DEPARTMENT OF MATHEMATICS
UNIVERSITY OF THE PUNJAB

COURSES OF M.PHIL. / PH.D.
(Fall 2017-ONWARD)

Students of M.Phil. / Ph.D. programme will be offered four courses in the first semester and four courses in the second semester. Option in courses will be offered if the Department has sufficient relevant faculty.

a. Core Courses

MCO-501	Geometry	(3 credits)
MCO-502	Mathematical Techniques	(3 credits)
MCO-503	ODEs and Computational Linear Algebra	(3 credits)
MCO-504	Partial Differential Equations	(3 credits)
MCO-505	Integral Equations	(3 credits)
MCO-506	Group Theory	(3 credits)
MCO-507	Functional Analysis	(3 credits)
MCO-508	Advanced Mathematical Physics	(3 credits)

Optional Courses

b. Applied Mathematics

MA-601	General Relativity-I	(3 credits)
MA-602	General Relativity-II	(3 credits)
MA-603	Cosmology	(3 credits)
MA-604	Relativistic Astrophysics	(3 credits)
MA-605	Classical Field Theory	(3 credits)
MA-606	Electrodynamics-I	(3 credits)
MA-607	Electrodynamics-II	(3 credits)
MA-608	Magnetohydrodynamics-I	(3 credits)
MA-609	Magnetohydrodynamics-II	(3 credits)
MA-610	Fluid Dynamics	(3 credits)
MA-611	Elastodynamics	(3 credits)
MA-612	Plasma Physics	(3 credits)
MA-613	Advanced Course in Plasma Physics	(3 credits)

MA-614	Quantum Field Theory	(3 credits)
MA-615	Non-Newtonian Fluid Mechanics	(3 credits)
MA-616	Introduction to Bio Mathematics	(3 credits)

c. Computational Mathematics

MC-621	Theory of Spline Functions I	(3 credits)
MC-622	Theory of Spline Functions II	(3 credits)
MC-623	Theory of Spline Functions III	(3 credits)
MC-624	Subdivision Schemes	(3 credits)
MC-625	Approximation Theory	(3 credits)
MC-626	Numerical Solution of PDEs	(3 credits)
MC-627	Minimal Surfaces	(3 credits)
MC-628	Computer Graphics	(3 credits)
MC-629	Mathematical Modeling-I	(3 credits)
MC-630	Mathematical Modeling-II	(3 credits)
MC-631	Advanced Computational Methods	(3 credits)
MC-632	Sobolev Gradients and Differential Equations	(3 credits)

d. Pure Mathematics

MP-641	Rings and Modules	(3 credits)
MP-642	Operator Theory	(3 credits)
MP-643	Lie Algebras & Lie Groups	(3 credits)
MP-644	Field Extensions & Galois Theory	(3 credits)
MP-645	Linear Groups & Group Representations	(3 credits)
MP-646	General Topology	(3 credits)
MP-647	Homotopy Theory	(3 credits)
MP-648	Topological Groups	(3 credits)
MP-649	Homological Theory	(3 credits)
MP-650	Lattice Theory	(3 credits)
MP-651	Representation Theory	(3 credits)
MP-652	BCK Algebra	(3 credits)
MP-653	BCI Algebra	(3 credits)
MP-654	Advanced Theory of Rings and Modules	(3 credits)
MP-655	Spectral Theory in Hilbert Spaces – I	(3 credits)
MP-656	Spectral Theory in Hilbert Spaces – II	(3 credits)
MP-657	Harmonic Analysis	(3 credits)
MP-658	Banach Algebras-I	(3 credits)
MP-659	Banach Algebras-II	(3 credits)
MP-660	Advanced Measure Theory	(3 credits)
MP-661	Advanced Number Theory	(3 credits)
MP-662	Combinatorics	(3 credits)
MP-663	Introduction to Fuzzy Systems	(3 credits)
MP-664	Fuzzy Graph Theory	(3 credits)
MP-665	Graph Theory	(3 credits)
MP-666	Design Theory	(3 credits)

e. Reading and Research

MRR-671	Reading & Research-MA1	(3 credits)
MRR-672	Reading & Research-MA2	(3 credits)
MRR-673	Reading & Research-MC1	(3 credits)
MRR-674	Reading & Research-MC2	(3 credits)
MRR-675	Reading & Research-MP1	(3 credits)
MRR-676	Reading & Research-MP2	(3 credits)

f. Non-Credit Courses

MSA-681	Seminar Attendance	(0 credits)
MSA-682	Seminar Delivered-G	(0 credits)
MSA-683	Seminar Delivered-T	(0 credits)
MSA-684	Seminar Delivered-R	(0 credits)

g. M.Phil. Thesis

MTH-690	Thesis	(6 credits)

h. Ph.D. Thesis

MTH-699	Thesis	(48 credits)

COURSE OUTLINES

CORE COURSES:

MCO-501 GEOMETRY

Tangent vectors and Curves in R^3. Differential forms, Curves. The Frenet Formulas, Covariant derivatives, Frame fields, Isometries of R^3. The tangent map of an isometry, Surfaces in R^3. Patch computation, Differential forms on a surface, Mappings of surfaces. Integration of forms. Topological properties of surfaces, Manifolds. The shape operator, Normal and Gaussian curvature, Some global theorems. Isometries and local isometries, Riemannian geometry. Covariant derivative. Geodesics, The Gauss-Bonnet theorem and its applications, Smooth manifolds. Smooth mappings, Tangent vectors, Curves, Vector fields, One-forms, Submanifolds, Topology of manifolds, Integral curves, Tensors, Basic algebra, Tensor components, Contraction, Covariant tensors, Tensor derivations, Semi-Riemannian manifolds, Isometries, The Levi-Civita connection, Parallel translation. Geodesics, Curvature. Ricci and scalar curvature.

Recommended Books:

1. Barrett O'Neill.: Elementary Differential Geometry (Second Edition), Academic Press 1997.
2. Barrett O'Neill.: Semi-Riemannian Geometry with Applications to Relativity, Academic Press 1983.
3. Bioshop, R.L. and Goldberg, S.I.: Tensor Analysis on Manifolds (Dover Publication, Inc. N.Y.1980).
4. do Carmo, M.P.: Riemannian Geometry (Birkhauser, Boston, 1992).
5. Langwitz, D.: Differential and Riemannian Geometry (Academic Press, 1970).
6. Abraham, R., Marsden, J.E. and Ratiu, T.: Manifolds, Tensor Analysis and Applications (Addison Wesley, 1983).

MCO-502 MATHEMATICAL TECHNIQUES

Green's function method with applications to wave-propagation. Solution of algebraic equations by perturbation methods. Evaluation of integrals by expansion of integrands. Laplace methods. The method of stationary phase. The methods of steepest descent. Solution of the linear damped oscillator equation by perturbation methods. The WKB approximation. Variational problems with variable end points. Corner conditions. Sufficient conditions for minimum. The Ritz method and its applications. A survey of transform techniques. Wiener-Hopf technique with applications to diffraction problems.

Recommended Books:

1. Nayfeh, A.: *Perturbation Methods* (John Wiley & Sons, Inc.,1973).
2. Stakgold, I.: *Boundary Value Problems of Mathematical Physics, Vol I & II* (The Macmillan Company, New York, 1970).

3. Noble, B.: *Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations* (Pergamon Press, New York, 1958).
4. Mitra, R. and Lee, S.W.: *Analytical Techniques in the Theory of Guided Waves* (Macmillan, New York, 1971).
5. Sneddon, I.N.: *The Use of Integral Transforms* (McGraw- Hill, New York, 1972).
6. Clegg, J.C.: *Calculus of Variations* (John Wiley, New York, 1968).

MCO-503 ODEs AND COMPUTATIONAL LINEAR ALGEBRA

Introduction. Euler's method. The improved and modified Euler's method. Runge-Kutta method. Milnes method. Hammign's methods. Initial value problem. The special cases when the first derivative is missing. Boundary value problems. The simultaneous algebraic equations method. Iterative methods for linear equations. Gauss-Siedel method. Relaxation methods. Vector and matrix norms. Sequences and series of matrices. Graph Theory. Directed graph of a matrix. Strongly connected and irreducible matrices. Grerschgoin theorem. Symmetric and positive definite matrices. Cyclic-Consistently ordered matrices. Choice of optimum value for relaxation parameter.

Recommended Books

1. Ortega, J.M. and Poole, W.G.: *An Introduction to Numerical Methods for Differential Equations* (Pitman, 1981).
2. Leon, S.J.: *Linear Algebra with Applications* (Maxwell Macmillan International Edition, 1990).
3. Young, D.M. and Gregory, R.T.: *A Survey of Numerical Mathematics, Vol-II* (Addison-Wesley Publishing Company, 1973)
4. Gerald, C.F. and Wheatley, P.O.: *Applied Numerical Analysis* (Addison-Wesley Publishing Company, 1994).
5. Mathews, J.H.: *Numerical Methods for Mathematics, Science & Engineering*, (Prentice- Hall, 1987).
6. Demidovich, B.P. and Maron, I.A.: *Computational Mathematics* (MIR, Moskva, 1987).

MCO-504 PARTIAL DIFFERENTIAL EQUATIONS

Classification of linear second order PDEs in two and more variables. Cauchy's problems for linear second order equations in *n* independent variables. Adjoint operator. Self adjoint equation and operator. Linear PDEs in *n* independent variables. Lagrange's identity. Green's theorem for self adjoint operator. Spherical and Cylindrical Waves. Heat equation. Wave equation. Laplace equation.

Recommended Books

1. Garabedian, P.R.: *Partial Differential Equations* (Chelsea Publication, 1998).
2. Dennemyer, R.: *Introduction to Partial Differential Equations and Boundary Value Problems* (McGraw Hill Book Co., 1968).
3. Chester, C.R.: *Techniques in Partial Differential Equations* (McGraw Hill Book Co., 1971).
4. McOwen, R.: *Partial Differential Equations* (Prentice Hall, 1996).

5. Ladyzhenskaya, O.A.: *The Boundary value Problem of Mathematical Physics* (Springer-Verlag, NY, 1985).

MCO-505 INTEGRAL EQUATIONS

Linear integral equations of the first kind. Linear integral equations of the second kind. Relationship between differential equation and Volterra integral equation. Newmann series. Fredholm Integral equation of the second kind with separable Kernels. Eigenvalues and eigenvectors. Iterated functions. Quadrature methods. Least square methods. Homogeneous integral equations of the second kind. Fredholm integral equations of the second kind. Fredholm integral equations of the first kind. Abel's integral equations. Hilbert Schmidt theory of integral equations with symmetric Kernels. Regularization and filtering techniques.

Recommended Books

1. Baker, C.T.H.: *Integral Equations* (Clarendon Press, 1977).
2. Smithies, F.: *Integral Equations* (Cambridge University Press, 1989).
3. Squire, W.: *Integration for Engineers and Scientists* (American Elsevier, New York, 1970).
4. Wazwaz, A.M.: *A First Course in Integral Equations* (World Scientific Pub., 1989).
5. Lovitt, W.V.: *Linear Integral Equations* (Dover Publication, 2005).

MCO-506 GROUP THEORY

Elementary concepts. Symmetric and alternating groups of finite degree. Order of a permutation. Orbits of the symmetric and alternating groups. Stabilizer subgroups and transitive groups. Free products of group. Group amalgams and their embeddability in groups. Generalized free product of groups. Permutational product of groups. Cartesian product of groups. Wreath product of groups. Multiplicative group of a finite field. Projective line over finite fields. Projective and linear groups through action.

Recommended Books

1. Kochendorffer, R.: *Group Theory* (McGraw Hill London, 1970).
2. Coxeter, H.S.M. and Moser, W.O.: *Generators and Relations for Discrete Groups* (Springer-Verlag, New York, 1984).
3. Rose, J.S.: *A Course in Group Theory* (Cambridge Univ. Press, 1978).
4. Magnus, W., Karrass, A. and Solitar, D.: *Combinatorial Group Theory* (Interscience, New York, 1969).
5. Johnson, D.L.: *Topics in the Theory of Group Presentations* (Cambridge Univ. Press, 1980).

MCO-507 FUNCTIONAL ANALYSIS

Separation properties. Hahn-Banach theorem. Banach algebras theorem (Introduction). Linear mappings. Finite dimensional spaces. Metrization. Boundedeness and continuity. Seminorms and local convexity. Baire category theorem. The Banach-Steinhaus theorem. The open mapping theorem. The closed graph theorem. Bilinear mappings. The normed dual of normed spaces. Adjoints.

Recommended Books

1. Kreyszig, E.: *Introductory Functional Analysis with Applications* (John Wiley, 1989).
2. Nachbin, L.: *Introduction to Functional Analysis: Branch Spaces and Differential Calculus* (Marcel Dekker, Inc. 1981).
3. Davis, E.B.: *Spectral Theory and Differential Operators* (Cambridge University Press, 1995).
4. Rudin, W.: *Functional Analysis* (McGraw-Hill Inc., 1973).
5. DeVito, C.L.: *Functional Analysis and Linear Operator Theory* (Addison Wesley Publishing Co., 1990).

MCO-508 ADVANCED MATHEMATICAL PHYSICS

Nonlinear ordinary differential equations, Bernoulli's equation, Riccati equation, Lane-Emden equation, Nonlinear Pendulum, Duffing's equation, Pinney's equation, Perturbation theory, Bogoliubov-Krilov method. Linear partial differential equations, classification, initial and boundary values problems, Fourier analysis, Heat equation, Wave equation, Laplace equation etc. Integral equations, classification, integral transform separable kernels, singular integral equations, Wiener-Hopf equations, Fredholm theory, series solutions. Variational methods, The Euler-Lagrange equations, Solutions to some famous problems, Sturm-Liouville Problem and variational principles, Rayleigh-Ritz Methods for partial differential equations. Matrix algebra, method of Faddeev, Caley-Hamilton' theorem function of matrices. Functions of matrices, Kronecker and Tensor product, special matrices.

Recommended Books

1. Stephenson, G and Radmore, P.M.: *Advanced Mathematical Methods for Engineering and Science Students* (Cambridge University Press, 1990).
2. Riley, K.F., Hobson, P.M. and Bence, S.J.: *Mathematical Methods for Physics and Engineering* (Cambridge University Press, 2006).
3. Tang, K.T.: *Mathematical Methods for Engineers and Scientists* Volumes 1,2,3 (Springer, 2007).
4. Stone, M. and Goldbart, P.: *Mathematics for Physics* (Cambridge University Press, 2009).
5. Arfken, G.B. and Weber, H.J.: *Mathematical Methods for Physicists* (Academic Press, 2005).

OPTIONAL COURSES:

APPLIED MATHEMATICS

MA-601 GENERAL RELATIVITY-I

Original formulation of Special Relativity. Velocity addition in 3-d formulation. 4-Vector formalism. Poincare group. The null cone. Review of Electromagnetism. 4-Vector formulation of Maxwell's equations. Special Relativity with small accelerations. The principles of General Relativity. The Einstein field equations. The stress-energy momentum tensor. The vacuum Einstein equations and the Schwarzschild solution. Birkhoff's theorem. The Reissner-Nordstrom solution and the generalized Birkhoff's theorem. The Kerr and the Kerr-Newmann solution. The Newtonian limit of Relativity. The Schwarzschild exterior solution and relativistic equations of motion. The classical tests of Relativity and their current status. The Schwarzschild interior solution. Linearized gravity and gravitational waves. Foliations.

Recommended Books

1. Qadir, A.: *Relativity: An Introduction to the Special Theory* (World Scientific, 1989).
2. Stephani, H.: *General Relativity: An Introduction to the Theory of Gravitational Field* (Cambridge University Press, 1990).
3. Wald, R.M.: *General Relativity* (The University of Chicago Press, 1984).
4. Misner, C.W., Thorne, K.S. and Wheeler, J.A.: *Gravitation* (W. H. Freeman and Co., 1973).
5. Plebanski, J. and Krasinski, A.: *An Introduction to General Relativity and Cosmology* (Cambridge University Press, 2006).

MA-602 GENERAL RELATIVITY-II (Pre-requisite of MA-601)

Black holes. Coordinate and essential singularities. Horizons. Coordinates passing through horizons. The Kruskal and the Carter-Penrose (CP) diagrams for the Schwarzschild geometry. The maximal extension. The Einstein-Rosen bridge. Wormholes. The CP diagram for the RN metric. The no-hair and cosmic censorship hypotheses. Gravitational forces about black holes. Black hole thermodynamics. Observational status and central black holes. Kaluza-Klein theory. Problems of quantum gravity. Quantization in curved space backgrounds and Hawking radiations. Isometries. Homotheties and their significance in Relativity.

Recommended Books

1. Wald, R.M.: *General Relativity* (The University of Chicago Press, 1984).
2. Stephani, H.: *General Relativity: An Introduction to the Theory of Gravitational Field* (Cambridge University Press, 1990).
3. Misner, C.W., Thorne, K.S. and Wheeler, J.A.: *Gravitation* (W. H. Freeman and Co., 1973).
4. Plebanski, J. and Kransinski, A.: *An Introduction to General Relativity and Cosmology* (Cambridge University Press, 2006).

5. Carroll, S.M.: *An Introduction to General Relativity: Spacetime and Geometry* (Addison Wesley, 2004).

MA-603 COSMOLOGY

Historical background: Astronomy, Astrophysics and Cosmology. Observational facts about the Universe and its contents. The strong and weak forms of the cosmological principle. The Einstein and de Sitter models of the Universe. The Hubble law and the Friedmann models. Steady state models. The hot big bang model. The microwave background. Discussion of the significance of the start of time. Fundamentals of High Energy Physics. The chronology and composition of the Universe. Non-baryonic dark matter. Problems of the standard model of cosmology. Structures and structure formation in the Universe. The cosmological constant. The inflationary solutions to the problems of the standard model. Later developments on the inflationary models. The Bianchi classification of homogeneous spacetimes. The Kasner (mixmaster) models of the Universe and the generic approach to the initial singularity. BKL-oscillations.

Recommended Books

1. Plebanski, J. and Kransinski, A.: *An Introduction to General Relativity and Cosmology* (Cambridge University Press, 2006).
2. Peebles, P.J.E.: *Physical Cosmology* (Princeton University Press, 1975).
3. Ryan, R. and Shepley, C.: *Homogeneous Cosmology* (Addison Wesley, 1982).
4. Abbott, L.F. and Pi, S.Y.: *Inflationary Cosmology* (World Scientific, 1986).
5. Joshi, P.S.: *Global Aspects in Gravitation and Cosmology* (Oxford University Press, 1993).

MA-604 RELATIVISTIC ASTROPHYSICS

Static stellar structure and the equilibrium conditions. Introduction to stellar modeling. The Hertzprung-Russel diagram and stellar evolution. Gravitational collapse and degenerate stars. White dwarfs, neutron stars and black holes. Systems of stars. Irregular and globular clusters. Galaxies superclusters and filaments. Astrophysical dark matter and galactic haloes.

Recommended Books

1. Chandrsekhar, S.: *An Introduction to the Study of Stellar Structure* (Dover Publications. Inc., 1967).
2. Richard, L., and Deeming, T.: *Astrophysics, Vol. I and II* (Jones and Barlett Publishers, Inc., 1984).
3. Schwarzschild, M.: *Structure and Evolution of Stars* (Dover Publications, New York, 1965).
4. Misner, C.W., Thorne, K.S., and Wheeler, J.A.: *Gravitation* (W.H, Freeman & Co., 1973).
5. Joshi, P.S.: *Global Aspects in Gravitation and Cosmology* (Oxford University Press, 1993).

MA-605 CLASSICAL FIELD THEORY

Review of continuum mechanics. Solid and fluid media. Constitutive equations and conservation equations. The concept of a field. The four dimensional formulation of fields and the stress-energy momentum tensor. The scalar field. Linear scalar fields and the Klein-Gordon equation. Non-linear scalar fields and fluids. The vector field. Linear massless scalar fields and the Maxwell field equations. The electromagnetic energy-momentum tensor. Electromagnetic waves. Diffraction of waves. Advanced and retarded potentials. Multipole expansion of the radiation field. The massive vector (Proca) field. The tensor field. The massless tensor field and the Einstein field equations. Gravitational waves. The massive tensor field. Coupled field equations.

Recommended Books

1. Scipio, L.A.: *Principles of Continua with Applications* (John Wiley, New York, 1969).
2. Landau, L.D. and Lifshitz, M.: *The Classical Theory of Fields* (Pergamon Press, 1980).
3. Jackson, J.D.: *Classical Electrodynmamics* (John Wiley & Sons, 1999).
4. Misner, C.W., Thorne, K.S. and Wheeler, J.A.: *Gravitation* (W.H. Freeman and Co., 1973).
5. Carroll, S.M.: *An Introduction to General Relativity: Spacetime and Geometry* (Addison Wesley, 2004).

MA-606 ELECTRODYNAMICS-I

Maxwell's equations. Electromagnetic wave equation. Boundary conditions. Waves in conducting and non-conducting media. Reflection and polarization. Energy density and energy flux. Lorentz formula. Wave guides and cavity. Resonators. Spherical and cylindrical waves. Inhomogeneous wave equation. Retarded potentials. Lenard Wiechart potentials. Field of uniformly moving point charge. Radiation from a group of moving charges. Field of oscillating dipole. Field of an accelerated point charge.

Recommended Books

1. Reitz, J.R. and Milford, F.J. and Christy, R.W.: *Foundations of Electromagnetic Theory* (Addison-Wesley, 1993).
2. Penofskey, W.K.H. and Philips, M.: *Classical Electricity and Magnetism* (Addison-Wesley, 1962).
3. Corson, D. and Lorrain, P.: *Introduction to Electromagnetic Fields and Waves* (W. H. Freeman and Company, 1962).
4. Jackson, J.D.: *Classical Electrodynamics* (John Wiley & Sons, 1999).
5. Ghosh, S.N.: *Electromagnetic Theory and Wave Propagation* (Narosa, 2002).

MA-607 ELECTRODYNAMICS-II (Pre-requisite of MA-606)

General angular and frequency distributions of radiation from accelerated charges. Thomson scattering. Cherenkov radiation. Fields and radiation localized oscillating sources. Electric

dipole fields and radiation. Magnetic dipole and electric quadruple fields. Multipole fields. Multipole expansion of the electromagnetic fields. Angular distribution sources of muiltipole radiation. Spherical wave expansion of a vector plane wave. Scattering of electromagnetic wave by a conducting sphere.

Recommended Books

1. Reitz, J.R., and Milford, F.J.: *Foundations of Electromagnetic Theory* (Addison-Wesley, 1969).
2. Petoskey, K.H., and Philips, M.: *Classical Electricity and Magnetism* (Addison-Wesley, 1962).
3. Corson, D., and Lorrain, P.: *Introduction to Electromagnetic Fields and Waves* (W.H. Freeman, 1962).
4. Jackson, J.D.: *Classical Electrodynamics* (John Wiley & Sons, 1999).
5. Ghosh, S.N.: *Electromagnetic Theory and Wave Propagation* (Narosa, 2002).

MA-608 MAGNETOHYDRODYNAMICS-I

Basic Equations: Equations of electrodynamics. Equations of Fluid Dynamics. Ohm's law. Equations of magnetohydrodynamics.

Motion of an Incompressible Fluid: Motion of a viscous electrically conducting fluid with linear current flow. Steady state motion along a magnetic field. Wave motion of an ideal fluid.

Small Amplitude MHD Waves: Magneto-sonic waves. Alfven's waves. Damping and excitation of MHD waves. Characteristics lines and surfaces.

Simples Waves and Shock Waves in Magnetohydrodynamics: Kinds of simple waves. Distortion of the profile of a simple wave. Discontinuities. Simple and shock waves in relativistic magnetohydrodynamics. Stability and structure of shock waves. Discontinuities in various quantities. Piston problem. Oblique shock waves.

Recommended Books

1. Cowling, T.G.: *Magnetohydrodynamics* (Interscience Publishers, 1963).
2. Kulikowsky, A.G., and Lyabinov, A.G.: *Magnetohydrodynamics* (A.Weslev, 1965).
3. Anile, A.M.: *Relativistic Fluids and Magneto-Fluids with Applications in Astrophysics and Plasma Physics* (Cambridge University Press, 1989).
4. Anderson, J.E.: *Magnetohydrodynamics, Shock Waves* (M.I.T. Press, 1975).
5. Punsley, B.: *Black Hole Gravitohyrodynamics* (Springer-Verlag, Berlin, 2001).

MA-609 MAGNETOHYDRODYNAMICS-II (Pre requisite of MA-608)

Flow of Conducting Fluid Past Magnetized Bodies: Flow of an ideal fluid past magnetized bodies. Fluid of finite electrical conductivity flow past a magnetized body.

Dynamo Theories: Elsasser's Theory. Bllard's Theory. Earth's field. Turbulent motion and dissipation. Vorticity anology.

Ionized Gases: Effects of molecular structure. Currents in a fully ionized gas. Partially ionized gases. Interstellar fields. Dissipation in hot and cool clouds.

Recommended Books

1. Cowling, T.G.: *Magnetohydrodynamics* (Interscience Publishers, 1963).
2. Kulikowsky, A.G., and Lyabinov, A.G.: *Magnetohydrodynamics* (A. Weslev, 1965).
3. Akhiezer et.al.: *Plasma Electrodynamics* (Pergamon Press, 1975).
4. Anderson, J.E.: *Magnetohydrodynamics, Shock Waves* (M.I.T. Press, 1975).
5. Punsley, B.: *Black Hole Gravitohyrodynamics* (Springer-Verlag, Berlin, 2001).

MA-610 FLUID DYNAMICS

Euler's equation of motion. Viscosity. Navier-Stoke's equations and exact solutions. Dynamical similarity and Reynold's number. Turbulent flow. Boundary layer concept and governing equations. Reynold's equations of turbulent motion. Magnetohydrodynamics. MHD equations. Fluid drifts. Stability and equilibrium problems.

Recommended Books

1. Batchelor, G.K.: *Fluid Dynamics* (Cambridge University Press, 1967).
2. Allen, T. and Ditsworth, R. L.: *Fluid Mechanics* (McGraw- Hill, Inc., New York, 1972).
3. Curie, I.G.: *Fundamental Mechanics of Fluids* (McGraw- Hill, Inc., 1993).
4. Acheson, D.J.: *Elementary Fluid Dynamics* (Oxford University Press, Inc., New York, 2000).
5. Meyer, R.E.: *Introduction to Mathematical Fluid Dynamics* (Dover Publication, 2007).

MA-611 ELASTODYNAMICS

Some topics in theory of elasticity. Equation of wave motion in an elastic medium. Reflection and transmission at a plane interface. Surface waves. Dispersion. Waves in infinite media. Half-space problems. Diffraction and scattering due to irregular structures.

Recommended Books

1. Achenbach, J.D.: *Wave Propagation in Elastic Solids* (North-Holland Publishing Company, Oxford, 1984).
2. Dieulesant, D. and Royer, F.: *Elastic Waves in Solid* (John Wiley & Sons, New York, 1980).
3. Graff, K.F.: *Wave Motion in Elastic Solids* (Clarendon Press, Oxford, 1991).
4. Hudson, J.A.: *The Excitation and Propagation of Elastic Waves, Vol. I & II* (Cambridge University Press, Cambridge, 1985).

MA-612 PLASMA PHYSICS

Introduction to plasma physics, occurrence of plasmas in nature, concept of temperature, Debye shielding, criteria for plasmas, applications of plasma physics. Single particle motion, motion of charged particles in uniform E and B fields, adiabatic invariants. Plasmas as fluids, relation of plasma physics to ordinary electromagnetic, the fluid equation of motion, equation of continuity, the complete set of fluid equations, plasma approximations. Waves in plasmas, representation of waves, group velocity, plasma oscillations, electron plasma waves, sound waves, ion waves, validity of plasma approximation, comparison of ion wave and electron wave, electrostatic electron oscillations perpendicular to B, electrostatic ion waves perpendicular to B, the lower hybrid frequency, EM waves with Bo=0, EM waves perpendicular to Bo, cutoffs and resonances, EM waves parallel to Bo, hydromagnetic waves, magnetosonic waves.

Books Recommended

1. Bittoncourt, J.A.: *Fundamentals of Plasma Physics* (3^{rd} edition) (Springer-Verlag, 2004).
2. Bellan, P.M.: *Fundamentals of Plasma Physics* (Cambridge University Press, 2008).
3. Chen, F.F.: *Introduction to Plasma Physics and Controlled Fusion* (second edition) (Plenum Press, 1984).
4. Goldston, R.J. and Rutherford, P.H.: *Introduction to Plasma Physics* (IOP Publishing Limited, 1995).
5. Gurnett, D.A. and Bhattacharjee, A.: *Introduction to Plasma Physics with Space and Laboratory Applications* (Cambridge University Press, 2005).
6. Dendy , R.O.: *Plasma Dynamics* (Clarendon Press, 1990).

MA-613 ADVANCED COURSE IN PLASMA PHYSICS

Diffusion and mobility in weakly ionized gases, decay of plasma by diffusion, steady state solutions, recombination, diffusion across a magnetic field, collisions in fully ionized plasmas, the single-fluid MHD equations, diffusion in fully ionized plasmas, solutions of the diffusion equation, Bohm diffusion and neoclassical diffusion. Equilibrium and stability – introduction, hydromagnetic equilibrium, the concept of B. diffusion of magnetic field into a plasma, classification of instabilities, two-stream instability, the "Gravitational" instability, resistive drift waves, the Weibel instability. Kinetic theory, the meaning of $f(v)$, equations of kinetic theory, derivations of fluid equations, plasma oscillations and Landau damping, a physical derivation of Landau damping, ion Landau damping, kinetic effects in a magnetic field. Nonlinear effects – introduction, sheaths, ion acoustic shock waves, the pondermotive force, parametric instabilities, plasmas echoes, nonlinear landau damping, equations of nonlinear plasma physics.

Books Recommended

1. Bittoncourt, J.A.: *Fundamentals of Plasma Physics* (3^{rd} edition) (Springer-Verlag, 2004).
2. Bellan, P.M.: *Fundamentals of Plasma Physics* (Cambridge University Press, 2008).
3. Chen, F.F.: *Introduction to Plasma Physics and Controlled Fusion* (second edition) (Plenum Press, 1984).

4. Goldston, R.J. and Rutherford, P.H.: *Introduction to Plasma Physics* (IOP Publishing Limited, 1995).
5. Gurnett, D.A. and Bhattacharjee, A.: *Introduction to Plasma Physics with Space and Laboratory Applications* (Cambridge University Press, 2005).
6. Dendy , R.O.: *Plasma Dynamics* (Clarendon Press, 1990).

MA-614 QUANTUM FIELD THEORY

Classical field theory, lagrangian mechanics, variational principle, vibrating stings, classical field theory, Lorentz transformations, Lorentz group, representations of Lorentz group, classical scalar fields, Klein-Gordon equation, complex scalar fields, energy-momentum tensor, electromagnetic field, Maxwell's equations, spinor field, Dirac equation, symmetries and conservation laws, Noether's theorem, translation invariance. Quantization of fields, canonical quantization of fields, quantization of scalar fields, particle interpretation of quantum field theory, normal ordering, non-Hermitian fields. Interacting Quantum Fields, interacting fields, perturbation theory, time ordering, S-matrix, cross section, decay rate of an unstable particle, higher order perturbation theory, Wick's theorem second order perturbation theory, Feynman rules and diagrams, renormalization, mass renormalization, coupling constant renormalization, field renormalization.

Books Recommended

1. Bogoliubov, N.N. and Shirkov, D.V.: *Introduction to the Theory of Quantized Fields* (Wiley, 1959).
2. Itzykson, C. and Zuber, J-B.: *Quantum Field Theory* (McGraw-Hill, 1980).
3. Ryder, L.H.: *Quantum Field Theory* (Cambridge University Press, 1999).
4. Kaku, M.: *Quantum Field Theory* (Oxford University Press, 1993).
5. Mandl, F. and Shaw, G.: *Quantum Field Theory* (Wiley, 1984).
6. Bjorken, J.D. and Drell, S.D.: *Relaivistic Quantum Field* (McGraw-Hill, 1965).
7. Weinberg, S.: *The Quantum Theory of Fields* Vol. I, II (Cambridge University Press, 1995).

MA-615 NON NEWTONIAN FLUID MECHANICS

Classification of non-Newtonian fluids. Rheological formulae (Time-independent fluids, thixotropic fluids and viscoelastic fluids). Variable viscosity fluids. Cross viscosity fluids. The deformation rate. Viscoelastic equation. Materials with short memories. Time dependent viscosity. The Rivlin-Ericksen fluid. Basic equations of motion in rheological models. The linear viscoelastic liquid. Couette flow. Poiseuille flows. The current semi-infinite field. Axial oscillatory tube flow. Angular oscillatory motion. Periodic transients. Basic equations in boundary layer theory. Orders of magnitude. Truncated solutions for viscoelastic flow. Similarity solutions. Turbulent boundary layers. Stability analysis.

Recommended Books

1. Harris, J.: *Theology and Non-Newtonian Flow* (Longman Inc., 1977).
2. Schowalter, W.R.: *Mechanics of Non-Newtonian Fluids* (Pergamon Press, 1978).
3. Bird, R.B., Armstrong R.C. and Hassager, O.: *Dynamics of Polymeric Liquids*, Vol. 1st, 2nd Edition (John Wiley & Sons, 1987).

4. Bird, R.B., Stewart, W.E. and Lightfoot, E.N.: *Transport Phenomena*, 2nd Edition (John Wiley & Sons, Inc., 2002).
5. Hughes, W.F. and Brighton, J.A.: *Fluid Dynamics*, Schaum's Outlines Series (Tata McGraw Hill, 2004).

MA-616 INTRODUCTION TO BIO MATHEMATICS

Continuous and discrete population models for single species. Models for interacting populations. Age-structured populations. Stochastic population growth. Dynamics of infectious diseases. Historical aside on epidemics. Simple epidemic models and practical applications. Modeling venereal diseases. Multi-Group model for gonorrhea and its control. Modeling of AIDS and HIV. Modeling the transmission. Dynamics of the human immunodeficiency virus (HIV). Modeling combination. Drug therapy. Delay model for HIV infection with drug therapy. Modeling the population dynamics of acquired immunity to parasite infection. Age-dependent epidemic model and threshold criterion. Existence and stability of equilibrium. The disease-free equilibrium and its stability. The endemic equilibrium and its stability. Local and global stability.

Recommended Books

1. Murray, J.D.: *Mathematical Biology I: An Introduction*, 3rd Edition (Springer, 2002).
2. Farkas, M.: *Dynamical Models in Biology (*Academic Press 2001).
3. Murray, J.D.: *Mathematical Biology II: Spatial Models and Biomedical Applications*, 3rd Edition (Springer-Verlag, 2002).
4. Shier, D.R. and Wallenius K.T.: *Applied Mathematical Modeling* (Chapman & HALL/CRC, 1999).
5. Velten, K.: Mathematical Modeling and Simulation: *Introduction for Scientists and Engineers* (Wiley-VCH Verlag GmbH & Co. KGaA Weinheim, 2009).

COMPUTATIONAL MATHEMATICS

MC-621 THEORY OF SPLINE FUNCTIONS I

Parametric Curves: Affine Maps: Translation, Rotation, Reflection, Stretching, Scaling and shear. Barycentric combination. Convex combination. Convex Hull. Forms of parametric curves: Algebraic form, Hermite form, Control point form, Bernstein Bezier form and their matrix forms. Algorithm to compute Bernstein Bezier form. Properties of Bernstein Bezier form: Convex Hull property. Affine invariance property, Variation diminishing property. Rational quadratic form. Rational cubic form. Tensor product surface.

Spline Functions: Natural splines. Cardinal splines. Periodic splines on uniform mesh. Representation of spline and its different forms. Natural spline and periodic spline in terms of polynomials and power truncated functions. Odd degree spline. Existence theorem. Existence and uniqueness of natural and periodic spline. Remainder theorems.

Recommended Books

1. Brannan, D.A.: *Geometry* (Cambridge University Press, 1999).
2. Farin, G.: *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide* (Academic Press Inc., 2002).
3. Bartels, R.H., Bealty, J.C. and Beatty, J.C.: *An Introduction to Spline for use in Computer Graphics and Geometric Modeling* (Morgan Kaufmann Publisher, 2006).
4. de Boor, C.: *A Practical Guide to Splines* (Springer Verlag, 2001).
5. Schumaker, L.L.: *Spline Functions: Basic Theory* (John Wiley, 1993).
6. Wang, R.H.*: Multivariate Spline Functions and Their Applications* (Mathematics and its Applications) (Science Press/ Kluwer Academic Publishers, 2005).

MC-622 THEORY OF SPLINE FUNCTIPONS II (Pre-requisite MC-614)

Interpolatory cubic splines. The representation of s in terms of the values $M_i=s^{(2)}(x_i)$, $i=0,1,2,\ldots,k$. The representation of s in terms of the values $m_i=s^{(1)}(x_i)$, $i=0,1,2,\ldots,k$. Quadratic Hermite spline. Theorems regarding error analysis. Theorems regarding to Convergence of the D1, D2, natural and periodic splines. End conditions for cubic Hermite spline interpolation. E(α)-cubic splines.

Recommended Books

1. Farin, G.: *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide* (Academic Press Inc., 2002).
2. Faux, I.D.: *Computational Geometry for Design and Manufacture* (Ellis Horwood, 1979).
3. Bartels, R.H., Bealty, J.C. and Beatty, J.C.: *An Introduction to Spline for use in Computer Graphics and Geometric Modeling* (Morgan Kaufmann Publisher, 2006).
4. de Boor, C.: *A Practical Guide to Splines* (Springer Verlag, 2001).
5. Schumaker, L.L.: *Spline Functions: Basic Theory* (John Wiley, 1993).
6. Wang, R.H.*: Multivariate Spline Functions and Their Applications* (Mathematics and Its Applications) (Science Press/ Kluwer Academic Publishers, 2005).

MC-623 THEORY OF SPLINE FUNCTIONS III (Pre-requisite MC-615)

End conditions for interpolatory spline with unequally spaced knots. Superconvergence (Equally-spaced knots). Cubic spline collocation for two point boundary value problems. B-spline representation in terms of divided differences. The B-spline representation of spline functions. Computational considerations: The representation of B-splines (Method based on the recursive definition of divided differences). Method of additional knots. The computation of s(x). The computation of derivatives.

Recommended Books

1. Farin, G.: *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide* (5^{th} edition) (Academic Press Inc., 2002).
2. Faux, I.D.: *Computational Geometry for Design and Manufacture* (Ellis Horwood, 1979).
3. Bartels, R.H., Bealty, J.C. and Beatty, J.C.: *An Introduction to Spline for use in Computer Graphics and Geometric Modeling* (Morgan Kaufmann Publisher, 2006).
4. de Boor, C.: *A Practical Guide to Splines* (Springer Verlag, 2001).
5. Schumaker, L.L.: *Spline Functions: Basic Theory* (John Wiley, 1993).
6. Wang, R.H.*: Multivariate Spline Functions and Their Applications* (Mathematics and Its Applications) (Science Press/ Kluwer Academic Publishers, 2005).

MC-624 SUBDIVISION SCHEMES

Subdivision Curves: Notion of subdivision: Piecewise polynomial curves. Definition of B-splines. Refinability of B-splines. Refinement for spline curves. Subdivision for spline curves. Discrete convolution. Analysis of Subdivision: Invariant neighborhoods. Eigen analysis. Convergence of subdivision. Invariance under affine transformations. Geometric behavior of repeated subdivision. Size of the invariant neighborhood. Continuous and discrete Fourier transform. Corner cutting schemes. x-point schemes (3-point, 4-point). Analysis of subdivision curves.

Subdivision Surfaces: Primal and dual schemes. Mask of the scheme. Stationary, Non-Stationary, Linear, Non-linear, Uniform, Non-uniform, Interpolating, approximating, Quadrilateral, Triangular and Hexagonal schemes. Natural parameterization of subdivision surfaces. Subdivision matrix. Smoothness of surfaces. Analysis of subdivision surfaces. Catmull-Clark scheme. Doo-Sabin scheme. Loop scheme. Modified Butterfly scheme.

Recommended Books

1 Warren, J. and Weimer, H.: *Subdivision Methods for Geometric Design: A Constructive Approach* (The Morgan Kaufmann Series in Computer Graphics, 2004).
2. Farin, G.: *Curve and Surfaces for Computer Aided Geometric Design: A Practical Guide*(Academic Press, Inc., 1990).
3. Barrtels, R.H et al.: *An Introduction to Spline for use in Computer Graphics and Geometric Modeling* (Morgan Kaufmann Publisher, Inc., 2006).
4. Schumaker, L.L.: *Spline Functions: Basic Theory* (John Wiley, 1993).
5. Faux, I.D.: *Computational Geometry for Design and Manufacture* (Ellis Horwood, 1979).

6. de Boor, C.: *A Practical Guide to Splines* (Springer Verlag, 2001).

MC-625 APPROXIMATION THEORY

The Approximation problem. Function spaces and related theorems. Existence theorem for linear approximation problem. Uniqueness theorem. Chebyshev approximation. Least squares approximation. Gram-Schmidt orthonormalization process. Orthogonal function. Pade approximation. Remex algorithm.

Recommended Books

1. Rice, J.R.: *The Approximation of Functions Vol. I* (Addison-Wesley Publishing Co., 1964)
2. Rivlin, T.J.: *An Introduction to the Approximation of Functions* (Dover, 1981).
3. Powell, M.J.D.: *Approximation Theory and Methods* (Cambridge University Press, 1988).
4. Achieser, N.I.: Theory of approximation (Dover, 2004).
5. Cheney, E.W.: *Introduction to Approximation Theory* (American Mathematical Society, 2000).

MC-626 NUMERICAL SOLUTIONS OF PDEs

Classification of linear second order PDEs in two and more variables. Laplacian Equations. Heat Equation. Review of analytical methods of solving Laplace, Heat and Wave equations. Finite difference and finite element methods. Error analysis of methods. Numerical Solutions for Elliptic, Parabolic and Hyperbolic PDEs. Irregular Boundries.

Recommended Books

1. Gerake, C.F.: *Applied Numerical Analysis* (Addison Wesley, 1970).
2. Smith, D.G.: *Numerical Solution of Partial Differential Equations: Finite Difference Methods* (Oxford Press, 1990).
3. Ames, W.F.: *Numerical Methods for Partial Differential Equations* (Academic Press, 1997).
4. Henwood, D. and Bonet, J.: *Finite Elements* (Macmillan Press, 1996).
5. Becker, E.B., Carey, G.F. and Oden, J.T.: *Finite Elements* (Prentice Hall, 1981).
6. Bickford, W.B.: *A First Course in Finite Elements* (Irwin Inc., 1994).

MC-627 MINIMAL SURFACES

Regular surfaces: Differentiable functions on surfaces. The tangent plane. Geometric definition of area. Gaussian and mean curvature. Curvature in local coordinates. Ruled and minimal surfaces: Historical survey and introduction to the theory of minimal surfaces. Basic minmal surface properties. Topological and physical properties. Stable
and unstable minimal surfaces. Two dimensional minimal surfaces in three dimentional space. Helecoid, catenoid and conoid. Harmonic approximation to area. Nambu- Goto action.

Recommended Books

1. Carmo, D.: *Differential Geometry* (Prentice Hall, 1982).
2. Fomenko, A.T.: *The Plateau Problem* (Gorden and Breach Science Publishers, 1990).
3. Dierkes, U.: *Minimal Surfaces* (2^{nd} edition) (Springer, 2005).
4 Massari, U.: *Minimal Surfaces of Codimention I* (Elsevier Science Ltd., 1984).
5. Yang, K.: *Minimal surfaces of finite Total Curvature* (Springer, 1994).
6. Nitsche, J. C. C.: *Lectures on Minimal Surfaces* (Cambridge University Press, 1989).

MC-628 COMPUTER GRAPHICS

Introduction to computer graphics and its applications. Overview of raster graphics and transformation pipeline, i.e. transformations between different coordinate systems which involve modelling coordinate system. Device coordinate system. World coordinate system. Normalized coordinate system. Display window coordinate system and screen coordinate system. Graphics output primitives in drawing of lines, polygons, triangles, etc. Draw polylines with different line joining methods. Attributes of graphics primitives like colour, line style and fill style. 2D and 3D transformations and viewing. Describing and using viewing parameters to change the shape of the object, using viewport to change the ratio of clipping window. Differences in viewing and modelling transformations. Window clipping by Cohen-Sutherland algorithm.

Recommended Books

1. Donald, H. and Baker, M. P.: *Computer Graphics with OpenGL* (Prentice Hall, 2003).
2. James, D. Foley et al.: I*ntroduction to Computer Graphics* (Addison-Wesley, 1993).
3. Richard, S. Wright, Benjamin Lipchak: *OpenGL SuperBible* (Sams, 2004.

MC-629 MATHEMATICAL MODELLING-I

Introduction to Modelling. Collection and interpretation of data. Setting up and developing models. Checking models. Consistency of models. Dimensional analysis. Discrete models. Multivariable models. Matrix models. Continuous models. Modelling rates of changes. Limiting models. Graphs of functions as models. Periodic models. Modelling with difference equations. Linear, Quadratic and Non-Linear Models.

Recommended Books

1. Edwards, D. and Hamson, M.: *Mathematical Modelling Skills* (Macmillan Press Ltd., 1996).
2. Giordano, F.R., Weir, M.D. and Fox, W.P.: *A First Course in Mathematical Modelling* (Thomson Brooks/Cole, 2003).
3. Law, A.M. and Kelton, W.D.: *Simulation Modelling and analysis* (McGraw-Hill, 1982).
4. Spriet, J.A. and Vnsteenkiste, G.C.: *Computer Aided Modelling and Simulation*, (Academic Press, 1982).
5. Aris, R.: *Mathematical Modelling Techniques* (Dover Publication, 1995).

MC-630 MATHEMATICAL MODELLING-II (Pre-requisite of MC-622)

Modeling with Differential Equations: Exponential growth and decay. Linear, non-linear systems of differential equations. Modeling with integration. Modeling with random numbers: Simulating qualitative random variables. Simulating discrete random variables. Standard models. Monte Carlo simulation. Fitting models to data. Bilinear interpolation and Coons patch.

Recommended Books

1. Edwards, D. and Hamson, M.: *Mathematical Modelling Skills* (Macmillan Press Ltd., 1996).
2. Giordano, F.R., Weir, M.D. and Fox, W.P.: *A First Course in Mathematical Modelling* (Thomson Brooks/Cole, 2003).
3. Law, A.M. and Kelton, W.D.: *Simulation Modelling and analysis* (McGraw-Hill, 1982).
4. Spriet, J.A. and Vnsteenkiste, G.C.: *Computer Aided Modelling and Simulation,* (Academic Press, 1982).
5. Aris, R.: *Mathematical Modelling Techniques* (Dover Publication, 1995).

MC-631 ADVANCED COMPUTATIONAL METHODS

Review of some fundamental concepts of computer programming. Convergence. Stability and accuracy of algorithms. Algorithms for solution of systems of linear equations. Iterative sparse methods and direct sparse methods. Solving systems of nonlinear equations. Numerical algorithms for ODEs. Automatic differentiation and sensitivity analysis. Introduction to spectral methods. Collocation and Galerkin methods. Spectral methods for PDEs. Differentiation in spectral space. Fourier and Chebyshev methods. Application to solve linear and nonlinear problems.

Recommended Books

1. Otto, S.R. and Denier, J. P.: *An Introduction to Programming and Numerical Methods in Matlab* (Springer, 2005).
2. Higham, N.J.: *Accuracy and Stability of Numerical Algorithms*, 2nd Edition. SIAM (Philadelphia, 2002).
3. Golub, G.H. and Van Loan, C. F.: *Matrix Computations*, 2nd Edition. The Johns Hopkins University Press (Baltimore, 1996).
4. Saad, Y.: *Iterative Methods for Sparse Linear Systems,* 2nd Edition (SIAM, 2003).
5. Laub, A.J.: *Computational Matrix Analysis* (SIAM, 2012).

MC-632 SOBOLEV GRADIENTS AND DIFFERENTIAL EQUATIONS

Introduction to Hilbert spaces. Review of approximation theory. Gradients in Hilbert spaces. Projections. Projections in discrete settings. Reisz representation theorem. Adjoint operators. steepest descent method. Prototypical problem in finite difference setting. Sobolev gradients. Solution of prototypical problem using Sobolev gradients. Comparison between the two methods. Introduction to spectral methods. Solving linear differential equations using a combination of Sobolev gradients and spectral methods.

Recommended Books

1. Neuberger, J.W.: *Sobolev Gradients and Differential Equations* (Springer, 2010).
2. Kreyszig, E.: *Introductory Functional Analysis with Applications* (John Wiley & Sons, 10^{th} Edition, 1989).
3. Sial, S.: *Notes on Sobolev Gradients and Control Problems* (2016).
4. Otto, S.R. and Denier, J. P.: *An Introduction to Programming and Numerical Methods in Matlab* (Springer, 2005).

Annexure-A

MC-633

Quantum Computation and Quantum Information

Introduction:

Quantum Computation and Quantum Information is an emerging and vibrant field of research. In contrast to classical computation which is based on Boolean algebra, quantum computation is based on the laws of nature working at the microscopic scale, namely, quantum mechanical scale. In addition, this course relies on Linear Algebra which is a fundamental course to learn for every graduate student of Mathematics. The motivation for using quantum computation originates from its superior computational efficiency over its classical counterpart. There are certain computational problems, for instance, prime factorization problem which may take hundreds of years on classical super-computer. However, in the domain of quantum computation, such tasks may be solved within hours. Keeping in view, this key factor, technology giants like Google, IBM and Microsoft are investing huge amounts in developing quantum computer which is still in early stages.

Student's performance will be evaluated on group projects handed out in class, drop quizzes and term exams.

Objectives:

1. To familiarize the students with classical computational concepts and their limitations.
2. To provide the preliminary ideas and concept used in quantum computation and information and to enable the students to compare classical and quantum computational paradigms.
3. To provide various applications and examples of quantum computation which may enhance the students understanding and insight regarding latest trends in the field of quantum computation and quantum information theory.

Preliminaries

- Classical computer technology and historical background
- Basic principles and postulates of quantum mechanics: Hilbert space, quantum states, evolution of quantum state, quantum measurement, superposition
- Operator function, density matrix representation, Schrodinger equation.

Quantum states and their properties

- Comparison of bits and qubits, pure and mixed quantum states, Bloch sphere representation of pure and mixed quantum states.
- Entangled and separable quantum states, quantum entanglement
- Measures of quantum entanglement, no-cloning theorem
- Schmidt decomposition, positive operator valued measurements (POVM), EPR states and Bell's inequality

Quantum Gates and Logic

- Single qubit operation, controlled operations, measurement
- Universal quantum gates, CNOT gate
- Quantum circuits

Applications of Quantum Computation

- Quantum teleportation, super-dense coding, quantum state discrimination, various distance measures between quantum states
- Quantum Fourier transform, quantum cryptography, Schor's algorithm uncertainty principle, polarization and spin basis
- BB84, BB90 and Ekert protocols
- Quantum image processing

Zahid Hussain

Recommended Books

1. R. Liboff, Introductory Quantum Mechanics, Pearson, 4th Edition, 2011.
2. M. A. Nielson and I. L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 10th Edition, 2010.
3. C. P. Williams and S. H. Clearwater, Exploration in Quantum Computation, Springer, 2nd Edition, 2011.
4. P. Bouwmester, A. K. Ekert and A. Zeilinger, The Physics of Quantum Information, Springer, 2010.
5. R. K. Brylinsky and G. Chen, Mathematics of Quantum Computation, CRC Press, 1st Edition, 2002.
6. E. G. Rieffel and W. H. Polak, Quantum Computing: A Gentle Introduction, The MIT Press, 2014.

Zahid Hussain

PURE MATHEMATICS

MP-641 RINGS AND MODULES

Elementary concepts of rings. Nilpotent element. Units. Prime ideals and maximal ideals. Nilradical and Jacobson radical. Operations on ideals. Extension and contraction. Modules and module homomorphism. Submodules and quotient modules. Operations on submodules. Direct sum and product. Finitely generated modules. Exact sequences. Projective and injective modules. Tensor product of modules. Exactness properties of the tensor product.

Recommended Books

1. Atiya, M.F. and Macdonal, I.G.: *Introduction to Commutative Algebra* (Addision Wesley Publishing Company London, 1969).
2. Burton, D.E.: *A First Course in Rings and Ideals* (Addision Wesley Pub. Co., 1968).
3. Zauki, O.S.P.: *Commutative Algebra* (London, 1963).
4. Kaplansky, I.: *Commutative Rings* (Univ. of Chicago Press, 1974).
5. Herstien, I.N.: *Non Commutative Rings* (Math Association of America).
6. Anderson, F.W. and Fuller, K.R.: *Rings and Category of Modules* (2^{nd} edition) (Springer-Verlag, New York, 1992).

MP-642 OPERATOR THEORY

Spectral Theory of Linear operators in Normal spaces: Spectral theory in finite dimensional normed spaces. Basic concepts. Spectral properties of bounded liner operators. Further properties of resolvent and spectrum. Use of complex analysis in spectral theory. Basic algebras. Further properties of Banach algebra.

Compact linear Operators on Normed Spaces and their Spectrum: Compact linear operators on normed spaces. Further properties of compact linear operators. Special properties of compact liner operators on normed spaces.

Spectral Theory of Bounded Self-Adjoint Operators linear operators: Spectral properties. Operations of bounded self adjoint linear operators. Positive operators. Square roots of positive operators. Pprojection operators.

Recommended Books

1. Kreyszig, E.: *Introductory Functional Analysis with Applications* (John Wiley, 1989).
2. Nachbin, L. *Introduction to functional Analysis: Branch Spaces and Differential Calculus* (Marcel Dekker, Inc. 1981).
3. Rudin, W.: *Functional Analysis* (McGraw-Hill Inc., 1973).
4. Davis, E.B.: *Spectral Theory and Differential Operators* (Cambridge University Press, 1995).
5. DeVito, C.L.: *Functional Analysis and Linear Operator Theory* (Addison Wesley Publishing Co., 1990).

MP-643 LIE ALGEBRAS & LIE GROUPS

Definition and construction of Lie and associative algebras: Weakly closed subsets of an associative algebra. Nil weakly closed sets. Engel's theorem. Primary components. Weight spaces. Lie algebras with semi-simple enveloping algebras. Lie's theorems. Application to abstract Lie algebras. Some counter examples.

Recommended Books

1. Jacobson, N.: *Lie Algebras* (Dover Publication, 1979).
2. Humphreys, J.E.: *Introduction to Lie Algebras and Representation Theory* (Springer-Verlag, 1997).
3. Gilmore, R.: *Lie Groups, Lie Algebras and Some of Their Applications* (Dover Publication, 2006).
4. Erdmann, K. and Mark, W.: *Introduction to Lie Algebras* (Springer, 2006).
5. Iachello, F.: *Lie Algebras and Applications* (Springer, 2006).

MP-644 FIELD EXTENSIONS AND GALOIS THEORY

Extension of a field. Degree of an extension. Finite extensions. Algebraic and transcendental elements, Algebraic extension. Roots of a polynomial. Splitting extension. Automorphisms of a field. Fixed field of a group of homomorphisms. Normal extensions. Galois extensions. Fixed field of a group. Galois group of a polynomial. The fundamental theorem of Galois Theory.

Recommended Books

1. Herstein, I. N.: *Topics in Algebra* (2^{nd} edition) (Lexington Xerox College Publishing, 1975).
2. Artin, E.: *Galois Theory* (2^{nd} edition) (Notre Dame Univ., Indiana, 1966).
3. Stewart, I.: *Galois Theory* (2^{nd} edition) (Chapman and Hall, 1989).
4. Ledermann, W.: *Introduction to Group Theory* (Oliver and Boyd, 1973).
5. Borceux, F. and Janelidze, G.: *Galois Theories* (Cambridge University Press, 2003).

MP-645 LINEAR GROUPS AND GROUP REPRESENTATIONS

Direct Products. Classification of Finite Abelian Groups. Semidirect Products. Extensions. Central and Cyclic Extensions. General Linear Groups. Special and Projective Linear Groups. Tvansvections. Generators of Linear Groups. Exceptional Isomorphism. Simple Linear Groups. Classification of Groups with at most 31 elements.

Recommended Books

1. James, G. and Lieback, M.: *Representations and Characters of Groups* (Cambridge University Press, 2001).
2. Coxeter, H.S.M. and Moser, W.J.: *Generators and Relations for Discrete Groups* (Springer Verlag, 1980)
3. Rotman, J.J.: *An Introduction to the Theory of Group* (Allyn and Bacon, 1994).
4. Lederman, W.: *Introduction to Group Character* (Cambridge University Press, 1987).
5. Alperin, J.L. and Bell, R.B.: *Groups and Representations* (Springer, 1995).
6. Humphreys, J.F.: *A Course in Group Theory* (Oxford University Press, 2004).

MP-646 GENERAL TOPOLOGY

Topological spaces: Product spaces. Weak topologies and quotient spaces.

Convergence: Inadequacy of sequence. Nets and filters. Product and quotient spaces. Covering axioms. Covering of space Para compacts spaces. C-compact spaces. Further theorems on connected spaces pathwise and local connectedness.

Recommended Books

1. Willard, S.: *General Topology* (Addision Wesley, New York, 1970).
2. Munkres, J.R.: *Topology: A First Course* (Prentice-Hall, 1975).
3. Kelley, J.L..: *General Topology* (Springer-Verlag, New York, 1975).
4. Newman, M.H.A.: *Elements of the Topology of Plane Sets of Points* (Cambridge University Press, 1964).
5. Gemignani, M.C.: *Elementary Topology* (Dover Publication, 1990).

MP-647 HOMOTOPY THEORY

Paths and path connected spaces. Homotopy of continuous mappings. Homotopy of paths. Homotopy classes. The fundamental group of a circle. Higher fundamental groups. The fundamental group of covering spaces. Torus. Orbit spaces. Punctured plane and surfaces.

Recommended Books

1. Kosniowski, C.: *A First Course in Algebraic Topology* (Cambridge University Press, 1980).
2. Munkres, K.R.: *Topology* (Prentice Hall of India, 1983).
3. Whitehead, G.W.: *Elements of Homotopy Theory* (Springer, 1995).
4. Selick, P.S.: *Introduction to Homotopy Theory* (American Mathematical Society, 1997).
5. Whitehead,.W.: *Homotopy Theory* (MIT Press, 1971).

MP-648 TOPOLOGICAL GROUPS

Fundamentals of topology and groups. General theory of topological groups. Topological algebraic structures. Topological groups. Separation axioms in topological groups subgroups and quotient groups. Metrizable complete topological groups. Locally compact groups.

General results on locally compact groups. Liner groups. Locally Euclidean groups. Lie groups. Continuous and open homeomorphisms in Topological groups.

Recommended Books

1. Hussain, T.: *Introduction to Topology Groups* (W.B. Saunders Company, New York, 1966).
2. Higgins, P.J.: *An Introduction to Topology Groups* (Cambridge University Press, 1975).
3. Chandrasekharan, K.: *A Course on Topology Groups* (Hindustan Book Agency, 1996).
4. Pontriagin, L.S.: *Topology Groups* (Princeton University Press, 1958).
5. McCarty, G.: *Topology: An Introduction with Applications to Topology Groups* (Dover Publications, 2006).

MP-649 HOMOLOGICAL THEORY

Homological Theory of Simplicial Complexes: The polyhedron. The bary centric subdivision. The Brouwer's fixed point theorem. The homology group of a simplicial complex. Chain complexes and chain maps. Simplicial map and associated chain map.

Singular Homology Theory: Singular homology group. Singular homology sequences. Linear chain complexes. Computations of some homology groups.

Recommended Books

1. Mayer, J.: *Algebraic Topology* (Prentice Hall Inc., 1972).
2. Osborne, M.S.: *Basic Homological Algebra* (Springer, 2000).
2. Weibel, C.A.: *An Introduction to Homological Algebra* (Cambridge University).
4. Wall, C.T.S.: *Homological Group Theory* (Cambridge University Press, 1980).
5. Hilton, P.J. and Stammbach, U.: *A Course in Homological Algebra* (Springer, 1997).

MP-650 LATTICE THEORY

Elementary Concepts: Definition of lattice. Some algebraic concepts. Polynomials. Identities. Inequalities. Free lattices. Special elements.

Distributive lattices: Distributive lattices, Characterization and representation theorems. Polynomials and freeness. Congruence relations. Boolean algebra.

Recommended Books

1. Grazer, G.: *Lattice Theory* (W.H. Freeman and Company, New York, 1971).
2. Maclane, S. and Birkhoff, G.: *Algebra* (Macmillan Company, New York, 1967).
3. Davey, B.A. and Priestley, H.A.: *Introduction to Lattices and Order* (Cambridge University Press, 2002).
4. Blyth, T.S. and Birkhoff, G.: *Lattices and Ordered Algebraic Structures* (Springer, 2005).
5. Donnellan, T.: *Lattice Theory* (Elsevier Science Ltd., 1968).

MP-651 REPRESENTATION THEORY

Representation of groups: Algebras and group modules reducibility and decomposition of representation. Linear representation. Representation of abelian groups. Permutation representation. Maschke's theorem and its application. Chifford's theorem. G-homomorphism. Sehur's lemma. A module and regular representation. Semi simple algebras. Theorems of Burnside. Frobenins. Sehur and wedderburn. Representation of direct and central product.

Group Character: Characters as class function. Character ring. Tensor product. Basic concepts and application. Character table. Burnside's criterion for solvable groups. The Frobenius Wieland theorem on existence of normal subgroups in a group. Theorems of Jordan, Burnside and Schur on linear groups. The lifting process. Brauer's theorem on induced characters and its application. The generalization induction theorems.

Recommended Books

1. Gorenstein, D.: *Finite Groups* (Harper and Row, 1968).
2. Curtis, C.W. and Reiner, I.: *Representations Theory of Finite Groups and Associative Algebras* (Interscience, 1962).
3. Lederman, W.: *Introduction to Group Characters* (Cambridge University Press, 1976).
4. Burrow, M.: *Representation Theory of Finite Groups* (Dover Publications, 1993).
5. James, G. and Liebeck, M.: *Representations and Characters of Group* (Cambridge University Press, 2001).

MP-652 BCK ALGEBRA

Definition of BCK algebra. Examples. General properties of BCK algebra. Commutative BCK algebra. Ideal theory of BCK algebras. Definition, types and examples of ideals in BCK algebra. Self maps of BCK algebra. Types of ideals. Definition of self maps. Right and left self maps. Left regular maps and their general properties. Kernels and annihilators in BCK Algebras. Definition of kernel, annihilator and related theorems.

Recommended Books

1. Meng, J. and Jun, Y.B.: *BCK Algebras* (Kyung Moon Sa. Co., Seoul., 1994).
2. Iseki, K. and Tanaka, S.: *An Introduction to the Theory of BCK Algebra*, Math. Japonica, 23, No.1(1978), 1-26.
3. Anton, H. and Rorres, C.: *Elementary Linear Algebra with Applications* (Wiley, 2005).
4. Anton, H..: *Elementary Linear Algebra* (Wiley, 2000).
5. Vivek, S. and Vikas, B.: *Algebra* (Narosa Publishing House, India, 2003).

MP-653 BCI ALGEBRA

Classification of BCI Algebras: Implicative, Positive implicative BCI algebras. S_1, S2, S3 and S4 algebras.

Classification of ideals in BCI Algebras: Ideals in BCI algebra, Strong and weak ideals. Obstinate ideals. Ideals in P-semi simple algebras. Regular and non regular ideals.

Quotient algebras. Quotient BCI algebras and BCI homomorphism.

Recommended Books

1. Meng, J. and Jun, Y.B.: *BCK Algebras* (Kyung Moon Sa. Co., Seoul., 1994).
2. Iseki, K. and Tanaka, S.: *An Introduction to the Theory of BCK Algebra*, Math. Japonica, 23, No.1(1978), 1-26.
3. Anton, H. and Rorres, C.: *Elementary Linear Algebra with Applications* (Wiley, 2005).
4. Anton, H..: *Elementary Linear Algebra* (Wiley, 2000).
5. Vivek, S. and Vikas, B.: *Algebra* (Narosa Publishing House, India, 2003).

MP-654 ADVANCED THEORY OF RINGS AND MODULES

Rings: Direct product of rings. Rings of polynomials. Fields of fractions. Factorization in integral domains. Sum and direct sum of ideals. Unique factorization domains. Principal ideal domains. Euclidean domains. Polynomial rings over UFD.

Modules: R-homomorphism and quotient Modules. Direct sums and exact sequences. Free modules. Free modules over PIDs. Finitely generated modules over PIDs. Projective and injective modules. Completely reducible modules. Representation of linear mappings. Rank of a linear mapping. Irreducible polynomials and Eisenstein criterion. Adjunction of roots.

Recommended Books

1. Atiya, M.F. and Macdonal, I.G.: *Introduction to Commutative Algebra* (Addision Wesley Publishing Company, 1969).
2. Burton, D.E.: *A First Course in Rings and Ideals* (Addision Wesley Publishing Company, 1968).
3. Kaplansky, I.: *Commutative Rings* (Univ. of Chicago Press, 1974).
4. Vivek, S. and Vikas, B.: *Algebra* (Narosa Publishing House, India, 2003).
5. Bhattacharya, P. B., Jain, S. K. and Nagpaul, S. R.: *Basic Abstract Algebra* (Cambridge University Press, 1995).

MP-655 SPECTRAL THEORY IN HILBERT SPACES – I

(Pre-requisite: Linear Algebra, Real and Complex analysis.)

The concept of Hilbert spaces. Finite dimensional Euclidean spaces. Inner product spaces. Normed linear spaces. The Hilbert spaces. The specific geometry of Hilbert spaces. Subspaces. Othogonal subspaces. Bases. Polynomial bases in L^2 Spaces. Isomorphisms. Bounded linear operators. Bounded linear mappings. Linear operators. Bilinear forms. Adjoint operators. Projection operators. The Fourier-Plancherel operators. General theory of linear operators. Adjoint operators (general case). Differentiation operators in L^2 Spaces. Multiplication operators in L^2 Spaces. Closed linear operators. Invariant subspaces of a linear operator. Eigenvalues of a linear operator. The Spectrum of a linear operator. The spectrum of a self-adjoint operators.

Recommended Books

1. Helmberg, G.: *Introduction to Spectral Ttheory in Hilbert spaces* (N. H. publishing Co., 1969).
2. Riesz, F. and Nagay.B.S.: *Functional Analysis* (Ungar Publishing Co., New York, 1955).
3 Akhiezer, N. I., Glazman, I.M.: *Theory of Linear Operators in Hilbert Spaces, Volume I* (Frederick Ungar, New York, 1963).
4. Debnath, L. and Mikusinski, P.: *Introduction to Hilbert Spaces with Applications* (3rd edition) (Saurabh Printers Noida, 2005).
5. Dunford, N. and Schwartz, J.T.: *Linear Operators,Part-I: General Theory* (Interscience Publishers, New York, 1958).
6 Rudin, W.: *Functional Analysis* (McGraw- Hill, 1973).

MP-656 SPECTRAL THEORY IN HILBERT SPACES – II
(Pre-requisite of MP-639)

Spectral analysis of compact linear operators. Compact linear operators. Weakly converging sequences. The spectrum of a compact linear operator. The spectral decomposition of a compact self-adjoint operator. Fredholm integral equations. Spectral analysis of bounded linear operators. The order relation for bounded self-adjoint operators. Polynomials in a bounded linear operator. Continuous functions of a bounded self-adjoint operator. Step functions of a bounded self-adjoint operator. The spectral decomposition of a bounded self-adjoint operator. Functions of a unitary operator. The spectral decomposition of a unitary operator. The spectral decomposition of a bounded normal operator. Spectral analysis of unbounded self-adjoint operators. The Cayley transform. The spectral decomposition of an unbounded self-adjoint operator. Limit points of a spectrum. Perturbation of the spectrum by the addition of a completely continuous spectrum. Continuous perturbation. Analytic perturbations.

Recommended Books

1. Helmberg, G.: *Introduction to Spectral Theory in Hilbert spaces* (N. H. publishing Co., 1969).
2. Riesz, F. and Nagay.B.S.: *Functional Analysis* (Ungar Publishing Co., New York, 1955).
3 Akhiezer, N. I., Glazman, I.M.: *Theory of Linear Operators in Hilbert Spaces, Volume, I* (Frederick Ungar, New York, 1963).
4. Debnath, L. and Mikusinski, P.: *Introduction to Hilbert Spaces with Applications* (Saurabh Printers Noida, 2005).
5. Dunford, N. and Schwartz, J.T.: *Linear Operators,Part-II: General Theory* (Interscience Publishers, New York, 1963).
6 Rudin, W.: *Functional Analysis* (McGraw- Hill, 1973).

MP-657 HARMONIC ANALYSIS
(Pre-requisite: Linear Algebra, Real and Complex analysis, Topology)

Topology. Sets and Topologies. Separation axioms and related theorems. The Stone-Weierstrass theorem. Cartesian products and weak topology. Banach spaces. Normed linear

spaces. Bounded linear transformations. Linear functionals. The weak topology for X*. Hilbert space. Involution on β (H). Integration. The Daniell integral. Equivalence and measurability. The real L^P -spaces. The conjugate space of $L^{P.}$ Integration on locally compact Hausdorff spaces. The complex L^P –spaces. Banach Algebras. Definition and examples. Function algebras. Maximal ideals. Spectrum; adverse Banach algebras; elementary theory. The maximal ideal space of a commutative Banach algebra. Some basic general theorems

Recommended Books

1. Katznelson, Y.*: An Introduction to Harmonic Analysis* (Dover, New York, 1976).
2 Lindenstrauss, J. and Tasfriri, L.: *Classical Banach Spaces-I* (Springer Verlag, 1977).
3. Lindenstrauss, J. and Tasfriri, L.: *Classical Banach Spaces-II* (Springer Verlag, 1979).
4. Dunford, N. and Schwartz, J.T.: *Linear Operators, Part-III: Spectral Operators* (Interscience Publishers, New York, 1971).
5. Wojtaszczyk, P.: *Banach Spaces for Aanalysts* (Cambridge University Press, 1991).
6. Riesz, F. and Nagay.B.S.: *Functional Analysis* (Ungar Publishing Co., New York, 1955).

MP-658 BANACH ALGEBRAS - I

(Pre-requisite: Algebra, Real and Complex analysis, Topology)

Fundamental Algebraic concepts: Definition and examples of rings. Rings with identity. Center. Ideals. The radical. homomorphism and isomorphism of rings. Regular representations of rings. Topological rings. Definition of a topological ring. Topological adjunction of the identity. Rings with continuous inverse. Resolvents in a ring with continuous inverse. Topological division rings with continuous inverse. Rings with continuous quasi-inverse.

Normed Rings: Definition of a normed ring. Adjunction of the identity. Banach rings with identity. Continuous homomrphisms of normed rings. Regular representations of a nomed ring.

Symmetric Rings: Definition and simplest properties of a symmetric ring. Positive functionals. Normed symmetric rings. Positive functionals in a symmetric Banach ring.

Books Recommended

1. Naimark, M. A.: *Normed Rings* (Noordhoff, 1959).
2. Loomis, L. H.: *An Introduction to Abstract Harmonic Analysis* (Van Nostrand, 1953).
3. Wojtaszczyk, P.: *Banach Spaces for Aanalysts* (Cambridge University Press, 1991).
4. Riesz, F. and Nagay.B.S.: *Functional Analysis* (Ungar Publishing Co., New York, 1955).
5. Alberta, E.: *Banach Algebras and their Applications* (Canada, 2003).
6. Helemskii, A. Y.: *A Banach and Locally Convex Algebras* (Oxford University Press, 1993).

MP-659 BANACH ALGEBRAS – II (Pre-requisite of MP-642)

Commutative Normed Rings: Factor-ring modulo a maximal ideal. Functions on maximal ideals, generated by elements of a ring. Toplogization of the set of all maximal ideals. The case of a ring without identity. System of generators of a ring. Analytic functions of ring elements. Analytic functions of several ring elements. Decomposition of a ring into the direct sum of ideals. Primary ideals. Homomorphism and isomorphism of commutative rings. Uniqueness of the norm in a semi simple ring. The case of symmetric rings.

Ring boundary: Definition and fundamental properties of the ring boundary .Extension of maximal ideals. Completely symmetric commutative rings. Definition of a completely symmetric ring. Criterion for complete symmetry. Application of Stone's theorem. The ring boundary of a completely symmetric ring.

Regular Rings: Definition of a regular ring. Normal ring of functions. Lattice space of a ring. Properties of regular rings. The case of a ring without identity. Sufficient condition that a ring be regular.

Completely regular Commutative rings: Definition and simplest properties of a completely regular ring. Realization of completely regular commutative rings. Generalization to pseudo-normed rings.

Recommended Books

1. Naimark, M. A.: *Normed Rings* (Noordhoff, 1959).
2. Loomis, L. H.: *An Introduction to Abstract Harmonic Analysis* (Van Nostrand, 1953).
3. Wojtaszczyk, P.: *Banach Sspaces for Aanalysts* (Cambridge University Press, 1991).
4. Riesz, F. and Nagay.B.S.: *Functional Analysis* (Ungar Publishing Co., New York, 1955).
5. Alberta, E.: *Banach Algebras and their Applications* (Canada, 2003).
6. Helemskii, A. Y.: *A Banach and Locally Convex Algebras* (Oxford University Press, 1993).

MP-660 ADVANCED MEASURE THEORY (Pre-requisite: Set Theory, Real analysis.)

Riemann-Stieltjes and Lebesgue integration. Classical Banach Spaces. Weierstrass' approximation theorem. Riemann-Stieltjes integration. Lebesgue measurable sets. Lebesgue measure. Lebesgue measurable functions. Lebesgue integral functions. Properties of the Lebesgue integral. Fubini's theorem. Absolutely continuous functions. Differentiation under the integral sign. Classical Banach Spaces. L^P–spaces. Convergence and completeness in L^P–spaces. Bounded linear functional on the L^P–spaces. General convergence theorem. Singed measures. The Radon-Nikodym theorem. Product measures. Inner measure. Extension by sets of measure zero. Caratheodory outer measure. Hausdorff measure

Recommended Books

1. Royden, H.L.: *Real Analysis* (3^{rd} edition) (Macmillan, New York, 1988).
2. Saks, S.: *Theory of the Integrals* Vol. VII (Hafner Publishing Company, 1937).
3. Halmos, P.R.: *Measure Theory* (Von Nostrand, New York, 1950).

4. Bartle, R.G.: *The Elements of Integration and Lebesgue Measure* (Wiley Classics Library, 1995).
5. Loomis, L. H.: *An Introduction to Abstract Harmonic Analysis* (Van Nostrand, 1953).
6. Riesz, F. and Nagay.B.S.: *Functional Analysis* (Ungar Publishing Co., New York, 1955).

MP -661 ADVANCED NUMBER THEORY

A spectacular development (Proof of Fermat's Last Theorem for n=3, 4). Gauss's Quadratic reciprocity law. The Jacobi symbol and applications of Gaussian integers. The ring $\mathbf{Z}[\sqrt{-5}]$, Finite fields. Primitive polynomials. Irreducibility. Gauss lemma and Eisenstein criterion of irreducibility. Number fields and integral dependence. Integers in number fields. Cyclotomic polynomials and fields Class groups. Discriminants. Some results from geometry of numbers. Dirichlet's theorem. Splitting of rational primes. The group of units and Norm-Euclidean number fields.

Recommended Books

1. Harry P. and Harold G.D.: *The Theory of Algebraic Numbers* (The Mathematical Association of America, 1975).
2. Hardy, G.H. and Wright, E.M.: *An Introduction to the Theory of Numbers* (Oxford University Press, 1981).
3. Daniel A.M.: *Number Fields* (Springer-Verlag, 1977).
4. Melvyn B.N.: *Methods in Number Theory (*Springer-Verlag, 2000).
5. Sukumar D.A.: *An Introduction to Commutative Algebra and Number Theory (*Narosa, 1999).
6. Grosswald, E.: *Topics from the Theory of Numbers* (The Macmillan Company, 1966).

MP-662 COMBINATORICS

Elementary concepts of several combinatorial structures. Recurrence relations and generating functions. Principle of inclusion and exclusion. Latin squares and SDRs. Steiner systems. A direct construction. A recursive construction. Paking and covering. Linear algebra over finite fields. Gaussian coefficients. The pigeonhole Principle. Some special cases. Ramsey's theorem. Bounds for Ramsey numbers and applications. Automorphism groups and permutation groups. Enumeration under group action.

Recommended Books

1. Bryant, V.: *Aspects of Combinatorics* (Cambridge University Press, 1993).
2. Graham, R.L., Rothschild, B.L. and Spencer, J.: *Ramsey Theory* (John Wiley & Sons, 1980).
3. Cameron, P.J.: *Combinatorics: Topics, Techniques, Algorithms* (Cambridge University Press, 1994).
4. Biggs, N.L. and White, A.T.: *Permutation Groups and Combinatorial Structures* (Cambridge University Press, 1979).
5. Richard A.B.: *Introductory Combinatorics* (Prentice Hall, 1999).

MP-663 INTRODUCTION TO FUZZY SYSTEMS

The basic concepts of type-1, 2 fuzzy sets (membership, cardinality, normality), set operations (union, intersection, complementation), distances between fuzzy sets (Hamming distance, normalized Hamming distance, Euclidean distance, normalized Euclidean distance), similarity measures, fuzzy relation and composition, fuzzy number, fuzzy function, probability and possibility, fuzzy logic, linguistic variable, fuzzy inference, defuzzification, fuzzy control and fuzzy expert systems. Intuitionistic fuzzy sets (IFSs), distances between IFSs, similarity measures between IFSs, level cut sets, IF relation and composition, IF fuzzy number, Triangular norms, soft sets and set operations, Applications of fuzzy systems in semigroups, groups, semirings, graphs and differential equations.

Recommended Books

1. Chen, G. and Pham, T.: *Introduction to fuzzy systems* (Chapman and Hall/CRC, 2005).
2. Zimmermann, H. -J.: *Fuzzy set theory and its application*, (4th ed. Springer, 2001).
3. Mordeson, J. N. and Malik, D.S.: *Fuzzy semigroups* (Springer, 2010).
4. Zadeh A., Fu, K.-S., Tanaka, K. and Shimura, M.: *Fuzzy sets and their applications to cognitive and decision process* (Academic Press, New York, 1975).
5. Mordeson, J. N., Bhutani, K. R. and Rosenfeld, A.: *Fuzzy Group Theory* (Springer, 2010).
6. Atanassov, K.T.: Intuitionistic fuzzy sets: *Theory and Applications* (Heidelberg, 2012).

MP-664 FUZZY GRAPH THEORY

Fuzzy graphs, operations on fuzzy graphs (Cartesian product, composition, union and join), paths and connectedness, fuzzy bridges and fuzzy cuts, fuzzy forests and fuzzy trees, fuzzy cycles, metric in fuzzy graphs, fuzzy bipartite graphs, domination in fuzzy graphs, automorphism of fuzzy graphs, regular fuzzy graphs, fuzzy planar graphs, fuzzy hypergraphs, intuitionistic fuzzy graphs, intuitionistic fuzzy cycles, intuitionistic fuzzy trees, intuitionistic fuzzy hypergraphs, bipolar fuzzy graphs.

Recommended Books

1. Mordeson, J.N., Nair, P.S.: *Fuzzy Graphs and Fuzzy Hypergraphs* (Heidelberg, 2001).
2. Nagoorgani, A. and Chandrasekaran, V.T.: A First Look at Fuzzy Graph Theory (Allied Publishers, India, 2010).
3. Zadeh A., Fu, K.-S., Tanaka, K. and Shimura, M.: Fuzzy sets and their Applications to Cognitive and Decision Process (Academic Press, New York, 1975).
4. Atanassov, K.T.: Intuitionistic Fuzzy Sets: Theory and Applications (Heidelberg, 2012).

MP-665 GRAPH THEORY

Fundamentals. Definition. Paths cycles and trees. Hamilton cycles and Euler circuits. Planer graphs. Flows, Connectivity and Matching Network flows. Connectivity and Menger's theorem. External problems paths and Complete Subgraphs. Hamilton path and cycles. Colouring. Vertex colouring Edge colouring. Graph on surfaces.

Recommended Books

1. Bollobas, B.: *Graph Theory* (Springer Verlag, NewYork, 1979).
2. Wilson, R.J.: *Introduction to Graph Theory* (Longman London, 1979).
3. Bollobas, B.: *Modern Graph Theory* (Springer Verlag, NY, 2002).
4. Biggs, N.: *Algebraic Graph Theory* (Cambridge University Press, 1974).
5. Gross, J.L and Yellen, J.: *Graph Theory and its Applications* (Chapman and Hall, 2005).

MP-666 DESIGN THEORY

Basic definitions and properties, related structure. The incidence matrix, graphs, residual structures. The Bruck-Ryser-Chowla theorem. Singer groups and difference sets. Arithmetical relations and Hadamard 2- designs. Projective and affine planes. Latin squares, nets. Hadamard matrices and Hadamard 20 design. Biplanes, strongly regular graphs. Cameron's theorem and Hadamard 3-desings. Steiner triple systems. The Mathieu groups.

Recommended Books

1. Hughes, D.R. and Piper, F.C.: *Design Theory* (Cambridge University Press, 1985).
2. Beth, T., Jungnickel, D. and Lenz, H.: *Design Theory* (2^{nd} edition)(Cambridge University Press, 2000).
3. Quinn, K., Webb, B., Rowley, C. and Holroyal, F.C.: Combinatorial *Designs and their Applications* (Chapman and Hall, 1999).
4. Cameron, P.J. and Lint, J. H.: *Designs, Graphs, Codes and their links* (Cambridge University Press, 1991).
5. Cameron, P.J.: *Permutation Groups* (Cambridge University Press, 1999).

MSA-681 Seminar Attendance

All students must attend the weekly Colloquium at Department of Mathematics. At least 80% attendance is necessary for a PhD student.

MSA-682 Seminar Delivered-G

A seminar delivered by a PhD student outside the thrust area chosen.

MSA-683 Seminar Delivered-T

A seminar delivered by a PhD student in the thrust area chosen but not from the student's PhD research area.

MSA-684 Seminar Delivered-R

A seminar delivered by a PhD student in the student's PhD research area.