controlled operations, Quantum circuits.
Applications of quantum computing: Deutsch algorithm, Deutsch-Jozsa algorithm, Simon's problem, Quantum phase estimation algorithm, Quantum Fourier transform and eigenvalue problem, Quantum search algorithm, Quantum cryptography and Shor's algorithm, Complexities of the quantum algorithms and quantum supremacy, Quantum error detection and correction, Implementation of quantum algorithms using quantum computing packages such as Qiskit and Pennylane.

Recommended Books:

1. Bouwmester, P. and Ekert, A. K. and Zeilinger, A., *The Physics of Quantum Information*, 1st Edition, Springer, 2010.

2. Brylinsky, R. K. and Chen, G., *Mathematics of Quantum Computation*, 1st Edition, CRC Press, 2002.

3. Kaye, P., Laflamme, R. and Mosca, M., *An Introduction to Quantum Computing*,1st Edition, Oxford University Press, 2007.

4. Nielson, M. A. and Chuang, I. L., *Quantum Computation and Quantum Information*, 10th Edition, Cambridge University Press, 2010.

5. Rieffel, E. G. and Polak, W. H., *Quantum Computing: A Gentle Introduction*, 1st Edition, The MIT Press, 2014.

Course Title: Optimization Techniques
Course Code: MATH- 555
Prerequisites: Linear Algebra
Credit Hours: 3
Course Overview:
The theory of optimization is a significant field of research in mathematics. This course focuses on some very useful optimization techniques used in diverse fields of science and engineering. The students will be able to formulate optimization problems. This course includes algorithms to determine optimal solutions of network models. The scope of optimization techniques in this course cover linear programming, integer programming and dynamic programming. The understanding and utilization of this course requires usage of computer packages to apply these techniques.
Course Outline:
Introduction: What is optimization? Exposure to classes of optimization problems(Linear/nonlinear, continuous/discrete, constrained/unconstrained), Mathematical formulation of optimization problems.
Linear Programming (LP): LP model in standard form, Algebraic determination of basic feasible solutions, Simplex method, Dual simplex method.
Network Models: Basic concepts and definitions, Applications of network models, Minimal spanning tree algorithm, Shortest-route algorithms for networks (Dijkstra's algorithm, Floyd's algorithm), Maximal-flow algorithm.
Integer Programming: Applications of integer programming, Pure integer programming, Mixed integer programming, Cutting-plane algorithms, Mixed cut, Branch-and-bound method.
Dynamic Programming: Elements of dynamic programming, Problem of dimensionality, Optimization using dynamic programming.

Recommended Books:

1. Chong, E. K. P. and Zak, S. H. An Introduction to Optimization, 4th edition, Wiley, 2013.
2. Gillett, B. E. *Introduction to Operations Research*, Tata McGraw Hill Publishing Company Ltd., New Delhi.
3. Harvey, C. M. *Operations Research*, North Holland, New Delhi, 1979.
4. Hillier F. S. and Lieberman, G. J. *Operations Research*, CBS Publishers and Distributors, New Delhi, 1974.
5. Taha, H. A. *Operations Research - An Introduction*, Pearson Education, 2018.

Course Title: Theory of Spline Functions
Course Code: MATH-556
Prerequisites: Calculus, Linear Algebra
Credit Hours: 3
Course Overview: This course deals with parametric curves and spline functions. The main focus of the course is to analyze different form of curves and spline functions. Spline interpolation is a widely used technique for constructing smooth curves through a set of data points, offering several advantages in terms of accuracy and computational efficiency. Its primary objective is to create a piecewise polynomial function that maintains continuity and smoothness, particularly useful in applications where data points are discrete.
Course Contents:
Parametric curves: Algebraic form, Hermite form, Control point form, Bernstein Bezier form and their matrix forms, Algorithm to compute Bernstein Bezier form, Properties of Bernstein Bezier form (Convex Hull property, Affine invariance property, Variation diminishing property), Rational quadratic form, Rational cubic form, Tensor product surface.
Spline functions: Cubic Hermite spline, Natural spline, Periodic splines on uniform mesh, Representation of spline and its different forms, End conditions of cubic splines, Clamped conditions, Natural conditions, 2nd derivative conditions, Periodic conditions, Not a knot conditions, Natural spline and periodic spline in terms of polynomials and truncated power functions, Odd degree spline, Existence theorem, Existence and uniqueness of natural and periodic spline, B-spline with uniform integer knots.

Recommended Books:

1. Bartels, R. H., John C. Bealty, and Barsky, B. A., *An Introduction to Spline for use in Computer Graphics and Geometric Modeling*, Morgan Kaufmann Publisher, 2006.

2. Carl de Boor, *A Practical Guide to Splines*, Springer Verlag, 2001.

3. Farin, G. *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide*, Academic Press. Inc., 5th edition, 2002.

4. Faux, I. D. *Computational Geometry for Design and Manufacture*, Ellis Horwood, 1979.

5. Schumaker, L. L., *Spline Functions: Basic Theory*, John Wiley and Sons, 1993.

Course Title: Minimal Surfaces
Course Code: MATH- 557
Prerequisites: Differential Geometry
Credit Hours: 3
Course Overview:
This course introduces minimal surfaces in differential geometry, based on the work of Euler and Lagrange, and examines their various applications in geometric analysis. It covers several approaches to minimal surfaces, including variational methods, the mean curvature approach, and complex analysis, particularly through the Weierstrass-Enneper parameterization, with applications across diverse fields of study.
Course Outline:
Theory of curves and surfaces: Parametrization of curves in Euclidean space, The moving triad of unit tangent, Unit normal, Unit binormal, Corresponding planes, Curvature and torsion, Serret-Frenet apparatus, Spherical indicatrices in Euclidean and non-Euclidean spaces, Parametrization of surfaces in Euclidean space, Regular surfaces, Tangent planes, Normal lines, Singular points, Coordinate curves on a surface, First and second fundamental forms, Geometric definition of area, Normal curvature, Principal curvature, Gaussian and mean curvature, Shape operator, Theorema Egregium, Gauss-Weingarten and Mainardi-Codazzi equations, The fundamental theorem of surface theory, Geodesic curvature, and geodesics.
Minimal surfaces: Minimal surfaces in Euclidean and non-Euclidean spaces, Minimal surface equation, Ruled minimal surfaces, Basic minimal surface properties, Two-dimensional minimal surfaces in three-dimensional space. Euler-Lagrange equation, Isothermal coordinates, Harmonic functions, Holomorphic and meromorphic functions, Weierstrass-Enneper parameterization for minimal surfaces, Hodographic coordinates, Constant mean curvature surfaces, Stable and unstable minimal surfaces, Examples such as the helicoid, catenoid, conoid, and Ennepers surface along with other notable surfaces. *Polynomial Minimal Surfaces*: Dirichlet's integral and Plateau-Bézier problem, Quasi-harmonic surfaces, Polynomial minimal surfaces (minimal surfaces admitting Bézier form).

Recommended Books:

1. Carmo, M. P., *Differential Geometry of Curves and Surfaces*, Dover Publications, 2016.
2. Dierkes, U., Hildebrandt, S. and Sauvigny, F., *Minimal Surfaces*, Springer, 2010.
3. La Torre, A. D. L., *Introduction to the Theory of Minimal Surfaces*, Springer, 2023.
4. Marques, F. C. and Neves, A., *Minimal Surfaces*, Princeton University Press, 2020.
5. Marques, F. C. and Neves, A., *Geometric Aspects of the Theory of Minimal Surfaces*, Springer, 2016.

Course Title: Scientific Computing
Course Code: MATH-558
Prerequisites: Linear Algebra, Differential Equations
Credit Hours: 3
Course Overview: This course provides an introduction to computer algebra. The main focus of the course is to analyze ordinary differential equations both qualitatively and quantitatively. Following a review of standard results concerning existence and uniqueness of solutions and their continuous dependence on parameters, we will learn linear system, stability theory computational methods to real-world problems in science and engineering.
Course Contents:
Introduction: Overview of scientific computing, Role of numerical methods, Problem-solving using computers.
Error analysis: Understanding round-off errors, Truncation errors, Error propagation in numerical computations.
Floating point arithmetic: Storing numbers in computer, IEEE standards single and double precision, Computer arithmetics.
Elements of ODEs: Existence and uniqueness of solutions, Dependence on parameters, Solving ODEs numerically, Stability and convergence in ODEs, Flows defined by differential equations.
Numerical linear algebra: Solving systems of linear equations (Gaussian elimination, LU decompositon), Matrix factorization (QR, SVD) with programming, Algorithms for iterative methods for root-finding (Golden search, Newton's method), system of nonlinear equations with programming.
Numerical methods for PDEs: Finite difference methods for heat equation and wave equation.

Recommended Books:

1. Danaila, I., Joly, P., Kaber, S. M. and Postel, M., *An Introduction to Scientific Computing*, Springer, 2007.

2. Heath, M. T., *Scientific Computing, an Introductory Survey*, SIAM, 2018.

3. Lynch, S., *Dynamical Systems with Applications using Maple*, 2nd Edition, Birkhäuser, 2010.

4. Otto, S. R. and Denier, J. P., *An Introduction to Programming and Numerical Methods in Matlab*, Springer, 2005.

5. Perko, L., *Differential Equations and Dynamical Systems*, 3rd Edition, Springer, 2008.

Course Title: Subdivision Methods for Geometric Design
Course Code: MATH-559
Prerequisites: Calculus and Linear Algebra
Credit Hours: 3
Course Overview: The benefits of subdivision are its simplicity and power. Subdivision is an exciting new area in computer graphics that allows a geometric shape to be modeled as the limit of a sequence of increasingly faceted polyhedra. The course will deal with subdivision curves, convergence of subdivision, corner cutting schemes and subdivision surfaces.
Course Contents:
Subdivision Curves: Notion of subdivision, Piecewise polynomial curves, Definition of B-splines, Refinability of B-splines, Refinement for spline curves, Subdivision for spline curves, Discrete convolution.
Analysis of subdivision: Invariant neighborhoods, Eigen analysis, Convergence of subdivision, Invariance under affine transformations, Geometric behavior of repeated subdivision, Size of the invariant neighborhood, Continuous and discrete Fourier transform, Corner cutting schemes, x-point schemes (3-point, 4-point), Analysis of subdivision curves.
Subdivision surfaces: Primal and dual schemes, Mask of the scheme, Stationary, Non-stationary, Linear, Non-linear, Uniform, Non-uniform, Interpolating, approximating, Quadrilateral, Triangular and Hexagonal schemes, Natural parameterization of subdivision surfaces, Subdivision matrix, Smoothness of surfaces, Analysis of subdivision surfaces, Catmull-Clark scheme, Doo-Sabin scheme, Loop scheme, Modified butterfly scheme.

Recommended Books:

1. Barrtels, R. H et al., *An Introduction to Spline for use in Computer Graphics and Geometric Modeling*, Morgan Kaufmann Publisher, Inc., 2006.

2. de Boor, C., *A Practical Guide to Splines*, Springer Verlag, 2001.

3. Farin, G., *Curve and Surfaces for Computer Aided Geometric Design: A Practical Guide*, Academic Press, Inc., 1990.

4. Faux, I. D., *Computational Geometry for Design and Manufacture*, Ellis Horwood, 1979.

5. Warren, J. and Weimer, H., *Subdivision Methods for Geometric Design: A Constructive Approach,* The Morgan Kaufmann Series in Computer Graphics, 2004.

Course Title: Spline Solutions of Boundary Value Problems
Course Code: MATH- 560
Prerequisites: Theory of Spline Functions
Credit Hours: 3
Course Overview:
The purpose of this course is to give a comprehensive introduction to the theory of spline functions, together with some applications to various fields, emphasizing the significance of the relationship between the general theory and its applications. This course deals with the solutions of higher order boundary value problems using different types of spline functions. These new proposed methods are very encouraging in dealing with boundary value problems.
Course Outline:
Interpolatory spline, Quadratic Hermite spline, Cubic Hermite spline, The representation of cubic spline in terms of first derivatives, The representation of cubic spline in terms of second derivatives, Theorems regarding error analysis, Theorems regarding to Convergence of the D1, D2, natural and periodic splines, End conditions for cubic Hermite spline interpolation, B-spline, Non-polynomial splines. Quadratic spline solutions of different order boundary value problems, Cubic spline solution of second and higher order boundary value problems, Quartic spline solution of higher order boundary value problems, B-spline solution of higher order boundary value problems, Non-polynomial spline solutions of higher order boundary value problems.

Recommended Books:

1. Kalyani, P. *Spline Solutions of Higher Order Boundary Value Problems*, Grin Verlag, 2020.
2. Micula, G. and Micula, S. *Handbook of Splines*, Springer Verlag, 1999.
3. Schumaker, L. L. *Spline Functions: Basic Theory*, John Wiley and Sons, 1993.
4. Shirokova, E. A. and Ivanshin, P. N. *Spline-Interpolation Solution of One Elasticity Theory Problem*, Bentham Science Publishers, 2011.
5. Zahra, W. *Numerical Treatment of Boundary Value Problems Using Spline functions: Numerical Treatment for a class of Boundary Value Problems Using nonpolynomial Spline functions*, LAP LAMBERT Academic Publishing, 2010.

Course Title: Numerical Methods of Partial Differential Equations
Course Code: MATH-561
Prerequisites: Partial Differential Equations
Credit Hours: 3
Course Overview: Numerical methods for partial differential equations (PDEs) is a branch of numerical analysis that studies the numerical solution of partial differential equations. This course includes finite difference and finite element methods for hyperbolic, parabolic and elliptic PDEs.
Course Contents:
Classification of linear second order PDEs in two and more variables, Introduction and finite difference formulae, Parabolic equations, Finite difference methods, Convergence and stability, Explicit method, Implicit method, Weighted average approximation, Local truncation error, Consistency, Gerschgorin's theorems, Von Neumann's method, Lax equivalence theorem, Finite difference approximations in cylindrical and spherical polar coordinates, Methods for improving accuracy, The Douglas equation, Use of three time-level difference equations, Deferred correction method, Richardson's deferred approach to the limit, Solutions of nonlinear parabolic equations, Newton's linearization method, Richtmyer's linearization method, Lee's three time-level method, Stability of three or more time-level difference equations, Hyperbolic equations and characteristics, Lax-Wendroff explicit method, Wendroff's implicit approximation, Analytical and numerical solutions for first and second order quasi-linear equations, Propagation of discontinuities, first-order and second-order equations, Elliptic equations, Systematic iterative methods (Jacobi, Gauss-Siedal, and SOR methods), A necessary and sufficient condition for convergence of iterative methods, Methods for accelerating convergence (Lyusternik's method, Aitken's method), Irregular boundaries.

Recommended Books:

1. Evans, G., and Blackledge, J., and Yardley, P., *Numerical Methods for Partial Differential Equations*, Springer London, 2012.

2. Lapidus, L., and Pinder, G. F., *Numerical Solution of Partial Differential Equations in Science and Engineering*, Wiley, 2011.

3. Mazumder, S., *Numerical Methods for Partial Differential Equations: Finite Difference and Finite Volume Methods*, Elsevier Science, 2015.

4. Smith, G. D., *Numerical Solution of Partial Differential Equations: Finite Difference Methods*, Northern Gate Book, 1993.

5. Thomas, J.W., *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.

Course Title: Numerical Methods for Ordinary Differential Equations
Course Code: MATH- 562
Prerequisites: Ordinary Differential Equations
Credit Hours: 3
Course Overview:
An ordinary differential equation (ODE) is a differential equation dependent on only a single independent variable. A number of problems in science and technology can be formulated into ODEs. The analytical methods of solving ODEs are applicable only to a limited class of equations. Quite often differential equations appearing in physical problems do not belong to any of these familiar types and one is obliged to resort to numerical methods. Numerical methods are used to find numerical approximations to the ODEs. These methods are of even greater importance when we realize that computing machines are now readily available which reduce numerical work considerably. This course covers some numerical schemes for solving ODEs.
Course Contents:
Initial value problems for ordinary differential equation, Runge-Kutta methods, Fehlberg method, Multi-step methods (Predictor corrector methods, Adams Bashforth method), Higher order and system of differential equations, Boundary-value problems, Linear Shooting method, Shooting method for non-linear problems, Finite difference method for linear problems, Finite difference method for non-linear problems, RayleighRitz method. Spectral methods (Galerkin and collocation methods, Fourier spectral methods, Chebyshev spectral methods), Error analysis, Convergence and stability analysis of numerical methods, Development of algorithms/ computer programming codes of the numerical methods for ODEs.
Recommended Books:

1. Butcher, J. C., *Numerical Methods for Ordinary Differential Equations*, Wiley, 2008.

2. Burden, R. L. and Faires, J. D., *Numerical Analysis*, Cengage Learning, 2015.

3. Griffiths, D. F. and Higham, D. J., *Numerical Methods for Ordinary Differential Equations*, Springer, 2010.

4. LeVeque, R. J., *Finite Difference Methods for Ordinary and Partial Differential Equations*, Society for Industrial and Applied Mathematics, 2007.

5. Trefethen, L. N., *Finite Difference and Spectral Methods for Ordinary and Partial Differential Equations*, University of Oxford, 1996.

Course Title: Sobolev Gradients and Differential Equations
Course Code: MATH-563
Prerequisites: Linear Algebra, Differential Equation
Credit Hours: 3
Course Overview:
Many problems of science and engineering can be formulated in terms of differential equations (DE). To solve these DE, one of the idea is to construct a functional such that the equation can be considered to be solved when the functional is minimal. The gradient based iterations are helpful in optimization problems with constrained as well as unconstrained. These methods are also helpful to find the minimum state of different energy functionals aarising in scientfic disciplines. These methods are of even greater importance due to their usage in machine learning algorithms.
Course Contents:
Preliminaries: Introduction to Hilbert spaces, Review of approximation theory, Gradients in Hilbert spaces.
Projections: Projections in discrete settings, Reisz representation theorem, adjoint operators.
Optimization: Unconstrained optimization (Gradient descent, Newton's method), Constrained optimization (Langrange, Newton), Least-square approximation.
Steepest descent method: Framework using prototypical problem in finite difference setting and in finite element setting.
Sobolev Gradients: Solution of prototypical problem using Sobolev gradients, Comparison between the two methods, solving linear differential equations using a combination of Sobolev gradients and spectral methods, Applications in machine learning.

Recommended Books:

1. Neuberger, J.W (2010). *Sobolev Gradients and Differential Equations*, Second Edition, Springer.
2. Kreyszig, E (1989). *Introductory Functional Analysis with Applications*, John Wiley & Sons.
3. Lu, J. (2022). *Gradient Descent, Stochastic Optimization, and Other Tales*, Amazon Digital Services LLC.
4. Sial, S. (2016). *: Notes on Sobolev Gradients and Control Problems.*
5. Otto, S.R. and Denier, J.P. (2005). *An Introduction to Programming and Numerical Methods in Matlab*, Springer.

Course Title: Theory of Differential Equations
Course Code: MATH-572
Prerequisites: Linear Algebra
Credit Hours: 3
Course Overview:
Differential equations play an important role in applied mathematics. They provide modeling tools to describe and simulate real-world (physical) problems. The purpose of this course is to provide a comprehensive introduction to classical methods of applied mathematics. The main focus is on the study of qualitative theory of systems of ordinary differential equations (ODEs). After a brief review of standard results on the existence and uniqueness of solutions and their continuous dependence on parameters, we start with linear systems, stability theory, invariant manifolds, and end with a survey of periodic and homoclinic solutions. The second goal of this course is to introduce students to modern symbolic and numerical computational methods that can be used in quantitative analysis.

Course Contents:
Elements of the General Theory of the Linear ODE: Lipschitz continuity (local and global) Grownswal inequality, Solving IVP equivalent to integral equation, Existence and uniqueness of Initial value problem (Picard Theorem of successive approximation, Peano's uniqueness theorem, Continuous dependence on initial condition dependence on parameters.
Preliminary Results from Algebra and Analysis: Existence results about solutions of homogeneous and non homogeneous linear systems, The characteristic polynomial for a matrix, Cayley-Hamilton theorem, Diagonalization, Norm of a matrix, System of ODEs, Coupled, Conversion of coupled into non coupled, Flows defined by differential equations.
Stability of Linear Systems: Properties of linear systems, Solutions with homogeneous systems with constant coefficients, Critical points and linearized stability, Lyapunov functions and nonlinear stability.
Hyperbolic Theory: Stable and unstable manifolds of dynamical systems, Linearization of hyperbolic systems, Center manifold and nonlinear stability, Normal forms, Existence and uniqueness theorem for partial differential equations (PDEs), Convergence and Stability analysis for PDEs.

Recommended Books:

1. Agarwal, R. P. and O'Regan, D., *An Introduction to Ordinary Differential Equations*, Springer, 2012.

2. Hsieh, P. F. and Sibuya, Y., *Basic Theory of Ordinary Differential Equations*, Springer, 2012.

3. Lynch, S., *Dynamical Systems with Applications using Maple*, Birkhäuser, 2010.

4. Liu, J. H., *A First Course in the Qualitative Theory of Differential Equations*, Prentice Hall, 2003.

5. Perko, L., *Differential Equations and Dynamical Systems*, Springer, 2008.

Course Title: Mathematical Modelling in Epidemiology
Course Code: MATH-565
Prerequisites: Differential Equations
Credit Hours: 3
Course Overview: In recent years our understanding of infectious-disease epidemiology and control has been greatly increased through mathematical modelling. This course is a practical course in learning and applying techniques in mathematical modelling of infectious disease dynamics. There is a strong emphasis on using programming and problem solving to address a series of key questions arising in the field of mathematical epidemiology. This course will acquire hands-on experience of manipulating mathematical models, implementing appropriate numerical methods and fitting models to data.
Course Contents:
Preliminaries: What is epidemiology? Classification of infectious diseases, Basic definitions in the epidemiology of infectious diseases.
Modeling: General approach to modeling, Introduction to basic epidemic models like SIR, SIER, SIERS and their extension to complex models, Difference between deterministic vs stochastic modelling, Multiscale modeling.
Stability criterion: Calculation of basic reproduction number and model equilibrium points, Local and global stability of equilibrium points, Data sources and collection.
Parameter estimation: Modeling fitting to data and parameter estimation methods, Sensitivity analysis.
Applications: Diseases models dynamics simulation and prediction, Adaptive management for improved intervention efficacy, Health economics for dynamic models and decision making.

Recommended Books:

1. Carstensen, B., *Epidemiology with R*, Springer, 2021.
2. Daley, D.J. and Gani, J., *Epidemic Modelling: An Introduction*, Cambridge University Press, 2001.
3. Diekmann, O., Heesterbeek, J.A.P. and Britton, T., *Mathematical Tools for Understanding Infectious Disease Dynamics*, Princeton University Press, 2013.
4. Martcheva, M., *An Introduction to Mathematical Epidemiology*, Springer, 2015.
5. Vynnycky, E. and White, R.G., *An Introduction to Infectious Disease Modelling*, Oxford University Press, 2010.

Course Title: Artificial Intelligence with Python
Course Code: MATH- 566
Prerequisites: Probability, Statistics, Linear Algebra
Credit Hours: 3
Course Overview:
This course is specifically designed for mathematics students, providing a strong foundation in the mathematical principles that underlie artificial intelligence (AI) techniques. Throughout the course, students will explore key AI algorithms and tools while delving deep into their mathematical origins, such as linear algebra, probability theory, and optimization techniques. A special emphasis will be placed on fuzzy logic, a critical tool for handling uncertainty in AI systems. By integrating fuzzy logic with neural networks, students will learn how to develop sophisticated models that are capable of tackling complex, real-world problems.
Course Outline:
Preliminaries: Introduction to AI, Machine learning and deep learning.
Python for AI and Mathematics: Basic Python programming and libraries for AI (numpy, scipy, matplotlib), Introduction to pandas and scikit-learn.
Linear Algebra and Optimization: Vectors, Matrices, Linear algebra for AI.
Gradient Descent and Optimization: Cost functions and optimization algorithms, Mathematical derivation of gradient descent.
Statistical Inference and AI: Random variables, Expectation, Variance, Hypothesis testing and AI model evaluation, Linear regression and its variants (Simple linear regression, Multiple linear regression, Ridge and Lasso regression, regularization techniques).
Supervised Learning: Logistic regression for classification, Sigmoid function, Binary classification, Mathematics behind the logistic regression model, Decision trees and random forests, Splitting criteria (entropy, Gini index), Random forests and ensemble methods, Support vector machines (SVM), Hyperplanes, margins, and the kernel trick, Mathematics behind SVM for classification and regression.
Neural networks: Introduction to neural networks, Biological inspiration, Perceptron model, Feed forward neural networks, Mathematical formulation. *Back Propagation and Training Neural Networks*: Chain rule for gradients, Back propagation algorithm, Mathematical details of training a neural network.
Activation Functions: Mathematical behavior of activation functions and their influence on learning, Deep learning with Python, Building and training neural networks with Python, Hands-on coding session with simple examples.
Fuzzy Neural Networks: Introduction to fuzzy neural networks (FNN), Combining fuzzy logic with neural networks for uncertainty handling, Fuzzy neural networks mathematical model, Mathematical formulation of fuzzy neural networks, Training fuzzy neural networks, Error backpropagation with fuzzy logic.
Implementation of Fuzzy Neural Networks in Python: Hands-on coding with fuzzy systems and neural networks, Implementing FNN models using Python, Advanced topics in neural networks, Recurrent neural networks, Clustering algorithms.
Recommended Books:

1. Aggarwal, C.C., *Neural Networks and Deep Learning*, Springer, 2023.

2. Chollet, F., *Deep Learning with Python*, Manning Publications, 2021.

3. Deisenroth, M. P., *Mathematics for Machine Learning*, Cambridge University Press, 2020.

4. Liu, P. and Li, H.-X., *Fuzzy Neural Network Theory and Application*, World Sci. Pub. Com., 2004.

5. Russell, S., and Norvig, P., *Artificial Intelligence: A Modern Approach*, Pearson, 2020.

Course Title: Data Science Fundamentals
Course Code: MATH- 567
Prerequisites: Probability & Statistics
Credit Hours: 3
Course Overview:
This course introduces the core concepts of data science, focusing on its practical applications and the mathematical foundations behind it. Students will explore the data science lifecycle, including data acquisition, processing, visualization, and machine learning, using Python and key libraries like Matplotlib, NumPy, SciPy, and Pandas. The course emphasizes statistical analysis, probability, linear algebra, and calculus, preparing students to handle real-world data and build predictive models.
Course Contents:
Introduction: Overview of data science and its applications.
Data Science Life Cycle: Phases including business problem definition, Data acquisition, Processing, EDA, Model creation, Evaluation, Deployment and monitoring, Overview of industry roles and salary trends.
Python Programming Basics: History of Python, Installing and using Jupyter/Colab, Python syntax, Variables, Control structures, Functions and packages.
Container Types: Overview of lists, Arrays, Tuples, Dictionaries, Sets and conversions between them.
Python Libraries for Data Science: Matplotlib, NumPy, SciPy, SymPy, Pandas.
Mathematics for Data Science: Key topics including probability, Statistics, Linear algebra and calculus.
Dimensionality Reduction: Techniques like PCA, t-SNE, and LDA for simplifying data while preserving key features.
Time Series Analysis: Basics of time series data, Seasonality, ARIMA models and forecasting.

Recommended Books:

1. Cielen, D., Meysman, A., and Ali, M., *Introducing Data Science*, Manning, 2016.

2. Dietel, P., and Dietel, H., *Python for Programmers*, Pearson, 2019.

3. Führer, C., Solem, J. E., and Verdier, O., *Scientific Computing with Python 3*, Packt Publishing, 2021.

4. Hazrat, R., *A Course in Python*, Springer, 2023.

5. Seroul, R., *Programming for Mathematicians*, Springer, 2000.

Course Title: Fundamentals of Cryptography
Course Code: MATH-568
Prerequisites: Number Theory
Credit Hours: 3
Course Overview:
Cryptography is about the mathematical, algorithmic, and implementational aspects of information security. It is one of the core technologies for securing the emerging information infrastructure. Its applications range from (conceptually) simple tasks such as encryption, authentication, and key management to sophisticated tasks such as Internet security, electronic cash payments(using smart cards), and electronic voting. This course is a comprehensive introduction to modern cryptography that is aimed primarily at those interested in applications.
Course Contents:
Mathematical Foundations: Number theory fundamentals, Prime numbers, modular arithmetic, and properties of congruences, Greatest Common Divisor (GCD) and the Euclidean Algorithm, Factoring problems.
Classical Cryptography: Hash functions and data integrity, Symmetric key cryptography (private key cryptography), Classical ciphers, One-time pad, Stream ciphers, Asymmetric key cryptography / public key cryptography (RSA, Elgamal, Elliptic curve, in brief), Digital signatures schemes(RSA, DSA) Key Establishment and Key Management (key transport and key agreement, symmetric and asymmetric techniques), Crypt analysis, Algorithem development with Python.

Recommended Books:

1. Buchmann, J. A., *Introduction to Cryptography*, Springer, 2004.
2. Buell, D., *Fundamentals of Cryptography*, Springer, 2021.
3. Katz, J. and Lindell, Y., *Introduction to Modern Cryptography*, Chapman and Hall/CRC, 2015.
4. Stinson, D.R., *Cryptography Theory and Practice*, Chapman and Hall/CRC,, 2006.
5. Salomaa, A., *Public-Key Cryptography*, Springer, 1996.

Course Title: Modular Forms and Elliptic Curves
Course Code: MATH- 570
Prerequisites: Algebra, Real and Complex Analysis, Algebraic Geometry
Credit Hours: 3
Course Overview:
This course is a deep interconnection of number theory and algebraic geometry. The modular form is a complex analytic function on the upper half-plane that satisfies certain periodicity and growth conditions. These functions are invariant under the action of the module group $SL(2, Z)$. They play a vital role in various areas of mathematics, including number theory, combinatorics, and mathematical physics.
Course Contents:
Elliptic Functions: Review of complex Integration, residues and singularities, Basic construction of Elliptic functions, The field of elliptic functions, The Weierstrass elliptic function, Abels theorem.
The Modular Group: The moduli space of complex tori and group actions, Topology of the orbit space of the G-action, Modular curves as compact Riemann surfaces.
The Algebra of Modular Forms: Modular forms and cusp forms, Eisenstein series and the algebra of modular forms of G, Generalization to Hecke congruence subgroups, *Elliptic Curves:* Embedding tori as plane cubic hypersurfaces, Elliptic curves over subfields of $\mathbb{C}$, Elliptic curves over finite fields.

Recommended Books:

1. Bruinier, J. H., Geer, G. V. D., Harder, G., et al., *The 1-2-3 of Modular Forms*, Springer, 2008.
2. Lozano-Robledo, A., *Elliptic Curves, Modular Forms, and Their L-functions*, American Mathematical Society, 2011.
3. Neal, K. *Introduction to Elliptic Curves and Modular Forms*, Springer, 1993.
4. Silverman, J.H., *The Arithmetic of Elliptic Curves*, Springer, 2009.
5. Wehler., J., *Modular Forms and Elliptic Curves*, LMU Edition, 2021.

Course Title: Mathematical Statistics
Course Code: MATH-570
Prerequisites: Set Theory
Credit Hours: 3
Course Overview:
Mathematical statistics is a branch of mathematics that deals with models involving random, unpredictable components. The statistical methods used in practice are based on fundamentals of statistical theory. One branch of the theory uses probabilistic tools to establish important distributional results used throughout statistics. There are diverse real-world applications for making informed decisions in the face of uncertainty. This course provides an introduction to probability, distribution theory, and Statistical inference.
Course Contents:
Data visualization and Descriptive Statistics: Basics of data visualization, Common measures of central tendency and dispersion.
Probability Theory: Axioms of probability, Conditional Probability, Bayes's rule, Random Variables.
Functions of Random Variables: Distribution function technique, Transformation technique for one variable and several variables, Moment-generating function technique.
Sampling distributions: The distribution of mean for both finite and infinite populations, Central limit theorem, The Chi-Square distributions and its properties, The t distribution and its properties, The F distribution and its properties.
Regression and Correlation: Simple, multiple regression and least squares method, Simple, partial and multiple correlation, Normal regression analysis, Normal correlation analysis.
Statistical Inferences: Estimation, Types of estimation, Confidence interval, Tests of Hypothesis.

Recommended Books:

1. Bijma, F. Jonker, M. and Van der Waat, A. W., *An Introduction to Mathematical Statistic*, Amsterdam University Press, 2017.
2. Freund, J. E., *Mathematical Statistics*, Prentice Hall Inc., 1992.
3. Hogg, R. V., McKean, J. W. and Craig, A. T.,*Introduction to Mathematical Statistics*,, Pearson, 2021.
4. Ross, S. A., *A First Course in Probability*, 8th Edition, Prentice Hall, 2010.
5. Walpole, R. E., *Introduction to Statistics*, Macmillan Publishing Company, 3rd edition, 1982.

Course Title: Financial Mathematics
Course Code: MATH-571
Prerequisites: Basic Mathematics, Probability
Credit Hours: 3
Course Overview:
Financial mathematics courses focus on the mathematical properties and relationships of financial and money market concepts in investing and other economic activities. The course is designed to inspire confidence in participants to solve problems related to the globalization of financial markets, the development and feasibility of financial transactions, increasingly complex investment portfolios, analysis and forecast market developments.
Course Contents:
Mathematical introduction, Growth and decay curves, Simple interest, Bank discount, Compound interest, Discrete compounding, Compounding frequency of interest, Economic equivalence, Method of comparison of alternatives, Project balance, Credit and loan, Cost of credit and amortization, Depreciation and depletion, Breakeven analysis, Leverage, Stocks and bonds, Valuation of stocks and bonds, Mutual funds, Options, Cost of capital and ratio analysis, Decision under risk and uncertainty, Risk premium, Portfolio diversification, Life Insurance, Endowment, and annuities, Insurance policies.

Recommended Books:

1. Brown, R. and Zima, P., *Schaum's Outline of Mathematics of Finance*, McGraw Hill, 2011.
2. Capinski, M. and Zastawniak, T., *Mathematics for Finance*, Springer, 2011.
3. Campolieti, G. and Makarov, R. N., *Financial Mathematics*, Chapman and Hall/CRC, 2014.
4. Petters, A. O. and Dong, X., *An Introduction to Mathematical Finance with Applications*, Springer, 2016.
5. Wahidudin, A. N., *Financial Mathematics and its Applications*, Ventus Publishing ApS, 2011.
6. Saari, D. G., *Mathematics of Finance*, Springer, 2019.

SYLLABI FOR Ph.D. MATHEMATICS
(SEMESTER SYSTEM PROGRAMME)

To be offered in Department of Mathematics, University of the Punjab, Lahore with effect from Admissions 2025 to onwards.

Programme	Ph.D. Mathematics
Duration	3 – 5 Years
Semesters	6 – 10
Credit Hours	66
Department	Mathematics
Faculty	Science

Introduction

The Department of Mathematics at the University of the Punjab is one of the university's oldest academic units, founded in 1921. Over the years, it has produced numerous distinguished graduates, many of whom have gained prominence in Mathematics. For a considerable period, the department operated on an inter-collegiate basis, with faculty members from local institutions such as Government College Lahore, Forman Christian College, Dyal Singh College, Islamia College Civil Lines, and M.A.O. College, Lahore, conducting M.Sc. classes at the university. In 1956, the department became an independent institution with the appointment of two full-time faculty members: one reader and one senior lecturer. Since then, the department has steadily grown and now offers a range of programs, including BS (4 years), BS (5 semesters), M.Phil., and Ph.D.
In 1982, the department established a computer center to enhance the computational capabilities of university students, faculty, and staff. Furthermore, the Department of Mathematics publishes the Punjab University Journal of Mathematics, with its inaugural issue released in 1967 under the editorship of Prof. Dr. Syed Manzur Hussain.
The department is home to four robust research groups: Computational Mathematics, Fuzzy Mathematics, Gravitation & Cosmology, and Pure Mathematics. These groups contribute to the academic strength of the department, fostering innovation and advancing knowledge in their respective fields.

Vision

The Department of Mathematics aims to be recognized as an internationally top-ranking center of excellence in both teaching and research.

Mission

In pursuit of our vision, the Department of Mathematics strives to provide quality education at both undergraduate and postgraduate levels, aiming to produce high-calibre graduates who will excel in their chosen careers in industry, the professions, and academia. Our students are selected based on intellectual merit, without discrimination based on gender, race, or physical disabilities. We are committed to fostering a diverse and well-balanced portfolio of research of the highest quality, encompassing a wide range of interests.

Objectives

The following objectives are designed to guide the Department of Mathematics toward achieving its vision of becoming an internationally recognized center of excellence in teaching and research.

- Expand and diversify the faculty to encompass all disciplines of Mathematics.
- Strengthen all existing academic programs, with particular emphasis on the MPhil/PhD programs to facilitate world-class research.
- Develop an industry-based Mathematics curriculum, fostering close collaboration between mathematics and industry.
- Encourage faculty engagement in research projects, paper presentations at international conferences, organizing conferences, international research collaborations, and postdoctoral training.
- Promote the "Punjab University Journal of Mathematics" as a premier research journal of international repute.
- Provide strong support for both individual researchers and research groups.
- Enhance the abilities and character of students, nurturing them into well-rounded individuals.
- Recognize and reward the department's staff, acknowledging them as its greatest asset.

Admission Eligibility Criteria

- Prior to admission into a PhD the student shall have been awarded his or her MS/MPhil/or its equivalent degree with minimum CGPA of 3.00 (out of 4.00) in the semester system from a recognized University. If CGPA < 3, additional courses of 9-12 CH of level 7 in zero semester and shall score minimum 3.00 out of 4.00 GPA.
- No third division in the whole career.
- Qualifying marks in the admission test conducted by the University shall be 60%.
- Qualifying marks in the interview shall be 50%.

Duration of PhD Programme

- The minimum period for completion of PhD requirements is three years, and the maximum period is five years.
- The minimum period for completion of course work is two semesters (one year).

Course Work PhD

- Minimum 18 credit hours of course work must be undertaken with at least CGPA of 3.0 out of band 4.0.
- Comprehensive examination is conducted after successful completion of 18 credit hours.
- The comprehensive examination covers all course work and consists of one composite paper. To pass the comprehensive examination, a student must get GPA not less than 3.0

Requirement for the Award of Ph.D. Degree

The following requirements shall be fulfilled before the award of Ph.D. degree:

- PhD research proposal approved by ASRB.
- After passing the comprehensive examination, but before the submission of his/her thesis, a PhD student will give at least one seminar on a topic relevant to his/her field of research.
- PhD candidates must publish at least One research paper based on the PhD research in category "W" or Two research papers in category "X" from HEC approved journals.
- The thesis should pass a plagiarism test before sending it for evaluation.
- The PhD Thesis must be evaluated by two Foreign relevant subject experts.
- Open defense of Thesis.
- Final submission of PhD Thesis.

Coding Scheme of Courses

The course code consists of two parts: letter code and numeric code. All courses use the alphabetical code "MATH" which contains 4 characters and the numeric code contains 3 digits.

SEMESTER-WISE WORKLOAD

A Doctor of Philosophy degree in Mathematics is structured to be comprised of Six regular semesters over a period of Three to Five years. The semester-wise workload is as under:

Semester I

Sr. No.	Course Title	Credit Hours
1	Course I	3
2	Course II	3
3	Course III	3
Total Semester Credits		**09**

Semester II

Sr. No.	Course Title	Credit Hours
1	Course IV	3
2	Course V	3
3	Course VI	3
Total Semester Credits		**09**

Semester III- VI

Course Code	Course Title	Credit Hours
MATH-850	Ph.D. Thesis	48
Total Semester Credits		**48**

List of Courses

Table 1: List of Courses

Course Code	Courses	Credit Hours
MATH-701	Applied Linear Algebra	3
MATH-702	Applied Graph Theory	3
MATH-703	Partial Differential Equations	3
MATH-704	Mathematical Methods for Physics	3
MATH-705	Numerical Approximation Theory	3
MATH-706	Integral Equations	3
MATH-707	Riemannian Geometry	3
MATH-708	Theory of Groups	3
MATH-709	Module Theory	3
MATH-710	Measure Theory	3
MATH-711	Lie Algebras	3
MATH-712	Lie Symmetries of Differential Equations	3
MATH-713	Linear Groups and Group Representations	3
MATH-714	Field Extensions & Galois Theory	3
MATH-715	Representation Theory of Finite Groups	3
MATH-716	Theory of Manifolds	3
MATH-717	Number Theory and Cryptography	3
MATH-718	Applied Combinatorics	3

Table 2: List of Courses

Course Code	Course Title	Credit Hours
MATH-719	Algebraic Geometry	3
MATH-720	Lie Groups	3
MATH-721	Category Theory	3
MATH-722	Fixed Point Theory	3
MATH-723	Knot Theory	3
MATH-724	Operator Theory	3
MATH-725	Banach Algebras	3
MATH-726	Functional Analysis	3
MATH-727	Harmonic Analysis	3
MATH-728	Advanced Topology	3
MATH-729	Algebraic Topology	3
MATH-730	Topological Groups	3
MATH-731	Homological Algebra	3
MATH-732	Commutative Algebra	3
MATH-733	Algebraic Graph Theory	3
MATH-734	Introduction to Fuzzy Systems	3
MATH-735	Fuzzy Decision Making Methods	3
MATH-736	Fuzzy Graph Theory	3
MATH-737	Fuzzy Fractional Differential Equations	3
MATH-738	Introduction to Biomathematics	3
MATH-739	Mathematical Cosmology	3

Table 3: List of Courses

Course Code	Course Title	Credit Hours
MATH-740	Stellar Dynamics	3
MATH-741	Elastodynamics	3
MATH-742	General Relativity	3
MATH-743	Mathematical Plasma Dynamics	3
MATH-744	Magnetohydrodynamics	3
MATH-745	Fluid Dynamics	3
MATH-746	Biofluid Mechanics	3
MATH-747	Classical Field Theory	3
MATH-748	Quantum Field Theory	3
MATH-749	Classical Electrodynamics	3
MATH-750	Non-Newtonian Fluid Mechanics	3
MATH-751	Relativistic Theory of Black Holes	3
MATH-752	Fractional Differential Equations	3
MATH-753	Quantum Information Theory	3
MATH-754	Quantum Computing	3
MATH-755	Optimization Techniques	3
MATH-756	Theory of Spline Functions	3
MATH-757	Minimal Surfaces	3
MATH-758	Scientific Computing	3
MATH-759	Subdivision Methods for Geometric Design	3

Table 4: List of Courses

MATH-760	Spline Solutions of Boundary Value Problems	3
MATH-761	Numerical Methods for Partial Differential Equations	3
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