Course Title: Tensor Analysis

Course Code: MATH-308

Course Type: Major Math

Prerequisites: Multivariable Calculus

Credit Hours: 3 (3 + 0)

Course Objectives: The objectives of this course in Tensor Analysis are to:

- Equip students with a thorough understanding of tensor algebra and calculus.
- Enable students to manipulate tensors of various orders effectively.
- Apply tensor concepts in mathematical and physical contexts.
- Master fundamental concepts such as tensor notation, covariant and contravariant transformations, and differential operators in tensor form.
- Enhance analytical skills through theoretical study and practical applications.

Course Contents:

Introduction: Historical background, Importance of tensor analysis in various fields, Review of linear algebra: vectors and matrices, Basics of multi-variable calculus: partial derivatives, gradients, and Jacobians.

Curvilinear Coordinates: Transformation of Coordinates, Orthogonal curvilinear coordinates, Unit vectors in curvilinear systems, Arc length and volume elements, The gradient, divergence and curl, Special orthogonal coordinate systems, cylindrical, spherical, parabolic cylindrical, paraboloidal, elliptic cylindrical, prolate spheroidal, oblate spheroidal, ellipsoidal, bipolar coordinates.

Tensor Fundamentals and Basic Operations: Spaces of $N-$dimension, coordinate transformations, Einstein summation convention, contravariant and covariant vectors, Tensors of different ranks, Contravariant, covariant and mixed tensors, The Kronecker delta and Levi-Civita symbol, Tensors of rank greater than two, Symmetric and skew-symmetric tensors, Fundamental operations with tensors, Addition, subtraction, inner and outer products of tensors, Contraction theorem, quotient law.

Tensor Transformation and Metric Tensor: Tensor transformation laws, The line element and metric tensor, Conjugate or reciprocal tensors, Associated tensors, Christoffel's symbols, Transformation laws of Christoffel symbols, Geodesic Equation.

Advanced Tensor Calculus and Applications: Differentiation of tensor fields: Covariant and intrinsic derivatives and its applications. Lie derivatives: Scalar field, Vector fields, and Tensor fields, Riemannian metrics, Riemann curvature tensor and its components, The Ricci tensor, The Ricci scalar.

Recommended Books:

1. Chorlton, F., *Vector and tensor methods*, Ellis Horwood Publisher, 1977.

2. Spiegel, M. R., *Vector analysis- An introduction to tensor analysis*, McGraw Hill Book Company, 1981.

3. Joshi, A. W., *Matrices and tensors in physics*, Wiley, 1991.

4. Grinfeld, P., *Introduction to tensor analysis and the calculus of moving surfaces*, Springer, 2013.

5. Nguyen-Schäfer, H., and Schmidt, J. P., *Tensor analysis and elementary differential geometry for physicists and engineers*, Springer, 2014.

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