UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Mathematics A-I [Calculus(I)] **TIME ALLOWED: 30 mins.**

Course Code: MATH-101 / MATH 11010 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION – I

Q.1	**MCQs (1 mark each)**			
(i)	$\int \cos^2 x\, dx = ?$			
	(a) $-\sin^2 x + c$	**(b)** $x - sin2x + c$	**(c)** $\frac{1}{2}(x + sin2x) + c$	**(d)** none of these
(ii)	The velocity at $t = 1$ for the function $S(t) = \frac{t}{t+1}$ is			
	(a) $-\frac{1}{2}$	**(b)** $\frac{1}{4}$	**(c)** $-\frac{3}{2}$	**(d)** $\frac{1}{3}$
(iii)	$\lim_{x\to 0} \frac{x}{tanx} = ?$			
	(a) 0	**(b)** 1	**(c)** 2	**(d)** ∞
(iv)	Every differentiable function is ________________.			
	(a) differentiable	**(b)** integrable	**(c)** continuous	**(d)** exponential
(v)	The critical point of a function $f(x)$ occurs			
	(a) positive	**(b)** undefined	**(c)** zero	**(d)** Both b&c
(vi)	If z is a complex number, then $\bar{z} - z$ is			
	(a) real	**(b)** complex	**(c)** zero	**(d)** prime
(vii)	$\lim_{x\to\infty} \frac{2-3x}{\sqrt[3]{3+8x^3}} = ?$			
	(a) $\frac{2}{3}$	**(b)** $\frac{-3}{2}$	**(c)** $\frac{-2}{81}$	**(d)** none of these
(viii)	$cos30° + i^2 sin60°$ is equal to			
	(a) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$	**(b)** $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$	**(c)** $\sqrt{2}$	**(d)** 0
(ix)	$\lim_{x\to 3} \frac{x^3 - 27}{x - 3} = ?$			
	(a) 0	**(b)** 27	**(c)** -27	**(d)** ∞
(x)	$(\sqrt{3} + i)^3$ is equal to			
	(a) $3\sqrt{3}$	**(b)** $8i$	**(c)** $-8i$	**(d)** none of these

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-I [Calculus(I)] **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-101 / MATH-11010 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	Evaluate $\left(\frac{-2+i\sqrt{3}}{\sqrt{5}-6i}\right)^2$	(4)
(ii)	Evaluate $\int_0^1 x(1-\sqrt{x})^2\,dx$.	(4)
(iii)	If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$.	(4)
(iv)	Find the area of the region bounded by the curves $y = 3 - x^2$ and $y = 1 - x$.	(4)
(v)	Solve $z^4 - 3z^2 + 2 = 0$, where z is a complex number.	(4)

SECTION – III

	LONG QUESTIONS	
Q.3	Find the equation of tangent and normal lines for the curve at (0, 4) of the function $y = 3x^3 + 18x^2 + 3x + 4$.	(3+3)
Q.4	Find the extreme values and inflection point of the function: $f(x) = x^{2/3}(x^2 - 4)$.	(6)
Q.5	Evaluate the integral: Evaluate $\int_0^{\pi/2} \frac{\cos\theta\,d\theta}{(2-sin\theta)(3-sin\theta)}$.	(6)
Q.6	Solve the integral: $\int \frac{3x^2-1}{(x+1)(x^2+x+1)}\,dx$	(6)
Q.7	Find the Maclaurin series of the function $f(x) = \frac{x^2}{x+1}$	(6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-I [Vectors & Mechanics (1)] **TIME ALLOWED: 30 mins.**

Course Code: MATH-102 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

NOTE: Attempt all questions from each section.

SECTION I

1. Which of the following is vector .... (1 mark)

(a) volume

(b) speed

(c) momentum

(d) energy

2. If $\vec{r}_1 = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{r}_2 = 2\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ and $\vec{r}_3 = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, then $\vec{r}_1 + \vec{r}_2 + \vec{r}_3$ .... (1 mark)

(a) $4\hat{\imath} - 4\hat{\jmath}$

(b) $4\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$

(c) $4\hat{\jmath} - 4\hat{k}$

(d) $4\hat{\imath} + 4\hat{\jmath}$

3. The magnitude of $\vec{A} \times \vec{B}$ is the same as the area of a .... with sides $\vec{A}$ and $\vec{B}$. (1 mark)

(a) square

(b) parallelogram

(c) parallelepiped

(d) none of these

4. A vector $\vec{V}$ is called solenoidal if its .... is zero. (1 mark)

(a) gradient

(b) divergence

(c) curl

(d) magnitude

(P.T.O.)

5. If $\vec{A}$ is differentiable vector function and ϕ is differentiable scalar function of position (x, y, z), then $\nabla \cdot (\phi\vec{A})$ = ... (1 mark)

(a) $\phi\nabla \cdot \vec{A}$

(b) $\nabla\phi \cdot \vec{A}$

(c) $(\nabla\phi) \cdot \vec{A} + \phi(\nabla \cdot \vec{A})$

(d) $(\nabla\phi) \cdot \vec{A} - \phi(\nabla \cdot \vec{A})$

6. .... states that the moment about a point O of the resultant of a system of concurrent forces is equal to the sum of the moments of the various forces about the same point O. (1 mark)

(a) (λ, μ)−theorem

(b) Lamy's theorem

(c) Varigon's theorem

(d) none of these

7. The effect of a couple upon a rigid body is .... if it is replaced by any other couple of the same moment lying in the same plane. (1 mark)

(a) altered

(b) unaltered

(c) zero

(d) increased

8. The direction of friction is .... to the direction in which the body moves. (1 mark)

(a) same

(b) opposite

(c) perpendicular

(d) normal

9. The moment of a force $\vec{F}$ about the origin O is .... where $\vec{r}$ is the position vector relative to O of any point on the line of action of $\vec{F}$. (1 mark)

(a) $\vec{r} \cdot \vec{F}$

(b) $\vec{r} \times \vec{F}$

(c) $\nabla \cdot (\vec{r} \times \vec{F})$

(d) none of these

10. A set of particles, subject to workless constraints, is in equilibrium iff zero virtual work is done by the .... in any arbitrary infinitesimal displacement consistent with the constraints. (1 mark)

(a) frictional forces

(b) forces of constraints

(c) applied forces

(d) all of the above

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-I [Vectors & Mechanics (1)] **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-102 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION II-Questions with Short Answers

1. Show that the projection of $\vec{A}$ on $\vec{B}$ is equal to $\vec{A}\ .\ \hat{b}$, where $\hat{b}$ is a unit vector in the direction of $\vec{B}$. (3 marks)

2. Determine a unit vector perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. (3 marks)

3. Show that $\vec{A}\ .\ \frac{d\vec{A}}{dt} = A\frac{dA}{dt}$, where A is the magnitude of $\vec{A}$. (2 marks)

4. If $\vec{A} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, then find curl $\vec{A}$. (3 marks)

5. Show that the moment of a couple does not depend upon a fixed point. (3 marks)

6. Define smooth and rough bodies. (3 marks)

7. Differentiate [redacted] virtual and real displacement. (3 marks)

SECTION III-Questions with Brief Answers

8. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (5 marks)

9. Show that $\vec{A}\ .\ (\vec{B} \times \vec{C})$ is in absolute value equal to the volume of a parallelepiped with sides $\vec{A}$, $\vec{B}$ and $\vec{C}$. (5 marks)

10. If $\vec{\mathbf{r}}_1 = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{r}}_2 = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\vec{\mathbf{r}}_3 = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\vec{\mathbf{r}}_4 = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, find scalars a, b, c such that $\vec{\mathbf{r}}_4 = a\vec{\mathbf{r}}_1 + b\vec{\mathbf{r}}_2 + c\vec{\mathbf{r}}_3$. (5 marks)

11. If the three forces acting on a particle are in equilibrium, then
    (a) The forces must be coplanar. (2 marks)
    (b) The magnitude of each force is proportional to the sine of the angle between the other two. (3 marks)

12. Forces of magnitude P, $2P$, $3P$, $4P$ act respectively along the sides **AB, BC, CD, DA** of a square $ABCD$ of side "a" and forces each of magnitude $8\sqrt{2}P$ act along the diagonal **BD, AC**. Find the magnitude of the resultant & the distance of its line of action from A. (5 marks)

13. Six equal uniform rods **AB, BC, CD, DE, EF, FA** each of weight "w" are freely jointed to form a regular hexagon. The rod **AB** is fixed in horizontal position and the shape of the hexagon is maintained by a light rod joining C and F. Show that the thrust in this rod is $\sqrt{3}w$. (5 marks)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018

Examination: B.S. 4 Years Programme in Physical Education

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 30 mins.**

Course Code: MATH-111 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION-I

Q. 1 **MCQs (1 Mark each)**

(i) The property, for any $x, y \in \mathbb{R} \Rightarrow x + y \in \mathbb{R}$, is called --------

(a) Closure property (b) Associative property

(c) Commutative property (d) None of these

(ii) If α, β are roots of $3x^2 + 5x + 9 = 0$ then equation whose rots are $\frac{1}{\alpha}, \frac{1}{\beta}$ is:

(a) $x^2 + 2x + 6 = 0$ (b) $9x^2 + 5x + 3 = 0$ (c) $9x^2 - 5x - 3 = 0$ (d) None of these

(iii) If $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = 0$, then $x =$--------

(a) 3 (b) ± 1 (c) 0 (d) None of these

(iv) If $\sqrt{x} + \frac{1}{\sqrt{x}} = 5$, what will be the value of $x^2 + \frac{1}{x^2}$:

(a) 927 (b) 727 (c) 527 (d) None of these

(v) The sum of n terms of an A.P is $3n^2 + 4n$. Find the nth term:

(a) 5n+2 (b) 6n+1 (c) 8n+3 (d) None of these

(vi) If $a = \frac{1}{2}, r = \frac{1}{2}$, then the sum to infinity of geometric series is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

(vii) The number of terms in the expansion of $(2x + 3y)^{10}$ is:

(a) 10 (b) 11 (c) 12 (d) None of these

(viii) The expansion of $(1 - 3x)^{\frac{2}{3}}$ is valid if

(a) $|x| < \frac{1}{3}$ (b) $|x| < \frac{1}{2}$ (c) $|x| < \frac{2}{3}$ (d) None of these

(ix) $\cos^2\theta + \sin^2\theta =$

(a) -2 (b) -1 (c) 0 (d) None of these

(x) If $cos\theta < 0$ & $sin\theta > 0$ then the terminal arm of the angle lies in ____quadrant.

(a) Ist (b) II nd (c) III rd (d) None of these

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme in Physical Education

Roll No. ....

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-111 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION-II
SHORT QUESTIONS

Q.2

(i) Find the value of $|z_1 + z_2|$, where $z_1 = 1+3i$ and $z_2 = 6-i$. (2)

(ii) If $\begin{vmatrix} k-2 & 1 \\ 5 & k+2 \end{vmatrix} = 0$ then find the value of k. (2)

(iii) Evaluate $\left(\frac{-1+\sqrt{-3}}{2}\right)^3 + \left(\frac{-1-\sqrt{-3}}{2}\right)^3$ (2)

(iv) Show that the roots of equation $2x^2 + (mx-1)^2 = 3$ are equal if $3m^2 + 4 = 0$. (2)

(v) Find the sum of 13 terms of an A.P. whose middle term 10. (2)

(vi) Find the term involving x^4 in the expansion of $(2x+3)^5$ (2)

(vii) Find the nth term of the H.P. $\frac{1}{9}, \frac{1}{12}, \frac{1}{15}, \dots$. (2)

(viii) Find r when $\theta = \frac{\pi}{7}$ radians, $l = 14$ cm. (2)

(ix) Prove that $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{\cot\theta + 1}{\cot\theta - 1}$. (2)

(x) Find the area of a sector with central angle of 0.5 radian in a circular region whose radius is 2 m. (2)

SECTION-III
LONG QUESTIONS

Q.3 If possible, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$ (6)

Q.4 Solve the system of linear equations (6)

$$2x + y + z = 1, \qquad 3x + y - 5z = 8, \qquad 4x - y + z = 5$$

Q.5 If α and β are the roots of $4x^2 - 2x + 7 = 0$ then find the equation whose roots are α^2, β^2. (6)

Q.6 Expand and simplify $(a+b)^4 + (a-b)^4$. (6)

Q.7 Prove the identity $\frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{\sin(\alpha+\beta) - \sin(\alpha-\beta)} = \cot\beta$. (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 30 mins.**

Course Code: MATH-111 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1 Tick (√) the correct answer in the following MCQs

(i) The property $a+b=b+a$ for all $a,b \in \mathbb{R}$ is called

(a) Closure property (b) Symmetric property
(c) Commutative property (d) Reflexive property

(ii) The order of the matrix $[2 \quad 5 \quad 7]$ is

(a) 3×3 (b) 1×1 (c) 3×1 (d) 1×3

(iii) If $\begin{vmatrix} -1 & 2 \\ x & 1 \end{vmatrix} = 0$, then $x =$ ________

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) None of these

(iv) Roots of $x^2 - x - 2 = 0$ are

(a) $2,-1$ (b) $-2,1$ (c) $-2,-1$ (d) None of these

(v) The sum of n terms of an A.P is $3n^2 + 4n$. Find the nth term:

(a) 5n+2 (b) 6n+1 (c) 8n+3 (d) None of these

(vi) If the 6th term of an A.P. is 12 and the 18th term is 72 then $a_n =$

(a) 5n+15 (b) $5n - 20$ (c) $5n - 18$ (d) None of these

(vii) The number of terms in the expansion of $(ax+b)^{10}$ is:

(a) 11 (b) 12 (c) 13 (d) None of these

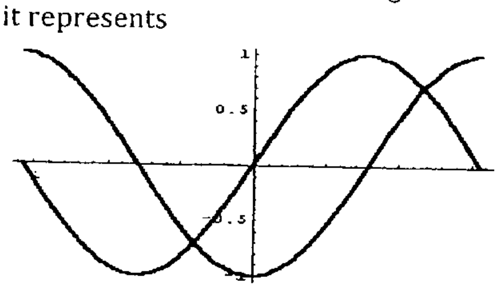
(viii) The expansion of $(1-x)^5$ is valid if

(a) $|x| < 1$ (b) $|x| > 1$ (c) $|x| > 2$ (d) None of these

(ix) $\cos^2\theta + \sin^2\theta =$ ________

(a) 1 (b) -1 (c) 0 (d) None of these

(x) Two trigonometric functions are drawn taking same scale from $-\pi$ to π in the following graph, it represents



(a) $\cos x$ and $\sec x$ (b) $\sin x$ and $\csc x$
(c) $\cos x$ and $\sin x$ (d) None of these

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-111 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION-II
SHORT QUESTIONS

Q.2

(i) Simplify and separate into real and imaginary parts $\frac{1+2i}{3-4i}$ (2)

(ii) Prove that $\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix} = 0$ (2)

(iii) Solve $\frac{1}{x-1}+\frac{2}{x-2}=1, x \neq -1, 2$. (2)

(iv) Show that the roots of equation $2x^2+(mx-1)^2=3$ are equal if $3m^2+4=0$. (2)

(v) Find the sum of the first 17 terms of the arithmetic series $4+9+14+....$ (2)

(vi) Find the term involving x^4 in the expansion of $(2x+3)^5$. (2)

(vii) Find the *n*th term of the H.P. $\frac{1}{9}, \frac{1}{12}, \frac{1}{15}, ...$ (2)

(viii) Find *r* when $\theta = \frac{\pi}{7}$ radians, $l = 14$ cm. (2)

(ix) Prove the identity $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$. (2)

(x) If the population of a town increases geometrically at the rate of 5% per year and the present population is 20000, what will be the population after 10 years from now? (2)

SECTION-III
LONG QUESTIONS

Q.3 Solve the system of linear equations (6)

$$2x-y-z=4, \quad 3x+4y-2z=11, \quad 3x-2y+4z=11$$

Q.4 Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$. (6)

Q.5 Solve $\sqrt{2x+8}+\sqrt{x+5}=7$ (6)

Q.6 Prove the identity $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$ (6)

Q.7 Expand and simplify $(x+y)^5+(x-y)^5$ (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018
Examination: B.S. 4 Years Programme

PAPER: Mathematics B-I [Vectors & Mechanics (1)] **TIME ALLOWED: 30 mins.**
Course Code: MATH-102 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

NOTE: Attempt all questions from each section.

SECTION I

1. Which of the following is vector .... (1 mark)

(a) volume

(b) speed

(c) momentum

(d) energy

2. If $\vec{r}_1 = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{r}_2 = 2\hat{i} - 4\hat{j} - 3\hat{k}$ and $\vec{r}_3 = -\hat{i} + 2\hat{j} + 2\hat{k}$, then $\vec{r}_1 + \vec{r}_2 + \vec{r}_3$ .... (1 mark)

(a) $4\hat{i} - 4\hat{j}$

(b) $4\hat{i} - 4\hat{j} - 4\hat{k}$

(c) $4\hat{j} - 4\hat{k}$

(d) $4\hat{i} + 4\hat{j}$

3. The magnitude of $\vec{A} \times \vec{B}$ is the same as the area of a .... with sides $\vec{A}$ and $\vec{B}$. (1 mark)

(a) square

(b) parallelogram

(c) parallelepiped

(d) none of these

4. A vector $\vec{V}$ is called solenoidal if its .... is zero. (1 mark)

(a) gradient

(b) divergence

(c) curl

(d) magnitude

(P.T.O.)

5. If $\vec{A}$ is differentiable vector function and ϕ is differentiable scalar function of position (x, y, z), then $\nabla . (\phi\vec{A})$ = ... (1 mark)

(a) $\phi\nabla . \vec{A}$

(b) $\nabla\phi . \vec{A}$

(c) $(\nabla\phi) . \vec{A} + \phi(\nabla . \vec{A})$

(d) $(\nabla\phi) . \vec{A} - \phi(\nabla . \vec{A})$

6. .... states that the moment about a point O of the resultant of a system of concurrent forces is equal to the sum of the moments of the various forces about the same point O. (1 mark)

(a) (λ, μ)–theorem

(b) Lamy's theorem

(c) Varigon's theorem

(d) none of these

7. The effect of a couple upon a rigid body is .... if it is replaced by any other couple of the same moment lying in the same plane. (1 mark)

(a) altered

(b) unaltered

(c) zero

(d) increased

8. The direction of friction is .... to the direction in which the body moves. (1 mark)

(a) same

(b) opposite

(c) perpendicular

(d) normal

9. The moment of a force $\vec{F}$ about the origin O is .... where $\vec{r}$ is the position vector relative to O of any point on the line of action of $\vec{F}$. (1 mark)

(a) $\vec{r} . \vec{F}$

(b) $\vec{r} \times \vec{F}$

(c) $\nabla .(\vec{r} \times \vec{F})$

(d) none of these

10. A set of particles, subject to workless constraints, is in equilibrium iff zero virtual work is done by the .... in any arbitrary infinitesimal displacement consistent with the constraints. (1 mark)

(a) frictional forces

(b) forces of constraints

(c) applied forces

(d) all of the above

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-I [Vectors & Mechanics (1)] **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-102 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION II-Questions with Short Answers

1. Show that the projection of $\vec{A}$ on $\vec{B}$ is equal to $\vec{A}\ .\ \hat{b}$, where $\hat{b}$ is a unit vector in the direction of $\vec{B}$. (3 marks)

2. Determine a unit vector perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. (3 marks)

3. Show that $\vec{A}\ .\ \frac{d\vec{A}}{dt} = A\frac{dA}{dt}$, where A is the magnitude of $\vec{A}$. (2 marks)

4. If $\vec{A} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, then find curl $\vec{A}$. (3 marks)

5. Show that the moment of a couple does not depend upon a fixed point. (3 marks)

6. Define smooth and rough bodies. (3 marks)

7. Differentiate [redacted] virtual and real displacement. (3 marks)

SECTION III-Questions with Brief Answers

8. Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point $(2,-1,2)$. (5 marks)

9. Show that $\vec{A}\ .\ (\vec{B}\times\vec{C})$ is in absolute value equal to the volume of a parallelepiped with sides $\vec{A}$, $\vec{B}$ and $\vec{C}$. (5 marks)

10. If $\vec{\mathbf{r}}_1 = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{r}}_2 = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\vec{\mathbf{r}}_3 = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\vec{\mathbf{r}}_4 = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, find scalars a, b, c such that $\vec{\mathbf{r}}_4 = a\vec{\mathbf{r}}_1 + b\vec{\mathbf{r}}_2 + c\vec{\mathbf{r}}_3$. (5 marks)

11. If the three forces acting on a particle are in equilibrium, then
    (a) The forces must be coplanar. (2 marks)
    (b) The magnitude of each force is proportional to the sine of the angle between the other two. (3 marks)

12. Forces of magnitude P, $2P$, $3P$, $4P$ act respectively along the sides **AB, BC, CD, DA** of a square $ABCD$ of side "a" and forces each of magnitude $8\sqrt{2}P$ act along the diagonal **BD, AC**. Find the magnitude of the resultant & the distance of its line of action from A. (5 marks)

13. Six equal uniform rods **AB, BC, CD, DE, EF, FA** each of weight "w" are freely jointed to form a regular hexagon. The rod **AB** is fixed in horizontal position and the shape of the hexagon is maintained by a light rod joining C and F. Show that the thrust in this rod is $\sqrt{3}w$. (5 marks)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 30 mins.**

Course Code: MATH-111 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1 Tick (√) the correct answer in the following MCQs

(i) The property $a+b=b+a$ for all $a,b\in\mathbb{R}$ is called

(a) Closure property (b) Symmetric property

(c) Commutative property (d) Reflexive property

(ii) The order of the matrix $[2\quad 5\quad 7]$ is

(a) 3×3 (b) 1×1 (c) 3×1 (d) 1×3

(iii) If $\begin{vmatrix}-1 & 2\\ x & 1\end{vmatrix}=0$, then $x=$ ________

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) None of these

(iv) Roots of $x^2-x-2=0$ are

(a) $2,-1$ (b) $-2,1$ (c) $-2,-1$ (d) None of these

(v) The sum of n terms of an A.P is $3n^2+4n$. Find the nth term:

(a) 5n+2 (b)6n+1 (c) 8n+3 (d) None of these

(vi) If the 6[th] term of an A.P. is 12 and the 18[th] term is 72 then $a_n=$

(a) 5n+15 (b) $5n-20$ (c) $5n-18$ (d) None of these

(vii) The number of terms in the expansion of $(ax+b)^{10}$ is:

(a) 11 (b) 12 (c) 13 (d) None of these

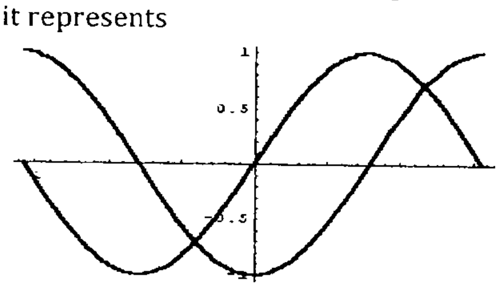
(viii) The expansion of $(1-x)^5$ is valid if

(a) $|x|<1$ (b) $|x|>1$ (c) $|x|>2$ (d) None of these

(ix) $\cos^2\theta+\sin^2\theta=$ ________

(a) 1 (b) -1 (c) 0 (d) None of these

(x) Two trigonometric functions are drawn taking same scale from $-\pi$ to π in the following graph, it represents



(a) $\cos x$ and $\sec x$ (b) $\sin x$ and $\csc x$

(c) $\cos x$ and $\sin x$ (d) None of these

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-111 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION-II
Q.2 **SHORT QUESTIONS**

(i) Simplify and separate into real and imaginary parts $\frac{1+2i}{3-4i}$ (2)

(ii) Prove that $\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix} = 0$ (2)

(iii) Solve $\frac{1}{x-1}+\frac{2}{x-2}=1,\ x \neq -1, 2$. (2)

(iv) Show that the roots of equation $2x^2+(mx-1)^2=3$ are equal if $3m^2+4=0$. (2)

(v) Find the sum of the first 17 terms of the arithmetic series $4+9+14+\ldots$ (2)

(vi) Find the term involving x^4 in the expansion of $(2x+3)^5$. (2)

(vii) Find the nth term of the H.P. $\frac{1}{9}, \frac{1}{12}, \frac{1}{15}, \ldots$ (2)

(viii) Find r when $\theta=\frac{\pi}{7}$ radians, $l=14$ cm. (2)

(ix) Prove the identity $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$. (2)

(x) If the population of a town increases geometrically at the rate of 5% per year and the present population is 20000, what will be the population after 10 years from now? (2)

SECTION-III
LONG QUESTIONS

Q.3 Solve the system of linear equations (6)

$2x-y-z=4,\quad 3x+4y-2z=11,\quad 3x-2y+4z=11$

Q.4 Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$. (6)

Q.5 Solve $\sqrt{2x+8}+\sqrt{x+5}=7$ (6)

Q.6 Prove the identity $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$ (6)

Q.7 Expand and simplify $(x+y)^5+(x-y)^5$ (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018
Examination: B.S. 4 Years Programme

PAPER: Business Mathematics | **TIME ALLOWED: 30 mins.**
Course Code: MATH-112 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.
Objective Type

Q-1 Encircle Correct Answer. **(10x1=10)**

1. Transpose of a column matrix is........ matrix.
   a) Zero b) Row c) Diagonal d) Square

2. If A is a symmetric matrix, then $A^t =$
   a) A b) 0 c) –A d) |A|

3. What will be the simple interest earned on an amount of Rs. 16,800 in 9 months at the rate of 6.4 % p.a?
   a) Rs. 787.50 b) Rs. 812.50 c) Rs 860 d) Rs. 887.50

4. In linear equation 'ax+by = c' a,b and c are considered as
   a) Variables b) Constants c) Zero d) Real Numbers

5. If every payment is made at the end then annuity is called
   a) Perpetuity b) Annuity due c) Ordinary Annuity d) None of these

6. Sum of n terms in G.P is
   a) $\frac{n}{2}[2a + (n-1)d]$ b) $\frac{a(1-r^n)}{1-r}$ c) $\frac{n}{2}(a + a_n)$ d) ar^{n-1}

7. If a = 2, d = 3, then nth term of A.P is
   a) 3n-1. b) 2n-2 c) 2n-3 d) 3n-3

8. Evaluate ${}^{100}C_{100}$
   a) 1000 b) 1000 c) 100 d) 1

9. 5^{th} term of G.P 3, 6, 12, ... is
   a) 15 b) 48 c) 2 d) 3

10. Find the value of $\log_4(\log_3 5)$.
    a) 1.460 b) 0.275 c) 1.273 d) 0.165

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Business Mathematics **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-112 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Subjective Type

Q-2 Answer the following short Questions. **(20)**

i. Sum of two consecutive integers is 27. Find the integers
ii. Define Linear equation.
iii. Write down the formula for annuity due.
iv. In how many ways the letters of the word PHILIPPINE can be arranged?
v. Find the determinant of $\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$
vi. What is meant by common ratio?
vii. What is symmetric matrix?
viii. What principal will earn Rs. 547 at 10.25% in 8 months?
ix. If 6 is added to a certain number the result is 13. What is the number?
x. What is scalar matrix?

Long Questions: **(30)**

Q-3 Solve $\frac{x-5}{2x} = \frac{x-4}{13}$ (6)

Q-4 If $A = \begin{bmatrix} 2 & -3 & 5 \\ K & 4 & 6 \\ 2 & 0 & 8 \end{bmatrix}$ is a singular matrix, then find the value of K. (6)

Q-5 Find the accumulated value of Rs.5, 000 invested at the end of each quarter for 5 years at 8% compounded quarterly. (6)

Q-6 The 10^{th} term of A.P is 20 and 20^{th} term is 40. Find the first term and common difference. (6)

Q-7 Prove that ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$ (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018
Examination: B.S. 4 Years Programme

PAPER: Calculus-I | **TIME ALLOWED: 30 mins.**
Course Code: MATH-121 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I

Q.1	**MCQs (1 mark each)**			
(i)	$\int \sin^2 x\, dx = ?$			
	(a) $-\cos^2 x + c$	(b) $x - \sin 2x + c$	(c) $\frac{1}{2}x - \frac{1}{4}\sin 2x + c$	(d) $\frac{1}{2}(x - \sin 2x) + c$
(ii)	If $f(x) = \cos x$, then $f'(\pi) = ?$			
	(a) $-\sin x$	(b) -1	(c) 1	(d) 0
(iii)	If $f(x)$ has a point of inflexion at "c", then			
	(a) $f''(x) > 0$	(b) $f''(x) < 0$	(c) $f''(x) = 0$	(d) $f(c) = 0$
(iv)	Every piece-wise continuous function is ________________.			
	(a) linear	(b) Integrable	(c) differentiable	(d) exponential
(v)	A local maximum point of a function f is always occurring where f'' is			
	(a) positive	(b) negative	(c) zero	(d) undefined
(vi)	Domain of $\sqrt{2x+4}$ is			
	(a) $x \leq -2$	(a) $x \leq 0$	(a) $x \leq 2$	(a) $x \geq -2$
(vii)	$\lim_{x\to\infty} \frac{2-3x}{\sqrt[3]{3+8x^3}} = ?$			
	(a) $\frac{2}{3}$	(b) $\frac{-3}{2}$	(c) $\frac{-2}{81}$	(d) none of these
(viii)	For what value of x, the inequality $-2x + 4 > 15x + 10$ is satisfied			
	(a) 2	(b) 1	(c) 0	(d) none of these
(ix)	$\lim_{x\to 2} \frac{x^2-4}{x-1} = ?$			
	(a) 0	(b) 1	(c) 2	(d) 4
(x)	Maclaurin series is centered at ____			
	(a) 3	(b) x	(c) 0	(d) None of these

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus-I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-121 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	Show that $\lim\limits_{n\to 0} \frac{\ln(n^2)}{n} = 0$.	(4)
(ii)	If $f(x) = \sqrt{x}$, $g(x) = 2 + \frac{1}{x^4}$, find the following functions and their domains: (i) fog (ii) gof	(4)
(iii)	Evaluate $\lim\limits_{x\to 0} \frac{\operatorname{cosec} x - \cot x}{x}$	(4)
(iv)	$\int \frac{x}{(x-1)(x^2+1)} dx = ?$	(4)
(v)	Find the absolute maximum and minimum values of $g(t) = 8t - t^4$ on [-2,1].	(4)

SECTION – III

LONG QUESTIONS		
Q.3	Find the equation of tangent and normal to the given curve at the points $(a, -2a)$ of the function: $y^2 = 4ax$.	(6)
Q.4	Find the first four terms of the Taylor series of the function $f(x) = \sqrt{x+1}$ at $a = 0$.	(6)
Q.5	Sketch the graph of the function $r = 3 + 4\cos\theta$.	(6)
Q.6	State and prove Rolle's theorem.	(6)
Q.7	Find the derivative of the function $f(x) = \arctan\left(\frac{x\sin\alpha}{1 - x\cos\alpha}\right)$.	(6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018
Examination: B.S. 4 Years Programme

PAPER: Applied Mathematics **TIME ALLOWED: 30 mins.**
Course Code: MATH-122 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

PART I (OBJECTIVE TYPE)

Q.1. Choose the correct answer. Cutting or over writing is not allowed.

1. The number 9 0.05950 $\times 10^3$ has – – – significant digits
   (a) 3 (b) 4 (c) 5 (d) 6
2. Order of convergence of Secant method is
   (a) 1 (b) 4.618 (c) 2 (d) 9
3. Jacobi method is a – – – method
   (a) Non-iteration (b) iteration (c) infinite (d) algebraic
4. If $f(y)$ is a real continuous function in $[x_0, x_1]$, and $f(x_0)f(x_1) < 0$, then for $f(y) = 0$, there is (are) – – – in the domain $[x_0, x_1]$.
   (a) one root (b) an undeterminable number of roots (c) no root (d) at least one root
5. If $x = 6$ is a root of $f(x) = 0$, then the factor of $f(x)$ is – – –.
   (a) $x + 6$ (b) 6 (c) $x - 6$ (d) x
6. The value of C for the density function $f(x){=}Cx$, $0 \le x \le 2$ is
   (a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{1}{2}$ (d) None
7. Bag contains 20 white and 40 read balls, one ball is drawn at random. What is the probability that ball is read
   (a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) Non of these
8. Mean of a binomial distribution is
   (a) np (b) $\frac{np}{q}$ (c) $np(1-q)$ (d) $np(1-p)$
9. There are 80 persons and we have to make a committee of 10 persons, then we have
   (a) $\frac{80!}{80!(80-10!)}$ (b) $\frac{80!}{10!(80-10!)}$ (c) $\frac{10!}{80!(80-10!)}$ (d) None
10. The convergence of which of the following method is sensitive to starting value?
    (a) False position (b) Gauss seidal method (c) Newton-Raphson method (d) All of these

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Applied Mathematics
Course Code: MATH-122

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

(SUBJECTIVE TYPE)

Part II Marks= 20

1. If A and B are events of sample space S, then prove that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. (2)

2. Prove that mean, median and mode are equal in normal distribution function. (3)

3. 400 passengers have made a reservation for an airplane flight. If the probability that a passenger will not show up is 0.02. Find the probability that exactly 4 will not show up. (2)

4. Find the value of C in the following p.d.f of a continuous r.v"y": (3)
$$f(y)=\begin{cases} C(3-y)(3+y), & 0\leq y\leq 3, \\ 0, & elsewhere. \end{cases}$$

5. Find the root of the function up to three decimal places by applying Newton-Rapson method $f(x)= x^3-4x+1$, taking an initial value x=1.5. (3)

6. Evaluate $\int_0^2 \frac{dx}{1+x+x^2}$ using Simpson Rule, for $n=6$. (3)

7. If S_x^2=10.0, S_y^2=485, 578.8, $\sum(X-\overline{X})$=159.45, $\sum(Y-\overline{Y})$=7, 767, 660 and $\sum(X-\overline{X})(Y-\overline{Y})$=28, 768.4, then find $Cov(x,y)$ and r_{xy}. (2)

8. State the fundamental laws of probability. (2)

Part III Marks= 30

1. (a) Solve the following system of equations by using Gauss-Seidel iterative method up to five iterations (6)

$$\begin{aligned} 9x_1+2x_2+4x_3 &= 20 \\ x_1+10x_2+4x_3 &= 6 \\ 2x_1-4x_2+10x_3 &= -15 \end{aligned}$$

   (b) For any two events A and B, prove that $P(A\cap B)=P(A).P(B)$.

2. (a) Find the root of the function correct up to three decimal places by applying the Secant method $f(x)=x^2-1$, take $x_o=1$.

   (b) Write the algorithm for the Bisection method for solving a non linear equation. (4)

3. Define correlation and correlation coefficient. Prove that correlation coefficient is independent of origin and scale. (10)

UNIVERSITY OF THE PUNJAB

Roll No. ....

First Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Calculus (IT)-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-131 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION – I

Q.1	**MCQs (1 mark each)**
(i)	If $f(x)$ has a local extremum at "c", then (a) $f'(x) > 0$ (b) $f'(x) < 0$ (c) $f'(x) = 0$ (d) none of these
(ii)	If $f(x) = \cos x$, then $f'(\pi) = ?$ (a) $-\sin x$ (b) -1 (c) 1 (d) 0
(iii)	If $f(x)$ has a local maximum at "c", then (a) $f''(x) > 0$ (b) $f''(x) < 0$ (c) $f''(x) = 0$ (d) $f(c) = 0$
(iv)	Every polynomial function is ________________. (a) nonlinear (b) trigonometric (c) differentiable (d) exponential
(v)	If slope of $f(x)$ is decreasing at "c", then (a) $f'(x) > 0$ (b) $f'(x) < 0$ (c) $f'(x) = 0$ (d) none of these
(vi)	A Saddle point of a function f is always occurring where f'' is (a) positive (b) negative (c) zero (d) undefined
(vii)	$\lim_{x\to\infty} \frac{2-3x}{\sqrt[3]{3+8x^3}} = ?$ (a) $\frac{2}{3}$ (b) $\frac{-3}{2}$ (c) $\frac{-2}{81}$ (d) none of these
(viii)	For what value of x, the inequality $-2x + 4 > 5x + 10$ is satisfied (a) 2 (b) 1 (c) 0 (d) none of these
(ix)	$\lim_{x\to 2} \frac{x^2-4}{x-2} = ?$ (a) 0 (b) 1 (c) 2 (d) 4
(x)	First order Differential equation has almost ____independent solutions (a) 0 (a) 1 (a) 2 (a) 3

UNIVERSITY OF THE PUNJAB

First Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-I | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-131 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	Evaluate $\lim_{x\to 0} \frac{tanx-sinx}{x^3}$.	(4)
(ii)	Evaluate $\int x^2\sin(x^3)\, dx$.	(4)
(iii)	Find the slope of the circle $x^2 + y^2 = 25$ at the point (3,-4).	(4)
(iv)	Find domain and range of the function $f(x) = 2 + \frac{x^2}{x^2+4}$.	(4)
(v)	Find the derivative of $y = \frac{t^2-1}{t^3+1}$.	(4)

SECTION – III

LONG QUESTIONS		
Q.3	Find the particular solution of $\frac{dy}{dx} + xy = x, \quad y(0) = 1$	(6)
Q.4	Discuss the points of continuity and differentiability of the function $f(x) = \|x\|$ at $x = 0$.	(6)
Q.5	A particle is moving along a horizontal coordinate line (positive to the right) with position function $s(t) = 2t^3 - 14t^2 + 22t - 5, \quad t \geq 0$. Find the velocity and acceleration and describe the motion of the particle.	(6)
Q.6	Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 8$.	(6)
Q.7	$\int x\sqrt{2x+1}\, dx = ?$	(6)

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A -II, [Plane Curves & Analytic Geometry] **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-103 / MTH-12309 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q. 2

Short Questions $10 \times 2 = 20$

1. Find centre and radius of the sphere $x^2 + y^2 + z^2 - 6x + 4z = 0$.
2. Find the points of relative extreme of the curve $y = 2x^3 - 15x^2 + 36x + 10$.
3. Find equation of the plane through the origin and perpendicular to the straight line $x = 2 + t, y = 2 - 3t, z = -2 + 2t$.
4. Find the traces of the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$ in $xz-$ plane and $yz-$ plane.
5. Determine whether the lines, give below, intersect or not L: $\frac{x-2}{-3} = \frac{y-1}{1} = \frac{z-3}{1}$ and M: $\frac{x-5}{-5} = \frac{y-1}{3} = \frac{z-2}{5}$.
6. Find equation of the sphere with center at $(4, 1, -6)$ and tangent to the plane $2x - 3y + 2z - 10 = 0$plane.
7. Find oblique asymptote of the curve $x^2y + xy^2 + xy + y^2 + 3x = 0$.
8. Find the radius of curvature on any point on the curve $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$.
9. Determine whether the origin is a node, a cusp or an isolated point for the curve $x^4 + y^3 - 2x^3 + 3y^2 = 0$
10. Write equation of the surface of revolution obtained by revolving the curve $x^2 + 2y^2 = 8$, $z = 0$, about the y-axis.

Q. 3

Subjective Questions $6 \times 5 = 30$

1. Prove that radius of curvature for the ellipse $b^2x^2 + a^2y^2 - a^2b^2 = 0$ is $\frac{a^2b^2}{p^3}$, where p denotes the length of the perpendicular from the center of the ellipse to the tangent at any point on the ellipse.
2. Find the measure of angle of intersection of the given curves $r = a\theta(1+\theta)$ and $r = \frac{a}{1+\theta^2}$.
3. Find the envelop of the family of lines $bx + ay - ab = 0$ where the parameters a and b are connected by the relation $a + b = c$.
4. Find the area of the region bounded by the loop of curve $(a + x)y^2 - x^2(a - x) = 0$.
5. Find an equation of plane passing through the line of intersection of the planes $2x - y + 2z = 0$ and $x + 2y - 2z - 3 = 0$ and at a unit distance from the origin.
6. Find an equation of the sphere passing through the points $(0, 0, 0)$,$(0, 1, -1)$, $(-1, 2, 0)$ and $(1, 2, 3)$

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A -II, [Plane Curves & Analytic Geometry] **TIME ALLOWED: 15 Mints.**
Course Code: MATH-103 / MTH-12309 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q. # 1: Encircle the correct answer **(10x1=10)**

1. Let the curve in polar form be given by $r = -5\csc\theta$. Then the angle ψ is given by
   (a) $\frac{\theta}{2}$ (b) $\theta - \pi$ (c) $\theta + \pi/2$ (d) 2θ
2. The equation $\lambda xy - 5x + 3y + 2 = 0$ represents two straight lines if $\lambda =$
   (a) 1 (b) 1/2 (c) 15/2 (d) 15
3. If for a curve, $f(x, y) = f(-x, y)$, then the curve is symmetric about
   (a) x-axis (b) the line $x = y$ (c) y-axis (d) both x and y axes
4. The curve $9y^2 = 14x$ is symmetric about
   (a) line x−axis (b) line $x = y$ (c) line y-axis (d) both x and y axes
5. The locus of centers of curvatures for a given curve is called its
   (a) involute (b) envelope (c) diameter (d) evolute
6. Let the curve be defined as $x^2y - (x-2)^2$, then its horizontal asymptote is
   (a) $x = -1$ (b) $x = 1$ (c) $y = 1$ (d) $y = -1$
7. The singular point $(0, 0)$ on the plane curve $y^2(a^2 - x^2) - x^2(b - x)^2$ is
   (a) node (b) cusp (c) critical point (d) none of these
8. A point through which there pass two branches of a curve is called
   (a) simple point (b) ordinary point (c) double point (d) corner point
9. A surface defined by an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$ is called
   (a) ellipsoid (b) hyperboloid of one sheet (c) hyperboloid of two sheets (d) paraboloid
10. Equation of plane passing passing through origin and perpendicular to $\mathbf{n} = [1, 2, 3]$ is
    (a) $4x - 6y + 2z = 0$ (b) $2x + 4y + 6z = 0$ (c) $2x - 3y + z = 1$ (d) $x - 3y + z - 2 = 0$

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-II [Mechanics (II)] TIME ALLOWED: 15 Mints.

Course Code: MATH-104 Part – I (Compulsory) MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q. # 1: Encircle the correct answer **(10x1=10)**

1. .... is the branch of dynamics which establishes the relation between the applied forces and the resulting motion. (1 mark)

   (a) Kinematics
   (b) Kinetics
   (c) Mechanics
   (d) Physics

2. The components of acceleration along x-axis and y-axis are ...., respectively. (1 mark)

   (a) $\frac{dx}{dt}$ and $\frac{dy}{dt}$
   (b) $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$
   (c) d^2x and d^2y
   (d) none of the above

3. The number of oscillations which the particle completes in a unit of time is known as .... (1 mark)

   (a) amplitude
   (b) frequency
   (c) time period
   (d) energy

4. The amount of work done by a force in moving the particle from P_1 to P_2 along the path of the particle is called .... (1 mark)

   (a) energy
   (b) force
   (c) torque
   (d) power

5. The .... of a rigid body is defined as a point through which the line of action of the weight of body always passes, whatever be the position of the body. (1 mark)

   (a) circular point
   (b) circumference
   (c) center of gravity
   (d) none of the above

(P.T.O.)

6. The velocity of a particle which starts from O at $t = 0$ such that its position at that time is given by $\vec{r} = at^2\ \hat{\imath} + 4at\ \hat{\jmath}$ is .... (1 mark)

(a) $2at\ \hat{\imath} + 4a\ \hat{\jmath}$

(b) $2at^2\ \hat{\imath} + 4at\ \hat{\jmath}$

(c) $2at\ \hat{\imath} + 4at\ \hat{\jmath}$

(d) $at\ \hat{\imath} + 4a\ \hat{\jmath}$

7. The rectilinear motion is the motion of a particle along .... (1 mark)

(a) circular arc

(b) parabola

(c) straight line

(d) none of the above

8. The first equation of motion that gives the velocity at any time "t" is given by .... (1 mark)

(a) $v = u + at$

(b) $x = ut + \frac{1}{2}at^2$

(c) $v^2 - u^2 = 2ax$

(d) none of the above

9. The normal component of acceleration can be defined as .... (1 mark)

(a) zero

(b) v

(c) v^2

(d) none of the above

10. The area under the curve of a velocity-time graph gives the .... traveled by the particle. (1 mark)

(a) distance

(b) velocity

(c) acceleration

(d) time period

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-II [Mechanics (II)] **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-104 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q. 2 Questions with Short Answers

i. State principle of angular momentum. (2 marks)

ii. State Kepler's first law of planetary motion. (2 marks)

iii. A uniform rod AB is $4ft$ long and weight 6 lb and weights are attached to it as follows: 1 lb at A, 2 lb at $1ft$ from A and 3 lb at $2ft$ from A, 4 lb at $3ft$ from A and 5 lb at B. Find the distance from A of the center of gravity of the system. (4 marks)

iv. A particle projected vertically upwards at $t = 0$ with velocity u, passes a point at a height h at $t = t_1$ and $t = t_2$, show that $t_1 + t_2 = 2u/g$ & $t_1 t_2 = 2h/g$. (4 marks)

v. Show that the transverse component of acceleration of a moving particle varies as the radial component of its velocity if its angular velocity about the fixed origin is constant. (4 marks)

vi. From a semi-circular lamina of radius 2 a, a circular lamina of radius a is removed. Prove that the center of mass of the remainder is at a distance $\frac{16a}{3\pi} - a$ from the diameter. (4 marks)

Questions with Brief Answers

Q#3: A particle is moving along the parabola $x^2 = 4ay$ with constant speed "v". Determine the tangential and the normal components of its acceleration when it reaches the point whose abscissa is $\sqrt{5}a$. (6 marks)

Q#4: Two particles travel along a straight line. Both start at the same time and are accelerated uniformly at different rates. The motion is such that when the particle attains the maximum velocity V, its motion is retarded uniformly. The two particles come to rest simultaneously at a distance "x" from the starting point. If the acceleration of the first is "a" and that of 2^{nd} is $\frac{1}{2}a$. Find the distance between the points where the two particles attain their maximum velocities. (6 marks)

Q#5: State and prove Kepler's third law of planetary motion. (6 marks)

Q#6: Find the centroid of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ lying in the first octant. (6 marks)

Q#7: A particle describes simple harmonic motion with frequency "N". If the greatest velocity is "v", find the amplitude and maximum value of the acceleration of the particle. Also, show that the velocity "v" at a distance "x" from the center of motion is given by $v = 2\pi N\sqrt{a^2 - x^2}$, where "$a$" is the amplitude. (6 marks)

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics
Course Code: MATH-105 / MTH-12311 Part – I (Compulsory)

TIME ALLOWED: 15 Mints.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q1. Encircle the correct answer (1x10=10)

1. The statement form $p \vee \sim p$ is ....
   a) Tautology b. contradiction c. negation d. All of these

2. If A= {a,b,c,d}, then the number of elements in P (A) =.....
   a. 2^4 b. 2^5 c. 2^6 d. 2^7

3. Consider the relation R={(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,4)} on set A={1,2,3,4} is
   a. Symmetric b. Reflexive c. Transitive d. All of these

4. The inverse of conditional statement *If today is Friday, then 2 + 3 = 5* is
   a. If today is not Friday, then $2 + 3 \neq 5$.
   b. If today is Friday, then $2 + 3 \neq 5$.
   c. If today is not Friday, then $2 + 3 = 5$.
   d. If today is not Friday, then $2 + 3 < 5$.

5. $1, 10, 10^2, 10^3, 10^4, 10^5, \ldots$ is
   a. Arithmetic series b. Geometric series c. Arithmetic sequence d. Geometric sequence

6. The total number of one-to-one functions, from a set with two elements to a set with two elements is..........
   a. Zero b. 2 c. 9 d. None of these

7. The order pairs which are not present in a relation, must be present in
   a. Inverse of that relation b. Composition of relation c. Complementary relation of that relation d. None of these

8. The negation of the implication "If Ali lives in Pakistan then he lives in Lahore." is
   a. Ali lives in Pakistan and he does not live in Lahore.
   b. If Ali does not live in Pakistan then he does not live in Lahore
   c. Ali does not live in Pakistan and he does not live in Lahore.
   d. None of the above

9. A collection of rules indicating how to form new set objects from those already known to be in the set is called
   a. Base b. Restriction c. Recursion d. Fallacy

10. Bill Gates is an American
    He is very rich
    "He is very rich" is a statement with truth-value
    a. Ture b. False c. None of these

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-105 / MTH-12311 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q2. Solve the following short questions (2x10=20)

1. Show that $\sim(p \rightarrow q) \rightarrow p$ is a tautology without using truth tables.
2. For any two sets A and B prove that $A - (A - B) = A \cap B$
3. What is a statement?
4. Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
5. Let R be the relation on the set of integers Z defined as: for all $a,b \in Z$, $(a,b) \in R \Leftrightarrow a > b$. Is R irreflexive?
6. Find four binary relations from $X = \{a,b\}$ to $Y = \{u,v\}$ that are not functions.
7. Find the number of ways that a party of seven persons can arrange themselves in a row of seven chairs.
8. Find the sum of first *n* natural numbers.
9. Define recursion.
10. Find the first three terms of the following recursively defined sequence.
    $b_1 = 3$
    $b_k = b_{k-1} + k$, for all integers $k \geq 2$.

Q3. Solve the following **Long Questions.** (5x6=30)

1. How many integers from 1 through 1000 are neither multiples of 3 nor multiples of 5?
2. Which term of the geometric sequence is 1/8 if the first term is 4 and common ratio ½.
3. Find the sum of all two digit positive integers which are neither divisible by 5 nor by 2.
4. Let "D" be the "divides" relation on Z defined as: for all $m, n \in Z$, $m\ D\ n \Leftrightarrow m|n$. Determine whether D is reflexive, symmetric or transitive. Justify your answer.
5. Show by mathematical induction $1 + nx \leq (1 + x)^n$ for all real numbers $x > -1$ and integers $n \geq 2$
6. Suppose $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ and both of these are one-to-one and onto. Prove that $(gof)^{-1}$ exists and that $(gof)^{-1} = f^{-1}og^{-1}$.

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-I (Algebra)
Course Code: MATH-111 / MTH-12107 Part – II

TIME ALLOWED: 2 Hrs. & 45 Mints.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Q.2 Answer the following short questions. **(10x2=20)**

i. If $\sec\theta = 2$ and the terminal side of the angle is not in the 1st quadrant, then find the remaining trigonometric values.

ii. If the matrix $\begin{bmatrix} k & 2 \\ 5 & 5 \end{bmatrix}$ is singular, find the value of k?

iii. The roots of $x^2 + kx + 9 = 0$ are equal. Find k?

iv. Find the nth term of the sequence $5, x, 2x - 5, \dots$.

v. Solve the equation $\frac{3}{2}(2x + 1) = \frac{1}{3}$.

vi. Express $\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$ as single trigonometric ratio.

vii. Simplify $\frac{3-2i}{1+5i}$.

viii. Write two consecutive integers whose sum is 41.

ix. Solve $2x^2 + 7x + 5 = 0$.

x. How many terms of an A.P: 7 + 5 + 3 +..., are required to make a sum of -825?

Answer the long questions.

Q. 3 Find the coefficient of x^3 in the expansion of $\left(x - \frac{2}{3x^2}\right)^{12}$. [6]

Q.4 Find the values of x and y, if [6]

$$\begin{bmatrix} x-1 & 1 \\ 1 & y+5 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -3 & -1 \\ 2 & 3y \end{bmatrix} = \begin{bmatrix} y & 0 \\ 2 & -x \end{bmatrix}$$

Q. 5 Solve the equation $x^{1/3} - x^{1/6} - 6 = 0$. [6]

Q.6 Find the cube roots of 8. [6]

Q.7 Solve the system of equations by Cramer's rule [6]

$$2x + y + z = 1,$$

$$3x + y - 5z = 8,$$

$$4x - y + z = 5.$$

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-I (Algebra) **TIME ALLOWED: 15 Mints.**

Course Code: MATH-111 / MTH-12107 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q.1 Tick on the correct option. **(10x1=10)**

i) What is the value of a, if 5 is the A.M. between a and 7.

a) 0 b) 1 c) 2 d) 3

ii) Which of the following is an irrational number?

a) $\sqrt{4}$ b) $\sqrt{9}$ c) $\sqrt{11}$ d) $\sqrt{16}$

iii) The polar form of complex number $\sqrt{3} - i$ is

a) $2cis\left(\frac{\pi}{6}\right)$ b) $2cis\left(-\frac{\pi}{6}\right)$ c) $cis\left(\frac{\pi}{6}\right)$ d) $cis\left(-\frac{\pi}{6}\right)$

iv) $\frac{1}{9}:\frac{5}{36} =$

a) $\frac{1}{4}$ b) $\frac{1}{5}$ c) $\frac{4}{5}$ d) $\frac{5}{4}$

v) Find the 36% of Rs. 145000

a) 52000 b) 52100 c) 52200 d) 52300

vi) $(1 - \cos x)(1 + \cos x) =$

a) $\frac{1}{\sin^2 x}$ b) $\frac{1}{\cos^2 x}$ c) $\frac{1}{\csc^2 x}$ d) $\frac{1}{\sec^2 x}$

vii) If $a_n - a_{n-1} = 8$ and $a_1 = 7$, then $a_3 =$

a) 14 b) 17 c) 20 d) 23

viii) For what value of k, the equation $kx^2 + 2x + 81 = 0$ has a perfect square.

a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) $\frac{1}{81}$ d) $\frac{1}{243}$

ix) The order of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \end{bmatrix}$ is

a) 3×2 b) 2×3 c) 3×1 d) 1×3

x) $\frac{\sin 45^o}{\cos 45^o + \tan 45^o} =$

a) $\sqrt{2} - 1$ b) $\sqrt{2} + 1$ c) $\sqrt{2}$ d) $\frac{1}{\sqrt{2}}$

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus-II **TIME ALLOWED: 15 Mints.**

Course Code: MATH-123 / MTH-12333 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Question no: 1 **(10x1=10)**

Attempt all MCQs and chose the best answer.

1. If n is any positive integer then 1+3+5+...+(2n-1) =

A. n

B. n+1

C. 2n+1

D. n^2

2. The series obtained by adding the term of arithmetic sequences is called

A. harmonic series

B. geometric series

C. arithmetic series

D. infinite series

3. The A.M between $1-x+x^2$ and $1+x+x^2$ is

A. $2-x^2$

B. $2+x^2$

C. $1-x^2$

D. $1+x^2$

4. The 5th term of the G.P 3,6,12,... is

A. 15

B. 48

C. 2

D. 3

5. The fifth term of the sequence $a^n = 2n+3$ is

A. −13

B. −7

C. 7

D. 13

(P.T.O.)

6. If A and H are arithmetic and harmonic mean between 2 and 3, then A + H =

A. 49/20
B. 49/10
C. 49/5
D. 32/4

7. 2,4,6,8,10,12,... is

A. G.P
B. A.P
C. Geometric series
D. arithmetic series

8. No terms of a Harmonic sequence can be

A. 1
B. 2
C. 3
D. 0

9. If n is the total number of harmonic mean between a and b then mth harmonic mean between a and b is

A. (n+1)ab/(n+1)+n(a-b)
B. (n+1)ab/(n+1)b+m(a-b)
C. (n+1)b+m(a-b)
D. (n+1)ba

10. The second term of the sequence with general term $n^2 - 4/2$ is

A. 3
B. −3
C. 1
D. 0

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus-II **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-123 / MTH-12333 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Question no: 2

Attempt all short questions. (2 X 10 = 20)

1. Solve the given question to find out point of inflection:

$$x = (y-1)(y-2)(y-3)$$

2. Locate the points of relative extreme (Critical points) for the given function:

$$f(x) = \frac{\ln(x)}{x} \quad ; (0<x<\infty)$$

3. Define stationary points or extreme points. What do you know about point of inflection?
4. Define increasing and decreasing functions. Write proper definition and labeled diagram to elaborate your answer. What information do we get about slope of a line and first derivative test?
5. The nth term of a sequence is given. Determine whether the sequence converges or diverges. If it converges find its limit:

$$\frac{3n^4+1}{4n^2-1}$$

6. Prove that if a positive term series $\sum_1^\infty a_n$ converges then the series $\sum_1^\infty \sqrt{a_n a_{n+1}}$ converges.
7. Define sequence and series. Explain with examples.
8. Determine whether the sequence converges or diverges:

$$\sum_1^\infty n \cdot \left(\frac{\pi}{n}\right)^n$$

9. Define direct comparison and Limit comparison test.
10. Find dy/dx in the $3(x^2+y^2)^2 = 25(x^2-y^2)$.

Question no: 3

Attempt all long questions. (3 X 10 = 30)

1. Solve the given series by limit comparison test:

$$\sum_1^\infty \frac{1}{(2n-1)^{1/3}}$$

2. The nth term of a sequence is given. Determine whether the sequence converges or diverges. If it converges find its limit:

$$\frac{(2n)!}{(n!)^2}$$

3. Define the following terms:
   - Limit of a sequence.
   - Results of a Null sequence.
   - n^{th} – term test for divergence.

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Analytical Geometry **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-124 / MTH-12118 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Question No.2: Answer the following short questions. **(5x4=20)**

i. Write the equation of the surface defined by $\frac{(z-1)^2}{4} - \frac{(y+2)^2}{10} = 4(x-4)$ relative to new set of parallel axis with origin at (4,-2,1).

ii. Show that $S: x(y^2 + z^2) = 1$ is a surface of revolution. Find a generatix and axis of revolution.

iii. Find the center and radius of the sphere $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

iv. Find an equation of the plane through the three given points (-1,1,1), (5,-8,-2), (4,1,0)

v. Find the intercepts of the given surface on the co-ordinate axes.

$$x^2 + 4y^2 + 5xz - 2x + y - 3 = 0$$

Section-III

(Long Question)

Q.3. (10)

a. Find an equation of the plane through (5,-1,+4) and perpendicular to each of the planes x+y-2z-3=0 and 2x-3y+z=0

b. Find parametric equation of the line containing the point (2,4,-3) and perpendicular to the plane 3x+3y-7z=9

Q.4. (10)

a. Show that the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is perpendicular to the plane 4x+8y+12z+19=0

b. A variable line in two adjacent position has direction cosines l, m, n *and* $l + l + \delta l, m\delta, n + \delta n$ show that the measure of the small angle $\delta\theta$ between the two positions is given by $(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$

Q.5. (10)

a. Find an equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - 4x + 2 - 6z = 0$ at the point P(3,2,5)

b. Show that the shortest distance between the lines $x + a = 2y = -12z$ *and* $x = y + 2a = 6(z - a)$ *is* $2a$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester - 2018

Examination: B.S. 4 Years Programme

PAPER: Analytical Geometry **TIME ALLOWED: 15 Mints.**
Course Code: MATH-124 / MTH-12118 **Part – I (Compulsory)** **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the best answer out of the choices given for each question. (10x1=10)

i. The distance between $P_1(2,1,5)$ and $P_2(-2,3,0)$ is
   a. $5\sqrt{3}$
   b. $6\sqrt{5}$
   c. $3\sqrt{5}$

ii. The mid-point of the segment joining $P_1(3,-2,0)$ and $P_2(7,4,4)$ is
   a. (5,1,2)
   b. (6,1,3)
   c. (4,7,2)

iii. The magnitude length of $A = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{x}$ is
   a. $\sqrt{a_1^2 + a_2^2 + a_3^2}$
   b. $a_1^2 + a_2^2 + a_3^2$
   c. $a_1 + a_2 + a_3$

iv. An equation in any two of the three Cartesian co-ordinates defines a _______ parallel to the axis of the third co-ordinate.
   a. Cone
   b. Cylinder
   c. Surface

v. Cylindrical co-ordinates represent a point in space by ordered triples.
   a. (x,y,z)
   b. $(\rho,\theta,\emptyset)$
   c. (r,θ,z)

vi. If $f(x,y,z) = 0$ implies $f(-x,-y,-z) = 0$, the surface is symmetric with respect to
   a. yz-plane
   b. origin
   c. y-axis

vii. The direction cosines of x-axis are
   a. (0,1,1)
   b. (1,0,0)
   c. (0,0,1)

viii. The straight line 'L' and the plane 's' intersects in one point if and only if
   a. $ac_1 + bc_2 + cc_3 \neq 0$
   b. $ac_1 + bc_2 + cc_3 = 0$

ix. The intercept form of equation of plane is
   a. $\frac{x}{A} + \frac{y}{B} + \frac{z}{c} = 1$
   b. $\frac{x}{A} + \frac{y}{B} + \frac{z}{c} = 0$
   x. $\frac{x}{A} + \frac{y}{B} + \frac{z}{c} \neq 0$

x. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$ represents a surface of the type
   a. an ellipsoid
   b. elliptic cone
   c. circular parabolic

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-II **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-132 / IT-12392 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Question no: 2

Attempt all short questions. (4 X 5 = 20)

1. Show that the divergence of a curl of a vector field is zero.
2. Show that the curvature of a line is always zero.
3. Define parametric equation of lines.
4. Find a, b, c if $F = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational.
5. Prove that div (grad φ) = ∇^2 φ

Question no: 3

Attempt all long questions. (3 X 10 = 30)

1. Prove that $F = (y^2 \cdot \cos x + z^3)\mathrm{i} + (2y \cdot \sin x + 4)\mathrm{j} + (3xz^2)\mathrm{k}$ is irrotational?
2. If $F = grad\ (x^3 + y^3 + z^3 - 3xyz)$. Find Curl F.
3. If **A** and **B** vectors are irrotational. Show that **A x B** is Solenoidal.

UNIVERSITY OF THE PUNJAB

Second Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-II **TIME ALLOWED: 15 Mints.**

Course Code: MATH-132 / IT-12392 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Question no: 1

Attempt all MCQs and chose the best answer.

1. Gradient of a function is a constant. State True/False.

a) True b) False c) can be both d) information not complete

2. The mathematical perception of the gradient is said to be

a) Tangent b) Chord c) Slope d) Arc

3. Divergence of gradient of a vector function is equivalent to

a) Laplacian operation b) Curl operation c) Double gradient operation d) Null vector

4. The gradient of $xi + yj + zk$ is

a) 0 b) 1 c) 2 d) 3

5. Curl of gradient of a vector is

a) Unity b) Zero c) Null vector d) Depends on the constants of the vector

6. Find the divergence of the vector yi + zj + xk.

a) -1 b) 0 c) 1 d) 3

7. Find whether the vector is solenoidal, $E = yz\,i + xz\,j + xy\,k$

a) Yes, solenoidal b) No, non-solenoidal

c) Solenoidal with negative divergence d) Variable divergence

8. Identify the nature of the field, if the divergence is zero and curl is also zero.

a) Solenoidal, irrotational b) Divergent, rotational

c) Solenoidal, irrotational d) Divergent, rotational

9. The curl of a curl of a vector gives a

a) Scalar b) Vector c) Zero value d) Non zero value

10. A field in which a test charge around any closed surface in static path is zero is called

a) Solenoidal b) Rotational c) Irrotational d) Conservative

UNIVERSITY OF THE PUNJAB

Third Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ………………….

PAPER: Mathematics A-III | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-201/MTH-21309 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE
SECTION – II

Q. 2	SHORT QUESTIONS	
(i)	For the given matrix find the basis of Column space. $$\begin{bmatrix} 1 & 2 & 2 & -1 \\ 3 & 6 & 5 & 0 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$	(4)
(ii)	If A is a nonsingular matrix whose inverse is $\begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$, find A.	(4)
(iii)	Find the reduced echelon form of the matrix $\begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$	(4)
(iv)	Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$	(4)
(v)	Check whether W is a subspace of V or not? $V = R^3,\ W = \{(a,b,c) \in V:\ a^2+b^2+c^2 \le 1\}.$	(4)

SECTION – III

LONG QUESTIONS		
Q.3	If possible, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$	(6)
Q.4	Determine whether the vectors are linearly independent or not? $v_1 = (1,-2,3), v_2 = (5,6,-1), v_3 = (3,2,1)$.	(6)
Q.5	Determine whether or not the set of vectors $\{(1,2,-1),(0,3,1),(1,-5,3)\}$ is a basis for R^3	(6)
Q.6	Find the real orthogonal matrix P for which $P^{-1}AP$ is orthogonal where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$	(6)
Q.7	Determine the values of a for which the system of linear equations has no solution, exactly one solution and infinitely many solutions. $x+y+7z=-7$ $2x+3y+17z=-16$ $x+2y+(a^2+1)z=3a.$	(6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Mathematics A-III | **TIME ALLOWED: 30 mins.**

Course Code: MATH-201/MTH-21309 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE

SECTION – I

Q.1	**MCQs (1 mark each)**
(i)	If $\left(\overline{A}\right)' = -A$ then A is called **(a)** Symmetric matrix **(b)** Skew symmetric matrix **(c)** Hermitian matrix **(d)** Skew Hermitian matrix
(ii)	The dimension of $KerT$ is called .... **(a)** Rank **(b)** Nullity **(c)** Column space **(d)** None of these
(iii)	A system of m homogeneous linear equations $Ax = 0$ in n variables has a non-trivial solution if and only if the rank of A is -------- **(a)** equal to n **(b)** less than n **(c)**greater than n **(d)** none of these
(iv)	A unit vector orthogonal to both $(1, 1, 2)$ and $(0, 1, 3)$ in R^3 is --------- **(a)** $\left(\frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ **(b)** $\left(\frac{-1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ **(c)** $\left(\frac{2}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}\right)$ **(d)** $\left(\frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$
(v)	The characteristic polynomial of the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ is............... **(a)** $p(\lambda) = (2-\lambda)^2$ **(b)** $p(\lambda) = (2-\lambda)(3-\lambda)$ **(c)** $p(\lambda) = \lambda^2$ **(d)** None of these
(vi)	The property $\forall$ a, b$\in$R then $a + b \in R$ is called **(a)** Associative property **(b)** Transitive property **(c)** Closure property **(d)** None of these
(vii)	The subspace of R^3 spanned by the vector (a, b, c) is -------- **(a)** $x = t, y = bt, z = ct$ **(b)** $x = -at, y = -bt, z = -ct$ **(c)** $x = at, y = bt, z = ct$ **(d)** None of these
(viii)	Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ is **(a)** Linear **(b)** Not Linear **(c)** Rational **(d)** None of these
(ix)	A linear transformation that is both one-one and onto is called **(a)** Isomorphism **(b)** Homomorphism **(c)** Basis **(d)** Bijective
(x)	A linear transformation $T : U \to V$ is one-to-one if and only if -------- **(a)** $N(T) = \{0\}$ **(b)** $N(T) \neq \{0\}$ **(c)** $N(T) = \{1\}$ **(d)** $N(T) = \{-1\}$

UNIVERSITY OF THE PUNJAB

Third Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-III [Calculus (II)] **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-202/MTH-21310 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

SECTION II-Questions with Short Answers

1. State the Root test for absolute convergence. (2 marks)

2. Define surface of revolution. (2 marks)

3. Show that $\sum_{1}^{\infty} \frac{n+5}{n^2+4}$ diverges. (4 marks)

4. If $z = \frac{\cos y}{x}$, $x = u^2 - v$, $y = e^v$, then find $\frac{\partial z}{\partial u}$. (4 marks)

5. Work out the critical points of $f(x, y) = x^3 + y^3 - 3axy$, $a > 0$. (4 marks)

6. Evaluate $I = \int_0^{2\pi} \int_0^{a(1-\cos\theta)} r^3 \cos^2\theta \, dr d\theta$. (4 marks)

SECTION III-Questions with Brief Answers

7. Find the limit of the sequence $\left\{\frac{\ln n}{n}\right\}$ as $n \to \infty$. (5 marks)

8. Determine the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-e)^n \ln n}{e^n}$. (6 marks)

9. If $U = f(x, y)$ is homogeneous function of degree n then prove that

$$x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy} = n(n-1)f.$$

(6 marks)

10. A topless box having a volume of 12 cubic meters is to be made of material costing Rs. 100 per square meter. Find the dimensions of the box that minimize the cost. (7 marks)

11. Work out the point of the plane $x - y + 2z = 6$ nearest to the origin. (6 marks)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Mathematics B-III [Calculus (II)]
Course Code: MATH-202/MTH-21310

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

OBJECTIVE

NOTE: Attempt all questions from each section.

SECTION I

1. If $\{a_n\}, \{b_n\}$ and $\{c_n\}$ are sequences such that $a_n \leq b_n \leq c_n$ for all n and if $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} c_n$, then .... (1 mark)

   (a) $\lim_{n\to\infty} b_n = L$

   (b) $\lim_{n\to\infty} b_n = 1$

   (c) $\lim_{n\to\infty} b_n = a_n c_n$

   (d) none of the above

2. A bounded monotonic sequence is .... (1 mark)

   (a) divergent

   (b) convergent

   (c) increasing

   (d) decreasing

3. If the series $\sum_{1}^{\infty} a_n$ converges then .... (1 mark)

   (a) $\lim_{n\to\infty} a_n = 0$

   (b) $\lim_{n\to\infty} a_n =$ constant

   (c) a_n is constant

   (d) both (a) and (b)

4. If $u = f(x, y)$ is a homogeneous function of x, y of degree n, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ .... (1 mark)

   (a) 0

   (b) n

   (c) u

   (d) nu

P.T.O.

5. A function f can have a relative extrema only at its .... (1 mark)

(a) intersecting point

(b) critical point

(c) saddle point

(d) point of inflection

6. Two surfaces are said to be tangent at a common point P if each has the same .... at P. (1 mark)

(a) tangent plane

(b) saddle point

(c) gradient

(d) both (a) and (c)

7. If an arc s, a portion of a plane curve, is rotated about a straight line, then the arc generates a surface called a .... (1 mark)

(a) surface of revolution

(b) torus

(c) anchor ring

(d) none of the above

8. Let $\{a_n\}$ be a sequence and f a continuous function defined on $[0, \infty[$ such that $f(n) = a_n$. Then .... (1 mark)

(a) $\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x)$

(b) $\lim_{n\to 0} a_n = \lim_{x\to 0} f(x)$

(c) $\lim_{n\to 0} a_n = \lim_{x\to\infty} f(x)$

(d) all of the above

9. Let C be a curve in a plane and L be a line not in the plane. The union of all lines that intersect C and are parallel to L is called a .... (1 mark)

(a) cone

(b) sphere

(c) cylinder

(d) none of the above

10. The equation $\frac{d^2y}{dx^2} + y = 0$ has solution .... with A and B are arbitrary constants. (1 mark)

(a) $Ax + B$

(b) $A\cos x + B\sin x$

(c) $A\ln x + B\ln x$

(d) none of the above

UNIVERSITY OF THE PUNJAB

Third Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Graph Theory **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-205/MTH-21312 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

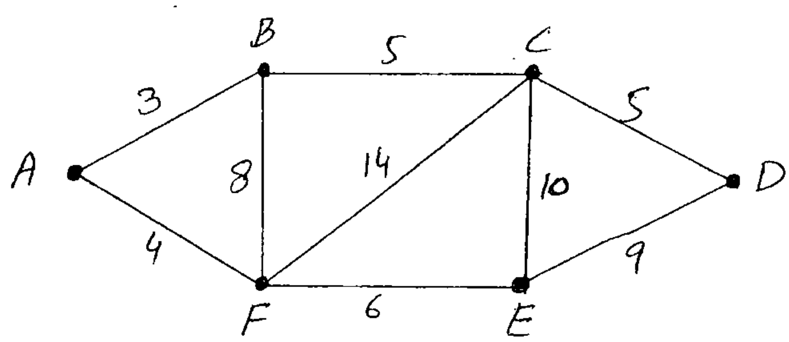
SUBJECTIVE

Q#2: Solve the following Short Questions. **(2×10=20)**

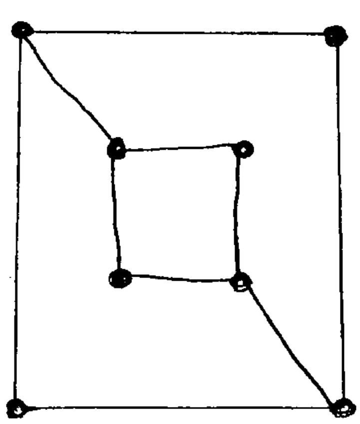
i) Write adjacency matrix representation of wheel W_5.
ii) Define Complete bipartite graphs and give example.
iii) Define vertex-induced subgraph and give example.
iv) Explain why a graph must have even number of vertices of odd degrees?
v) Determine the number of edges and vertices in G-e and G-v and $G\backslash e$?
vi) Define self-complementary graph witrh example.
vii) Can a tree be construct with 7 vertices and 9 edges? Explain your answer.
viii) Draw a graph which is Hamiltonian but not Eulerian.
ix) Draw 2 spanning trees of cube Q_3.
x) Define Hamiltonion graphs with example.

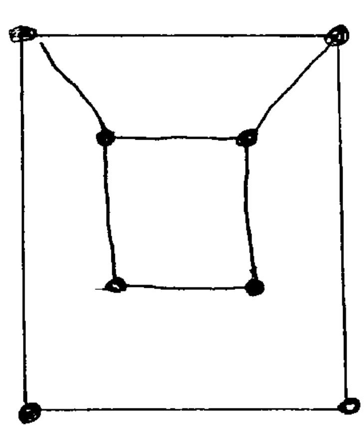
Q# 3: Solve the following Long Questions. **(5×6=30)**

i) Prove that in a tree every edge is a bridge.
ii) In a Petersen graph, find a trail of length 5, a path of length 8, and all cutsets with 3 edges.
iii) Find the labelled tree corresponding to the sequence {1,2,3,4}.
iv) Find the shortest path between vertices a and b in the following graph.



v) Define Isomorphic graphs and explain why the following two graphs are not isomorphic.





UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Graph Theory **TIME ALLOWED: 30 mins.**

Course Code: MATH-205/MTH-21312 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

(OBJECTIVE)

Q#1: Tick or circle the correct answer of the following. Multiple choice questions.

i) A pendant vertex has degree
(a) 0 (b) 1 (c) 2 (d) >2

ii) The number of vertices of odd degree in a non-trivial graph must be :
(a) 1 (b) odd (c) even (d) none

iii) A path graph P_n is
(a) regualr (b) closed (c) tree (d) none

iv) The sum of elements in each row of ________ is equal to degree of corresponding vertex of graph
(a) adjacency matrix only (b) incident matrix only (c) both (d) none

v) If degree of each vertex is odd then graph cannot be
(a)Eulerian (b) connected (c) disconnected (d) Hamiltonian

vi) In a directed graph sum of indegrees is equal to
(a) sum of out degrees (b) no. of edges (c) both (a) &(b) (d) none

vii) If vertex set of a subgraph is equal to vertex set of graph then subgraph is called
(a) spanning subgraph (b) vertex-induced subgraph (c) edge-induced subgraph

viii) If A is the adjacency matrix of graph, then entry a_{ij} of A^2 determines the no. of _______ of length 2 between vertex i and vertex j
(a) paths (b) walks (c) vertices (d)edges

ix) A path that traverse every vertex of graph is called _____________ path
(A) Eulerian (b) Semi-Eulerian (C) Closed (D) Hamiltonion

x) The complement of null graph is
(a)Graph itself (b) wheel graph (c) complete graph (d) disconnected graph

UNIVERSITY OF THE PUNJAB

Third Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Mathematics-II (Calculus) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-211/MTH-21107 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Short Questions

Q2: write the answers of the following questions **(5 X4)**

i) Solve $x^2-3x>10$

ii) Evaluate $\lim_{x\to 0} \frac{\ln(\tan x)}{\ln x}$

iii) Find dy/dx if $y=x^{\ln x}$

iv) Evaluate $\int x\sec^2 x\,dx$

v) Evaluate $\int_0^1 \frac{dx}{(1-x)^{\frac{1}{3}}}$

Long Questions

Q3 **(10)**

Let $f(x)=|x|$ if $x\neq 0$ and $f(0)=0$. Discuss the continuity and differentiability of f at x=0

Q4

Differentiate w.r.t x (5,5)

a) $y=\sqrt{(x^3+\csc x)}$

b) $y=(1+x^5\cot x)^{-8}$

Q5

Evaluate

a) $\int \frac{x^2}{\sqrt{x^3+1}}dx$

b) $\int_0^{\pi/8} \sin^5 2x \cos 2x \ dx$ (5,5)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Elementary Mathematics-II (Calculus) **TIME ALLOWED: 30 mins.**

Course Code: MATH-211/MTH-21107 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE **(Tick the correct statement)** **(10)**

1) If $f(x)=\sin\sqrt{x}$ then the natural domain of f is

a)$(-\infty, +\infty)$ b) $[1,+\infty)$

c)$(0, +\infty)$ d)$[2,+\infty)$

2) The solution of the inequality $-4<x-3<4$

a) (1,7) b) (-1,7)

c) (-1,-7) d) none of these

3) $\text{Lim}_{x\to\infty}(1+x)^{1/x}$

a) e b) -e

c) 0 d) ∞

4)$d \ln|x| /dx =$

a)$1/(x \ln e)$ b) $1/(x\ln a)$

c) $\pm x$ d) none above

5)$1/x^2+1$ is the derivative of

a)$\sin^{-1}x$ b) $\cos^{-1}x$

c) $\tan^{-1}x$ d)$\cot^{-1}x$

6)$\int \left(\frac{1}{x+1}\right) dx$

a)ln x b) $1/x \ln a$

c) $-1/x^2$ d)none of these

7)$\int \cos x\, dx$

a) sinx+c b) lncosx +c

c) ln sinx +c d)none of these

8)$-\int \left(\frac{1}{\sqrt{(1-x2)}}\right) dx$

a)$\sin^{-1}x$ b) $\cos^{-1}x$

c) $\tan^{1}x$ d)$\cot^{-1}x$

9)$\int_0^1 \left(1/\sqrt{(1-x2)}\right)$

a)0 b) 30

c) 60 d)90

10)$\int \sec x \tan x\, dx$

a)-cscx +c b) secx +c

c) cotx +c d)tan x+c

UNIVERSITY OF THE PUNJAB

Third Semester 2018
Examination: B.S. 4 Years Programme Roll No. ....

PAPER: Differential Equations-I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-221/MTH-21334 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

(Subjective)

Section-II (Short Questions) Marks=20

1. Verify that one-parameter family of solution

$$y = e^{-x^2} \int_0^x e^{t^2} dt + c_1 e^{-x^2},$$

satisfies differential equation $\frac{dy}{dx} + 2xy = 1$.

2. Solve initial-value problem (IVP)

$$\left(e^{2y} - y\right) \cos(x) \frac{dy}{dx} = e^y \sin(2x), \qquad y(0) = 0.$$

3. Solve

$$(x+1) \frac{dy}{dx} + y = \ln x, \qquad y(1) = 10.$$

4. Given that $x(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t)$ is the general solution of $\frac{d^2x(t)}{dt^2} + \omega^2 x(t) = 0$ on the interval $(-\infty, +\infty)$, show that a solution satisfying the initial conditions $x(0) = x_0$, $x'(0) = x_1$ is given by

$$x(t) = x_0 \cos(\omega t) + \frac{x_1}{\omega} \sin(\omega t).$$

5. Solve

$$x^4 \frac{d^4y}{dx^4} + 6x^3 \frac{d^3y}{dx^3} + 9x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = 0.$$

Section-III Marks=30

1. Solve the differential equation by using undetermined coefficients

$$\frac{d^2x(t)}{dt^2} + \omega^2 x(t) = F_0 \sin(\omega t), \qquad x(0) = 0, \; x'(0) = 0.$$

2. Solve differential equation

$$\frac{dy}{dx} = \cos(x+y),$$

subject to the initial condition $y(0) = \frac{\pi}{4}$.

3. Solve

$$(y^2 \cos x - 3x^2 y - 2x)dx + (2y \sin x - x^3 + \ln y)dy = 0, \qquad y(0) = e.$$

4. Solve the system of linear differential equations

$$\begin{aligned} (D-1)\,x(t) + \left(D^2+1\right) y(t) &= 1, \\ \left(D^2-1\right) x(t) + (D+1)\,y(t) &= 2, \end{aligned}$$

where $D = \frac{d}{dt}, D^2 = \frac{d^2}{dt^2}$.

5. Solve

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = \frac{1}{x+1},$$

by using variation of parameters.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Differential Equations-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-221/MTH-21334 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Instructions. Attempt all questions

Section-I (Objective) **Marks=10**

Fill in the blank or answer true/false.

1. The piecewise-defined function $y = \begin{cases} -x^2; & x < 0 \\ x^2; & 0 \leq x \end{cases}$ is a solution of the differential equation $x\frac{dy}{dx} - 2y = 0$ on the interval $(-\infty, \infty)$. (True/False)
2. $x\frac{dy}{dx} + y = \frac{1}{y^2}$ is a first-order linear ordinary differential equation. (True/False)
3. $y^3 + 3y = 1 - 3x$ is an implicit solution of $\frac{d^2y}{dx^2} = 2y\,(y')^3$. (True/False)
4. $y = \pm 2$ are two constant solutions of $\frac{dy}{dx} = y^2 - 4$. (True/False)
5. The set of functions $f_1(x) = 2+x, f_2(x) = 2+|x|$ is linearly dependent on interval $(-\infty, \infty)$.(True/False)
6. The set of functions $f_1(x) = x, f_2(x) = x-1, f_3(x) = x+3$ is linearly independent on interval $(-\infty, \infty)$. (True/False)
7. $\left(\frac{d^2}{dx^2} + b^2\right)\cos(bx) =$ ....
8. $W(\cos 3x, \sin 3x) =$ ....
9. $\frac{dy}{dx} = xy^{1/2}$ is a first order linear ordinary differential equation. (True/False)
10. $\frac{dy}{dx} = -\frac{x}{y}$ is an non linear ordinary differential equation. (True/False)

UNIVERSITY OF THE PUNJAB

Third Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Pure Mathematics **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-222/MTH-21119 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q.2 Answer the following short questions. [20]

i. Write all partitions of $S = \{1,2,3\}$.
ii. Find x and y given $(2x, x + y) = (6,2)$.
iii. Find the domain and range of the function $f(x) = \sqrt{1 - x^2}$.
iv. Find inverse function of $f(x) = \frac{2x-3}{2}$.
v. What is the difference between discrete and indiscrete topologies? Also give a suitable example.
vi. What is the difference between coarser and finer topologies? Also give a suitable example.
vii. What is the difference between tautologies and contradictions? Also give a suitable example.
viii. Find the domain and sketch the function $f(x) = \frac{1}{x^3}$.
ix. Define absurdity with suitable example.
x. Define Continuity. Also give an example.

Answer the long questions.

Q. 3 Determine the validity of the following arguments: [6]
If 7 is less than 4, then 7 is not a prime number.
7 is not less than 4.
____
7 is a prime number.

Q. 4 Prove that any open ball in the usual metric space $\mathbb{R}$ is open interval. [6]

Q. 5 A subset A of a metric space X is closed if and only if its compliment $X - A$ is open. [6]

Q. 6 Show that the propositions $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent. [6]

Q. 7 Prove that the sum of the first n positive integers is $\frac{1}{2}n(n + 1)$. [6]

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2018
Examination: B.S. 4 Years Programme

PAPER: Pure Mathematics **TIME ALLOWED: 30 mins.**
Course Code: MATH-222/MTH-21119 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1 Tick on the correct option. **[10]**

i) The number of elements in the power set of the set $\{\{1,2\},3\}$ is

a) 2 b) 4 c) 6 d) 8

ii) Power set of empty set has exactly_______________ subset/s.

a) zero b) one c) two d) three

iii) If $\mathcal{T}$ is a topology on non-empty set X, then arbitrary ___________ of member of $\mathcal{T}$ belong to $\mathcal{T}$.

a) Union b) Intersection c) product d) compliment

iv) In a Cartesian product $A \times B$, the $n(A \times B) =$

a) $n(A) + n(B)$ b) $n(B)/n(A)$ c) $n(A) \cdot n(B)$ d) $n(A)/n(B)$

v) Any two propositions can be combined by the word "and" to form a compound proposition called

a) Negation b) conjunction c) disjunction d) none of these

vi) The A.M between $3\sqrt{5}$ and $5\sqrt{5}$ is

a) $\sqrt{5}$ b) $2\sqrt{5}$ c) $3\sqrt{5}$ d) $4\sqrt{5}$

vii) Evaluate $\lim_{x\to 1} \frac{x^2-1}{x^2+3x-4}$

a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

viii) A subset of $A \times A$ is called a

a) relation from A to B b) relation in A c) relation in B

ix) The contrapositive of $p \to q$ is

a) $q \to p$ b) $\sim q \to \sim p$ c) $q \to \sim p$ d) $\sim q \to p$

x) Let $X = \{1,2,3,4,5\}$ with $\tau = \{X, \phi, \{2\}, \{4\}, \{2,4\}\}$. Let $A = \{1,2,5\}$. Then A^o=

a) $\{x\}$ b) $Z - \{x\}$ c) $\{\pm 1\}$ d) Z

UNIVERSITY OF THE PUNJAB

Third Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics (IT) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-231/IT-21404 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q2. Solve the following short questions (2x10=20)

1. Draw two 3-regular graphs with six vertices.
2. Construct a truth table for the statement form $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$.
3. What is a compound statement?
4. Let X is a non-empty set. Prove that the identity function on X is bijective.
5. How many integers from 1 through 1000 are multiples of 3 or multiples of 5?
6. Find the sum of all two digit positive integers which are neither divisible by 5 nor by 2.
7. Define a binary relation P from R to R as follows:
   for all real numbers x and y $(x, y) \in P \Leftrightarrow x = y^2$. Is P a function? Explain.
8. Find x and y given $(2x, x + y) = (6, 2)$.
9. Suppose that f is defined recursively by $f(0) = 3$, $f(n + 1) = 2f(n) + 3$. Find $f(2)$.
10. Find the number m of ways that nine toys can be divided among four children if the youngest child is to receive three toys and each of the others two toys.

Q3. Solve the following **Long Questions** (5x6=30)

1. Define a relation R on the set of all integers Z as follows:
   for all integers m and n, $m\ R\ n \Leftrightarrow m \equiv n \pmod 3$.
   Prove that R is an equivalence relation.
2. Given any two distinct rational numbers r and s with $r < s$. Prove that there is a rational number x such that $r < x < s$.
3. Prove that if n is an odd integer, then $n^3 + n$ is even.
4. Let S be the function such that $S(n)$ is the sum of the first n positive integers. Give a recursive definition of $S(n)$.
5. There are 15 girls and 25 boys in a class. How many students are there in total?
6. For the complete graph K_n, find
   (i) the degree of each vertex
   (ii)the total degrees
   (iii)the number of edges.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Discrete Mathematics (IT) **TIME ALLOWED: 30 mins.**
Course Code: MATH-231/IT-21404 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q1. Encircle the correct answer (1x10=10)

1. The inverse of the conditional statement $p \to q$ is
   a. $\neg p \to q$ b. $\neg p \to \neg q$ c. $q \to p$ d. $\neg q \to \neg p$
2. If A= {1,2,3,4}, then the number of elements in P (A) =.....
   a. 2^4 b. 2^5 c. 2^6 d. 2^7
3. Consider the relation R={(1,1),(1,2),(1,4),(2,1),(2,2),(3,3)} on set A={1,2,3,4} is
   a. Symmetric b. Reflexive c. Transitive d. None of these
4. A graph of a function *f* is one-to-one if and only if every horizontal line intersects the graph in .... point.
   a. at most one b. exactly one c. at least one d. none of these
5. 5, 9, 13, 17, ...is
   a. Arithmetic series b. Geometric series c. Arithmetic sequence d. Geometric sequence
6. The total number of one-to-one functions, from a set with three elements to a set with four elements is..........
   a. 24 b. 16 c. 12 d. 9
7. If $f(x) = 2x+1$ then its inverse =...........
   a. $x-1$ b. $\frac{x-1}{2}$ c. $1+x$ d. None of these
8. The inverse of given relation R = {(1,1),(1,2),(1,4),(3,4),(4,1)} is
   a. {(1,1),(2,1),(4,1),(2,3)}
   b. {(1,1),(1,2),(4,1),(4,3),(1,4)}
   c. {(1,1),(2,1),(4,1),(4,3),(1,4)}
   d. None of these
9. If a graph has vertices of degrees 1, 1, 4, 4 and 6. How many edges does the graph have?
   a. 8 b. 10 c. 12 d. 14
10. Which term of the sequence 4,1,-2,... is -77
    a. 26 b. 27 c. 28 d. None of these

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-IV **TIME ALLOWED: 15 Mints.**

Course Code: MATH-203 / MTH-22309 Part – I (Compulsory) MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q.1	MCQs (1x10=10)
(i)	Integrating factor of $\frac{dr}{dQ} = 500Q^n - \frac{r}{Q}$ is (a) 1/Q (b) 2Q (c) Q (d) None of these
(ii)	The Annihilator for $(x^2e^{-3x} + 15xe^{-3x} + 2e^{-3x})$ is given by________ (a)) D^3 (a)) $(D+3)^3$ (a)) $(D-3)^3$ (a)) D^2
(iii)	The initial value problem $y' = y,\ y(0) = 1$ has two solutions $y = 0$ and y=________ (a) $y = e^x$ (b) $y = e^{-x}$ (c) Ce^x (d) None of these
(iv)	**Classify the following differential Equation** $\frac{du}{dt} = 1 + u + t + ut$. (a) Separable (b) Linear (c) Exact (d) Reducible to Linear
(v)	**The function** ***P(x)*** **in the given linear 1st order ODE.** $\frac{dy}{dt} = \frac{y+t^2-y\sqrt{t}}{t}$ (a) x (b) 0 (c) 1 (d) None of these
(vi)	If y_1 and y_2 be the solutions of a differential equation then $y_3 = 5y_1 + 10\,y_2$ is (a) Solution (b) Not a Solution (c) Singular (d) None of These
(vii)	$y' + P(x)y = y^n f(x)$ is ________ (a) Bernoulli (b) Inhomogeneous (c) Linear (d) None of these
(viii)	The Singular point of $(x^3 - 27)y'' - 2xy' + y = 0$ is given by______ (a) 3 (b) 9 (c) $\sqrt{5}$ (d) None of these
(ix)	The general solution of a Non-homogenous third order differential equation involves______ arbitrary constants (a) 2 (b) 3 (c) 1 (d) None of these
(x)	$2xy''' + y^2 = 2x^2$ is a _____ order differential equation (a) 2 (b) 1 (c) 0 (d) None of these

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics A-IV
Course Code: MATH-203 / MTH-22309 Part – II

TIME ALLOWED: 2 Hrs. & 45 Mints.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Q. 2	SHORT QUESTIONS	
(i)	Solve the following differential equation $$dx + e^{3x}dy = 0$$	(4)
(ii)	Show that weather the following functions are linearly independent or not a) 1 b) Cos (x) c) Sin (x)	(4)
(iii)	Find the value of m so that the function $y=e^{mx}$ is a solution of D.E $$5y' = 2y$$	(4)
(iv)	Solve $y ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$.	(4)
(v)	Find the general solution of the following D.E $$x^2 + x - 27y'' - 2y' + xy = 0$$ Also radius of convergence if $x_0 = -3$.	(3+1)

	LONG QUESTIONS	
Q.3	Solve the following D.E. by using variation of parameter $$y'' - y = \frac{1}{x}$$	(6)
Q.4	Find the general solution of the following $$y''' + y'' = e^x cosx$$	(6)
Q.5	Solve the following Bernoulli equation $$3(1+t^2)\frac{dy}{dt} = 2ty(y^3 - 1)$$	(6)
Q.6	Solve the following differential equations $$y'' - 2y' + 2y = e^x \tan x$$	(6)
Q.7	Solve the following initial value problem by Power Series Method $$y'' - 2xy' + 8y = 0, \quad y(0) = 3,\ y'(0) = 0.$$	(6)

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-IV
(Metric Spaces & Group Theory)
TIME ALLOWED: 15 Mints.
Course Code: MATH-204 / MTH-22310 Part – I (Compulsory) MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q. 1 **(1x10=10)**

(i) If $A \cap B \neq \emptyset$ in a metric space then $d(A, B)$ is
a) zero b) greater than 1 c) 1 d) none

(ii) Every subset of discreet metric space is
a) open b) closed c) both open and closed d) none

(iii) If in a metric space (X,d), all singletons are open , then the derived set of any subset Y of X is
a) X itself b) empty set c) Y d) none

(iv) Suppose a sequence $\{x_n\}$ converges to a. Then a subsequence of $\{x_n\}$ converges to
a) Point different from a b) 0 c) a d) 1

(v) Suppose $f: X \to Y$ is continuous. Then pre image of closed set is
closed b) open c) may or may not be closed d) none

(vi) A group of prime order is always
a) Abelian but not cyclic b) cyclic c) non-abelian d) none

(vii) The intersection of all subgroups of a group is________.
a) {e} b) non-trivial subgroup c) group itself d) none

(viii) A non-abelian group of order 6 is
a) Z_6 b) S_6 c) S_3 d) none

(ix) The transposition is a cyclic permutation of length
a) 1 b) greater than 2 c) 2 d) none

(x) If $\alpha = (1\ 2)$ and $\beta = (4\ 6)$ are two permutations then order of product $\alpha\,\beta$ is
a) 2 b) 4 c) 3 d) none

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mathematics B-IV

(Metric Spaces & Group Theory)

Course Code: MATH-204 / MTH-22310 Part – II

TIME ALLOWED: 2 Hrs. & 45 Mints.

MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Q2. Solve the following short question. (2 marks for each question) **(2x10=20)**

(i) Define conjugate index with example. Also show that 2 is a self conjugate index.

(ii) Define closure of a subset. Also determine closure of set of natural numbers in usual metric space (R, d), where R is the set of real numbers.

(iii) Prove that every open ball in a metric space is open.

(iv) Prove that every Cauchy sequence in a metric space is bounded

(v) Find the limit points of an open interval and closed interval in real line with usual metric space.

(vi) Prove that $\overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$. What can you say about the equality? Justify your answer.

(vii) Show that the set $\{\bar{1}, \bar{2}, \bar{4}, \bar{5}, \bar{7}, \bar{8}\}$ under multiplication modulo 9 is a group. Also find the order of each element.

(viii) Determine whether the union of two subgroups is a subgroup or not? Justify your answer

(ix) Define transposition and detrermine whether the permutations $\alpha = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 5\,3\,2\,6\,4\,1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 3\,5\,2\,7\,6\,1\,4 \end{pmatrix}$ are even or odd?

(x) Let G be a group of order 1135. Can G have a subgroup of order 25? Justify your answer

Section –III **(6x5=30)**

Q3. Show that the space l_2 of all real sequences $x = \{x_n\}$ such that $\sum_{k=1}^{\infty} |x_k|^2 < \infty$ is a metric space.

Q4. State and prove Minkowski's Inequality

Q5. Prove that a function $f: X \to Y$ is continuous if and only for every open subset U in X, $f^{-1}(U)$ is open in X.

Q6. State Lagrange theorem and show that the converse statement of Lagrange theorem holds in case of finite cyclic group.

Q7. Let G be a group and $a \epsilon G$ have order n. Then show that for any integer$k$, $a^k = e$ if and only if $k = nq$, where q is an integer

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Number Theory **TIME ALLOWED: 15 Mints.**

Course Code: MATH-206 / MTH-22313 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q#1: **(1x10=10)**

i)A prime number has exactly __________ positive divisor.

a) 2 b) 1 c) 3 d) 4

ii) For an integer a, f $3 \nmid a^2$ then $a^2 =$ ______

a) $3k$ b) $3k+1$ c) $3k+2$ d) none

iii) Given integers a and b, not both zero then there exist integers x and y such that $\gcd(a,b)=$__________

a) $ab + xy$ b) $ax + by$ c) x+y d) xy

iv) The greatest common divisor of two consecutive integers is ____________

a) >2 b) 2 c) 1 d) 0

v) For any positive integers a, b, if $\gcd(a,b) = 1$, then

a) $\operatorname{lcm}(a,b) = ab$ b) $\operatorname{lcm}(a,b) = a$ c) lcm(a,b)=b d) lcm(a,b)=1

vi) The solution of Diophantine equation $33x + 18y = 22$

a) exist and unique b) exist but not unique c) does not exist d) none

vii) If a prime p divides product of n integers then

a) p divides exactly one integer b) p divides at least one integer c) p divides all n integers d) none

viii) If $5a \equiv 5b \pmod 7$, then

a) $a \equiv b \pmod 7$ b) $a \not\equiv b \pmod 7$ c) $a \equiv b+1 \pmod 7$ d) none

ix) The solutions of of $9x \equiv 21 \pmod{30}$

a) exist and unique b) exist and 3 in number c) does not exist d) none

x) The number 783423 is divisible by

a) 3 but not by 9 b) both by 3 and 9 c) neither by 3 nor by 9 d) divisible by 11

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Elementary Number Theory **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-206 / MTH-22313 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q#2: Solve the following Short Questions. **(2×10=20 Marks)**

i) Show that the fourth power of any integer is either of the form $5k$ or $5k+1$

ii) If $a \mid b$ and $a \mid c$ then show that $a \mid (bx + cy)$ for any integers x and y.

iii) Show that if $a \mid bc$ with $\gcd(a, b) = 1$ then $a \mid c$.

iv) For an arbitrary integer a, show that $3 \mid a(2a^2 + 7)$

v) If $a \equiv b \pmod n$, then show that $a^k \equiv b^k (mod\ n)$ for any positive integer k.

vi) Define Diophantine equation with an example. Also write the criterion for its solution.

vii) Determine whether the congruence $15x \equiv 6(mod\ 5)$ has a solution or not? Justify your answer.

viii) Find the remainder when 41^{65} is divisible by 7.

ix) If $ca \equiv cb(mod\ n)$, then show that $a \equiv b(mod\ n/d)$, where $d = \gcd(c, n)$.

x) Prove that if $\gcd(a,b)=1$ then $\gcd(2a + b, a + 2b)=1$ or 3

SECTION-II

Q#3: **Long Questions (5×6=30 Marks)**

i. State division algorithm and use it to show that $n(n + 1)(2n + 1)/6$ is an integer.

ii. . For $n \geq 1$, use mathematical induction to show that $5 \mid 3^{3n+1} + 2^{n+1}$.

iii. Find all incongruent solutions of the linear congruence $140x \equiv 133 \pmod{301}$.

iv. State and prove Chinese remainder theorem.

v. Use Euclidean Algorithm to obtain integers x and y satisfying $\gcd(56,72)=56x+72y$. Also find their lcm.

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Differential Equations-II **TIME ALLOWED: 15 Mints.**

Course Code: MATH-223 / MTH-22334 Part – I (Compulsory) MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Fill in the blank or answer true/false. **(1x10=10)**

1. $\mathcal{L}\{\int_0^t F(x)dx\} =$ ....
2. The only solution of the initial-value problem $\frac{d^2y}{dx^2} + x^2y = 0,\ y(0) = 0,\ y'(0) = 0$ is................................
3. $\mathcal{L}^{-1}\left\{\frac{1}{s^2+a^2}\right\} =$ ....
4. $x =$ ...., ..., are regular singular points of the second order linear differential equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$.
5. For Bessel function, $J_0'(x) =$ ....
6. $\int_0^x rJ_0(r)dr =$ ....
7. $\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} =$ ....
8. The equation $x^2(\frac{dy}{dx})^2 + 6y = 3x$ is a 2nd order linear differential equation. (True/False)
9. $y = 2x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 4y^2 = 0$. (True/False)
10. The general solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 1/4)y = 0$ is $y = c_1J_{1/2}(x) + c_2J_{-1/2}(x)$. (True/False)

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Differential Equations-II
Course Code: MATH-223 / MTH-22334 Part – II

TIME ALLOWED: 2 Hrs. & 45 Mints.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Section-II (Short Questions)

(4x5=20)

1. Show that

$$\int_0^x rJ_0(r)dr = xJ_1(x).$$

2. If $F(s)$ is the Laplace transform of the function $f(t)$, (i.e. $F(s) = \mathcal{L}\{f(t)\}$) and $n = 1, 2, 3, \ldots$, then show that

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{dt^n} F(s).$$

3. Determine the singular points of the given differential equations. Classify each singular point as regular and irregular:

$$(x^2 + x - 6)\frac{d^2y}{dx^2} + (x+3)\frac{dy}{dx} + (x-2)y = 0, \qquad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \nu^2)y = 0.$$

4. Use the Laplace transformation to solve the differential equation

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = t^3e^{2t}, \qquad y(0) = 0, y'(0) = 0.$$

5. Evaluate $\mathcal{L}^{-1}\left\{\tan^{-1}\left(\frac{1}{s}\right)\right\}$.

Section-III

(6x5=30)

1. Show that the substitution $y(x) = x^{-1/2}z(x)$ in Bessel's equation of order p, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - p^2)y = 0$, yields

$$\frac{d^2z}{dx^2} + \left(1 - \frac{p^2 - \frac{1}{4}}{x^2}\right)z = 0.$$

2. If $x = 0$ is a regular singular point of the given differential equation

$$x^2y'' - \left(x - \frac{2}{9}\right)y = 0.$$

Show that the indicial roots of the singularity do not differ by an integer. Use the method of Frobenius to obtain two linearly independent series solutions about $x = 0$. Form the general solution on $(0, \infty)$.

3. Use the Laplace transform to solve the given integral equation

$$f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau)e^{t-\tau}d\tau, \text{ for } f(t).$$

4. If $f(t)$ is piecewise continuous on $[0, \infty)$, of exponential order, and periodic with period P, then

$$L\{f(t)\} = \frac{1}{1 - e^{-sP}}\int_0^P e^{-st}f(t)dt.$$

5. The equation

$$\frac{dy}{dx} = P(x)y^2 + Q(x)y + R(x),$$

is called a Riccati equation. Suppose that one particular solution $y_1(x)$ of this equation is known. Show that the substitution

$$y(x) = y_1(x) + \frac{1}{u(x)},$$

transform the Riccati equation into the linear equation

$$\frac{du}{dx} + (Q(x) + 2P(x)y_1) = -P(x).$$

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years

Roll No. ....

PAPER: Linear Algebra **TIME ALLOWED: 15 Min.**

Course Code: MATH-224 / MTH-22120 Part – I (Compulsory) MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q. 1	MCQs (1x10 = 10 Marks)
(i)	The set of vectors $\{v_1, v_2, v_1+v_2, v_4, v_5, \dots, v_r\}$ in a vector space V over a field F is ... a) Linearly independent b) Linearly dependent c) Basis d) Subspace
(ii)	The set of vectors {(1,1,3), (2,1,1), (9,8,7), (5,6,7)} is ... a) Linearly independent b) Linearly dependent c) Basis d) Not Given
(iii)	Let V be a vector space over a field F. If there exists a finite subset S of V such that $L(S)=V$, where $L(S)$ denote the linear span of S, then V is said to be a) Trivial Space b) Infinite Dimensional c) Finite Dimensional d) Basis Free Space
(iv)	In a third order determinant, each element of the first column consists of the sum of two terms, each element of the second column consists of the sum of three terms and each element of the third column consists of the sum of four terms. Then it can decompose into n determinants, where n has value. (a) 1 (b) 9 (c)16 (d) 24
(v)	In an n-dimensional vector space V, any set of n linearly independent vectors always form a ... a) Row Space b) Column Space c) Null Space d) Basis
(vi)	Let $T: R^{11} \to R^{11}$ be a linear transformation. If the dimension of the null space of T is 9, then the dimension of $R(T)$ is a) 9 b) 11 c) 2 d) 22
(vii)	For a square matrix A, Rank $(A A^T)$= (a) Rank (A) (b) Rank (A^T) (c) Rank $(A^T A)$ d) All (a), (b) and (c) are true
(viii)	In the group (Z, o) of all integers where $aob = a + b - ab$ for $a, b \in Z$, the inverse of 2 is a) 1 b) 2 c) 3 d) 4 e) Not given
(ix)	The group $c_2 \times c_3$ is isomorphic to a) S_3 b) D_3 c) C_6 d) Not Given
(x)	Every one dimensional representation is a) Reducible b) Irreducible c) Both (a) and (b) d) Not Given

UNIVERSITY OF THE PUNJAB

Fourth Semester - 2018

Examination: B.S. 4 Years

Roll No. ....

PAPER: Linear Algebra **TIME ALLOWED: 2 Hrs. & 45 Min.**

Course Code: MATH-224 / MTH-22120 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q. 2	Short Questions (4x5 = 20 Marks)
(i)	Prove that every finite dimensional vector space contains a basis.
(ii)	Let $G = < a, b \mid a^2 = b^2 = (ab)^2 = e >$ and $C_2 = < g \mid g^2 = e >$ be two groups, then show that $G \times C_2$ is isomorphic to $C_2 \times C_2 \times C_2$.
(iii)	Find an orthonormal basis of the basis $\{(2,3), (4,5)\}$ using Gram-Schmidt orthonormalization process.
(iv)	Find the eigenvalues and eigenvectors of the matrix (if possible) $$\begin{bmatrix} 2 & 11 \\ 0 & 3 \end{bmatrix}$$
(v)	Prove or disprove that a one to one linear transformation preserves a basis.

Section-III

	Long Questions (6x5 = 30 Marks)
Q.3	For what values of a, will the system have no solution? Exactly one solution? Infinitely many solutions? $$x + 2y - 3z = 4$$ $$3x - y + 5z = 2$$ $$4x + y + (a^2 - 14)z = a + 2$$
Q.4	Let V be a vector space over a field F and B be a finite subset of V. Prove that the following statements are equivalent? (i) B is a basis for V. (ii) B is the maximal set of linearly independent vectors in V. (iii) B is the minimal set of generators for V.
Q.5	Show that $\{(1,2), (3,4)\}$ is a basis of R^2. Using Gram-Schmidt orthonormalization process, transform this basis into an orthonormal basis.
Q.6	Define reducible and irreducible representations with one example of each. Also state Schur's Lemma without proof.
Q.7	Define group homomorphism and isomorphism. Let H be a subset of a finite group G. If H contains all elements of G whose orders are finite. Prove that H is a subgroup of G.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fifth Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Real Analysis-I **TIME ALLOWED: 30 mins.**
Course Code: MATH-301 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1: Choose the best option. **Marks: 10**

(i) The union of rational and irrational numbers is:
(a) set of real numbers (b) set of complex numbers
(c) set of integers (d) none

(ii) Between any two rational numbers their lie rational number.
(a) one (b) two
(c) three (d) infinite

(iii) Let x, y, z $\in$ R, if x < y and y < z then x < z is called
(a) reflexive property (b) symmetric property
(c) transitive property (d) none

(iv) A real sequence $Sn \leq Sn + 1$ is, forall $n \geq 1$ is called"
(a) strictly increasing (b) strictly decreasing
(c) monotonically increasing (d) monotonically decreasing

(v) Every differentiable is also:
(a) continuous (b) discontinuous
(c) undefined (d) none

(vi) A bounded monotonic sequence ________ must be convergent:
(a) the integers (b) real numbers
(c) rational numbers (d) irrational numbers

(vii) If $\{t_a\}$ is bounded and $\{S_n\}$ in null sequence then $\{t_sS_n\}$ is"
(a) also a null sequence (b) bounded sequence
(c) not a sequence (d) none

(viii) Every subsequence of a convergent sequence is convergent and converge to the _ limit:
(a) same (b) different
(c) infinity (d) none

(ix) Find the value of the $\lim_{x\to 0} x \sin 1/x$:
(a) 0 (b) 1
(c) −1 (d) infinity

(x) There are types of discontinuity:
(a) 1 (b) 2
(c) 3 (d) 4

UNIVERSITY OF THE PUNJAB

Fifth Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Real Analysis-I
Course Code: MATH-301

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Questions with Short Answers.

Q.2: Answer the following short questions. All questions carry equal marks. (5×4=20)

(i) If r is rational and $r \neq 0$ and x is an irrational number then prove that rx is an irrational number.

(ii) If it exist the sup. Of a non empty subset of an ordered field is unique.

(iii) If p > 0, then $\lim_{x\to\infty} \frac{1}{n^p} = 0$

(iv) The series $\sum_{n=a}^{\infty} \frac{1}{n^p}$ converges if p > 1 and diverges if $p \leq 1$.

(v) Show that $f(x) = x \sin 1/x, x \neq 0$. at x = 0 is continuous.

Q.3: Do the following "Long Questions". (5×6=30)

(i) If x, y∈R and x < y then prove that there exists an irrational numbers μ such that x<μ<y.

(ii) Let x be a real number x < 1 and let p, q be rational numbers such that p > q > 0. show that

(iii) Let x, y, z $\in R^k$ then prove that //x.y//≤//x//.//y//.

(iv) Show that f(x) is defined by $f(x)\begin{bmatrix} \frac{x^2}{a} - a & 0 < x < a \\ 0 & at\ x = 0 \\ a - \frac{x^3}{a^2} & x > a \end{bmatrix}$ is continuous at x = 0.

(v) Suppose that f is differentiable at point c in the domains of f and g is differentiable at f(c), then the composite function h(x) = gof(x) = g(f(x)) is also differentiable at x = c and h'(c) = g'(f(c)).f'(c).

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fifth Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Group Theory-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-302 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q. 1 Encircle the correct answer **1 x 10**

1. The smallest non-abelian group is

   (a) Klien 4-group (b) Symmetric group S_3 (c) Group of Quaternions Q_8. (d) Z_3

2. If $a,b \in G$ and $(ab)^2 = a^2b^2$ then

   (a) a commutes with b (b) a does not commute with b (c) a and b are equal

3. Every group of prime order is

   a) Abelian b) Non-Abelian c) Cyclic d) both a and c

4. Any two conjugate subgroups of a group G have ____order.

   a) same b) distinct c) special d) all of these

5. If $G = \{\pm 1, \pm i\}$, then the generators of G are

   a) $\pm i$ b) ± 1 c) $-i$ d) i

6. A subgroup of index ________is always normal subgroup.

   a) 1 b) 2 c) 3 d) 4

7. The order of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ is

   a) 2 b) 3 c) 4 d) 0

8. If X is a complex in V_4 then $N_{V_4}(X) =$

   a) X b) V_4 c) φ d) $X \cap V_4$

9. Conjugate elements have _____order.

   a) same b) distinct c) special d) all of these

10. If G is abelian then

    a) $Z(G) \subset G$ b) $Z(G) \not\subset G$ c) $Z(G) = G$ d) $G \subseteq Z(G)$

UNIVERSITY OF THE PUNJAB

Fifth Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Group Theory-I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-302 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q-2 Solve the following 'SHORT' Questions. $(2 \times 10 = 20)$

1. Prove that every cyclic group is abelian.
2. Prove that the order of a cyclic permutation of length m is m.
3. Prove that if every element of a group G is idempotent then G is abelian.
4. Let G be a cyclic group such that O(G)=24, generated by a. Find the order of a^9.
5. Determine whether the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 5 & 1 \end{pmatrix}$ is even or odd?
6. Let G be a group of residue classes modulo 6 under addition, then prove that H=$\{\bar{0},\bar{2},\bar{4}\}$ is a subgroup of G. Also find all left cosets of H in G.
7. Give example of an abelian group which is not cyclic.
8. Let G be a group, then show that the derived group $G'$is a normal subgroup of G.
9. Find the centre of G = < a,b: a^3= b^2= $(ab)^2$= e >.
10. Give an example of non-abelian group whose all subgroups are normal.

Q-3 Solve the following 'LONG' Questions. $(10 \times 3 = 30)$

1. State and prove third isomorphism theorem.
2. a) A group G is abelian if and only if the factor group $G/_{Z(G)}$ is cyclic.

   b) A homomorphism $\varphi: G \longrightarrow G'$ is one-one if and only if kerφ =$\{e\}$.
3. State and Prove Lagrange Theorem.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fifth Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Complex Analysis-I **TIME ALLOWED: 30 mins.**

Course Code: MATH-303 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Question I. Tick the correct answer to each question. 1 x 10=10

1. The real part of $\tan(-i\log i)$ is ______.

   (a) $\tan 1$ (b) $\frac{\pi}{4}$ (c) $\frac{tan^{-1}2}{4}$ (d) $\frac{\pi}{2}$

2. If z represents a complex number then $\arg(z) + \arg(\bar{z})$=______.

   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{6}$

3. If $z = x^3 + iy^2$, for any $x, y \in \mathbb{R}$, then $\arg(e^z) =$ ______ .

   (a) $\sin y^2$ (b) x^3 (c) y^2 (d) x^3y^2

4. For any complex number $z = x + iy$, $|\sin z|^2 =$______.

   (a) $\sin^2 x + \sinh^2 y$ (b) $\sin^2 x + \cosh^2 y$ (c) $\cos^2 x + \sinh^2 y$ (d) $\cos^2 x + \cosh^2 y$

5. A curve composed of finite number of smooth arcs is called ______.

   (a) smooth curve (b) Jordan curve

   (c) piecewise smooth curve (d) closed curve

6. The inverse trigonometric functions are ______.

   (a) analytic (b) entire (c) multi-valued (d) none of these

7. The fixed points of the transformation $f(z) = z^2$ are

   (a) 0, 1 (b) 0, -1 (c) -1, 1 (d) $-i$, i

8. $w = e^{z+i}$ transforms origin of $z-$plane into $w-$plane to a ______.

   (a) point (0,1) (b) straight line (c) unit circle (d) point (0,e)

9. Radius of convergence of $\sum_{0}^{\infty} \frac{z^n}{n!}$ is ______.

   (a) 0 (b) ∞ (c) 1 (d) -1

10. $\int_C z^3 dz =$______, where $C : |z| = 1$.

    (a) 0 (b) z (c) 3 (d) 1

UNIVERSITY OF THE PUNJAB

Fifth Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Complex Analysis-I
Course Code: MATH-303

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SHORT QUESTIONS

Question II. Write the answer of the following short questions. **5 x 4=20**

1. Find the locus of the points $Arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$.
2. Prove that bilinear transformation is one-one.
3. Test the analyticity of the function $f(z) = \sin(37x + 35iy)$ without using Cauchy–Riemann equations.
4. Evaluate $\int_C \bar{z}dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by $z = t^2 + it$.

LONG QUESTIONS **10x3=30**

Question III. State and prove Morera's theorem.

Question IV. Evaluate $\int_C \frac{z^2 + 7z + 1}{(z-1)^7}dz$ where $C : |z| = 2$.

Question V. Prove that $\nabla^2 [\text{Re}f(z)]^2 = 2\,|f'(z)|^2$ where $f(z)$ is an analytic function.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fifth Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Vector and Tensor Analysis TIME ALLOWED: 30 mins.

Course Code: MATH-304 MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q.1 **Tick the correct option.**

(i) ____________ theorem converts line integral to surface integral.
(i) Green (ii) Gauss (iii) Divergence (iv) Stokes

(ii) The ____________ law is used for determining a quantity whether a tensor or not.
(i) Contraction (ii) Quotient (iii) Kronecker (iv) Product

(iii) How many components does a tensor of rank 2 in a 3-dimensional space?
(i) Zero (ii) Six (iii) Eight (iv) Nine

(iv) A contraction in a tensor of rank 2 yields
(i) A vector (ii) A scalar (iii) A tensor of rank 3 (iv) Zero vector

(v) A vector is solenoidal if its ____________ is zero
(i) Gradient (ii) Curl (iii) Divergence (iv) Directional angle

(vi) The scalar product of $3\hat{\imath}-\hat{\jmath}$, $\hat{\jmath}+2\hat{k}$, $\hat{\imath}+5\hat{\jmath}+4\hat{k}$ is ____________.
(i) -10 (ii) 20 (iii) 10 (iv) None of these

(vii) $\vec{A} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$, $z = \frac{12-2x-3y}{6}$ *and* $\hat{n} = \frac{1}{7}(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$, a unit normal to the surface which has the projection in the xy-plane for which $0 \le x \le 6, 0 \le y \le \frac{12-2x}{3}$. Then the surface integral $\iint \vec{A}\cdot\hat{n}\,dS =$ ____________.
(i) 24 (ii) 12 (iii) Zero (iv) None of these

(viii) The line integral $\int_{P_1}^{P_2} \vec{A}\cdot d\vec{r}$ appears to be independent of the curved path C in a region R joining the two points P_1 & P_2. Then what is true about the vector field $\vec{A}$?
(i) $\nabla\times\vec{A}=0$ (ii) $\nabla\cdot\vec{A}=0$ (iii) $\nabla\times\vec{A}\neq 0$ (iv) None of these

(ix) A unit vector vector normal to the surface $2x^2+4xy-5z^2=-10$ at the point (3,-1,2) is
(i) $12\hat{\imath}+8\hat{\jmath}-24\hat{k}$ (ii) $\frac{1}{7}(12\hat{\imath}+8\hat{\jmath}-24\hat{k})$ (iii) $\frac{1}{7}(3\hat{\imath}+2\hat{\jmath}-6\hat{k})$ (iv) $12\hat{\imath}+8\hat{\jmath}$

(x) A field $\vec{F}$ is conservative if
(i) $\nabla\times\vec{F}=0$ (ii) $\nabla\times\vec{F}\neq 0$ (iii) $\nabla\cdot\vec{F}=0$ (iv) None of these

UNIVERSITY OF THE PUNJAB

Fifth Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Vector and Tensor Analysis | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-304 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Section – I (Short Questions)

Q.2 Solve the following Short Questions. **(5 × 4 = 20)**

(i) State Stokes theorem and Gauss theorem of divergence .

(ii) For the general coordinates u_1, u_2, u_3, show that $\frac{\partial \vec{r}}{\partial u_1}, \frac{\partial \vec{r}}{\partial u_2}, \frac{\partial \vec{r}}{\partial u_3}$ *and* $\nabla u_1, \nabla u_2, \nabla u_3$ are reciprocal system of vectors.

(iii) Find scale factors for spherical coordinates.

(iv) Define inner product in tensors and prove that any inner product of the tensors A_r^p *and* B_t^{qs} is a tensor of rank 3..

(v) Evaluate the integral $I = \int \vec{A} \,.\, \vec{n} \; dS$ *where* $\vec{A} = z\hat{\imath} + x\hat{\jmath} - 3x^2y\hat{k}$ and S is the portion of the cylinder $x^2 + y^2 = 8$ lying in the first octant between $z=0$ and $z=4$.

Section – II (Long Questions) **(3 × 10 = 30)**

Q.3 Show that a necessary and sufficient condition that $F_1dx + F_2dy + F_3dz$ be an exact differential is that $\nabla \times \vec{F} = \vec{0}$ *where* $\vec{F} = F_1\hat{\imath} + F_2\hat{\jmath} + F_3\hat{k}$.

Q.4 Find covariant and contravariant components of a tensor in cylindrical coordinates if its covariant components in rectangular coordinates are $2x - z, x^2y, yz$.

Q.5 Define Christoffel symbols of first and second kind. Show that the contraction of the outer product of the tensors A^p *and* B_q is a scalar. **(4+6)**

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fifth Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Topology **TIME ALLOWED: 30 mins.**

Course Code: MATH-305 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

SECTION-I

Q. 1 **MCQs (1 Mark each)**

(i) Let $f:\mathbb{R}\to\mathbb{R}$ be the function defined by $f(x)=\begin{cases}x, & \text{if } x\neq 1\\ 3, & \text{if } x=1\end{cases}$.

Let $O=(2,4)$ be open in $\mathbb{R}$ the $f^{-1}(O)=$ ---------

(a) $(2,4)$ (b) $(1,4)$ (c) $\{1\}\cup(2,4)$ (d) None of these

(ii) The closure of the subset $(-1,4]\cup[7,11)$ of the real line $\mathbb{R}$ under the usual metric is ------

(a) $(-1,4]\cup[7,11)$ (b) $[-1,4]\cup[7,11]$ (c) $[-1,4]\cup[7,11)$ (d) None of these

(iii) The interior of the subset $(0,1)\cup\{2,3\}$ on the real line $\mathbb{R}$ under the usual topology is ------

(a) $(0,1)\cup\{2,3\}$ (b) $(0,1)$ (c) $[0,1]\cup\{2,3\}$ (d) None of these

(iv) In $(\mathbb{R},\tau)$ with usual topology τ on $\mathbb{R}$, then the interior set of $\mathbb{N}=\{1,2,3,...\}$ is -----

(a) $\{0\}$ (b) $\mathbb{N}$ (c) $\mathbb{R}$ (d) ϕ

(v) Let X an arbitrary space and Y a Hausdorff space. Let $f:X\to Y$ be a continuous function Then the graph $G=\{(x,y):y=f(x)\}$ is closed in ------------.

(a) $X\times X$ (b) $X\times Y$ (c) $Y\times Y$ (d) None of these

(vi) Let $(\mathbb{R},\tau)$ be a topological space with usual topology τ on $\mathbb{R}$ then the set $\mathbb{Q}$ is -------

(a) open (b) closed (c) both open and closed (d) neither open nor closed

(vii) Let X be infinite set with co-finite topology τ on X. Then (X,τ) is -------

(a) Compact and connected (b) Not Compact and connected

(c) Compact and not connected (d) Neither Compact nor connected

(viii) Every compact subset of a --------- space is normal

(a) regular (b) Hausdorff (c) Tychonoff (d) None of these

(ix) Let (X,τ) be a topological space and $A\subset X$. Then A is closed if and only if -------

(a) $F_r(A)\subset A$ (b) $F_r(A)\supset A$ (c) $A^{\circ}=A$ (d) None of these

(x) The set $\{\mathbb{Q}\cap(-\infty,r),\mathbb{Q}\cap(r,\infty)\}$ is a disconnection for $\mathbb{Q}$ where r is ---------

(a) integer (b) rational number (c) irrational number (d) real number

UNIVERSITY OF THE PUNJAB

Fifth Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Topology | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-305 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

SECTION-II

Q. 2 **SHORT QUESTIONS**

(i) Prove that $(0,1)$ is homeomorphic to (a,b). (4)

(ii) Let $X=\{a,b,c,d,e\}, \Im=\{\phi,\{a\},\{c,d\},\{a,c,d\},\{b,c,d,e\},X\}$. Find the frontier of the set $A=\{b,c,d\}$. (4)

(iii) Let $X=\{1,2,3\}$, $\tau=\{\phi,\{3\},\{1,2\},X\}$ be a topological space. Find the interior and exterior of the set $A=\{1,2\}$. (4)

(iv) Prove that every closed subspace of a normal space is a normal. (4)

(v) Let X be countably compact space. Then Show that every infinite subset of X has a limit point in X. (4)

SECTION-III

LONG QUESTIONS

Q.3 Prove that any uncountable set X with co-finite topology is not first countable and so not second countable. (6)

Q.4 Prove that the following statements are equivalent: (6)

(i) X is a Hausdorff space (ii) the diagonal $D=\{(x,x): x\in X\}$ is closed in $X\times X$.

Q.5 Let $f: X\to Y, g: X\to Y$ be continuous function from a space *X* to a Hausdorff space *Y* and suppose that $f(x)=g(x)$ for all x in a dense subsets D of *X*. Then prove that $f=g$ for all $x\in X$. (6)

Q.6 Prove that every compact subset of a Hausdorff space is closed. (6)

Q.7 Prove that a space X is connected if and only if there does not exist a surjective continuous function $f: X\to D$ where D is two point discrete space. (6)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Fifth Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Differential Geometry **TIME ALLOWED: 30 mins.**

Course Code: MATH-306 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE PART

Each MCQ carries 1 mark. Encircle and clearly mark the correct option only. Cutting, overwriting and use of ink remover are not allowed. [1x10=10]

1. (i) Let n be the unit normal to a given curve $r = r(s)$, the magnitude of the vector dn/ds, where s is the arc-length is
   (A) κ, (B) $\sqrt{\kappa^2 + \tau^2}$, (C) τ, (D) $\sqrt{\tau^2 - \kappa^2}$.

   (ii) The equation of the rectifying plane for a surface is given by (A) $(R - r) \cdot b = 0$, (B) $(R - r) \cdot n = 0$, (C) $(R - r) \cdot t = 0$, (D) $R = r$.

   (iii) If, for a curve τ is non-zero, then the curve is said to be a
   (A) maximal curve (B) plane curve (C) unique curve (D) twisted curve

   (iv) The distance between corresponding points of two involutes is equal to
   (A) ρ (B) κ (C) constant (D) zero.

   (v) The point on a surface $r = r(u, v)$ for which $r_u \times r_v = 0$, is called
   (A) an ordinary point (B) a singular point (C) a regular point (D) a double point.

   (vi) A surface $x = x(u, v)$ is called a minimal surface if at all of its points, the first curvature of the surface is
   (A) positive, (B) negative, (C) zero, (D) infinite.

   (vii) A curve $x(s) = x(u(s), v(s))$ on a given surface $x = x(u, v)$ whose tangents at all of its points are in the direction of principal curvature, is called
   (A) the skew curve (B) the line of curvature (C) the twisted curve (D) the rectifying plane

   (viii) The surface for which specific curvature is zero is said to be
   (A) a developable surface, (B) a maximal surface,
   (C) a surface of revolution, (D) a tangent plane.

   (ix) A curve $x(s) = x(u(s), v(s))$ on a given surface $x = x(u, v)$ whose tangents at all of its points are in the direction of principal curvature, is called
   (A) the skew curve, (B) the line of curvature,
   (C) the twisted curve, (D) the rectifying plane.

   (x) The product of principal curvatures of a surface is called the
   (A) first curvature, (B) Gauss curvature,
   (C) average curvature, (D) more information is needed.

UNIVERSITY OF THE PUNJAB

Fifth Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Differential Geometry | **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-306 | **MAX. MARKS: 50**

SUBJECTIVE PART

Note: Attempt this part of the Paper on Separate Sheet. Question 2 is worth a total of 20 marks and question 3 is worth a total of 30 marks.

SECTION-I (SHORT QUESTIONS)

Attempt the following questions. [4x5=20]

2. (i) Let the path traced by a particle be $\mathbf{x} = \mathbf{x}(t)$. Find the tangential and normal components of acceleration, hence show that acceleration vector lies in the osculating plane.

   (ii) Find the arc-length s as a function of θ along the epicycloid $x = (r_0 + r)\cos\theta$ - $r\cos\left(\frac{r_0+r}{r}\theta\right)$, $y = (r_0 + r)\sin\theta - r\sin\left(\frac{r_0+r}{r}\theta\right)$, where r and r_0 are constants.

   (iii) Prove that the tangent, principal normal and binormal to the locus of the centre of, osculating sphere C_1 are parallel to the binormal, principal normal and tangent to the given curve C.

   (iv) Show that the tangent plane at the point common to the surface $a(xy + yz + zx) = xyz$ and a sphere $x^2 + y^2 + z^2 = b^2$ whose centre is at origin, makes intercepts on the axes whose sum is constant.

   (v) Prove that at each point on a patch $\mathbf{r} = \mathbf{r}(u, v)$, $\mathbf{N}_u \times \mathbf{N}_v = K(\mathbf{r}_u \times \mathbf{r}_v)$, where K is the Gaussian curvature.

SECTION-II (LONG QUESTIONS)

Attempt the following questions. [3x10=30]

3. Define spherical image of the binormal. Prove that the curvature κ_1 and torsion τ_1 of the spherical indicatrix of the binormal are given by $\kappa_1 = \frac{\sqrt{\kappa^2+\tau^2}}{\tau}$ and $\tau_1 = \frac{\kappa'\tau - \tau'\kappa}{\tau(\kappa^2+\tau^2)}$.

4. State and prove Gauss-Weingarten equations for a surface $\mathbf{r} = \mathbf{r}(u, v)$.

5. What is a surface of revolution? When is a surface said to be minimal? Show that the condition of minimality for a surface of revolution $\mathbf{r} = (u\cos\alpha, u\sin\alpha, f(u))$ satisfies the differential equation $u\frac{d^2f}{du^2} + \frac{df}{du}\left(1 + \left(\frac{df}{du}\right)^2\right) = 0$.

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Real Analysis-II **TIME ALLOWED: 15 Mints.**

Course Code: MATH-307 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

SECTION I

1. If $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$ and if $f(x) = \Sigma_{n=1}^{\infty} f_n(x)$ $(a \leq x \leq b)$, the series converging uniformly on $[a,b]$, then (1 mark)

   (a) $\int_a^b f d\alpha < \Sigma_{n=1}^{\infty} \int_a^b f_n d\alpha$

   (b) $\int_a^b f d\alpha > \Sigma_{n=1}^{\infty} \int_a^b f_n d\alpha$

   (c) $\int_a^b f d\alpha = \Sigma_{n=1}^{\infty} \int_a^b f_n d\alpha$

   (d) none of the above

2. If f is a monotonic function in $[a,b]$, then f is a function of bounded variations in $[a,b]$ and .... (1 mark)

   (a) $V(f,[a,b]) > |f(b) - f(a)|$

   (b) $V(f,[a,b]) < |f(b) - f(a)|$

   (c) $V(f,[a,b]) = |f(b) - f(a)|$

   (d) none of the above

3. $\int_0^1 \frac{dx}{x^p}$ converges if and only if (1 mark)

   (a) $p < 1$

   (b) $p = 1$

   (c) $p > 1$

   (d) both (a) and (b)

4. The improper integral $\int_a^{\infty} \frac{C}{x^n}$, $a > 0$, where C is a positive constant, converges if and only if (1 mark)

   (a) $n > 1$

   (b) $n < 1$

   (c) $n \leq 1$

   (d) $n \geq 1$

5. A new class of functions, called functions of bounded variation, forms ...... setting for the discussion of Riemann-Stieltjes integrals. (1 mark)

   (a) a more restricted

   (b) an equal

   (c) a more general

   (d) none of the above

(P.T.O.)

6. A necessary and sufficient condition for the convergence of the improper integral $\int_a^b f dx$ at a, where f is positive in $[a, b]$ is that there exists a positive number M, independent of λ, such that (1 mark)

(a) $\int_{a+\lambda}^b f dx < M, 0 > \lambda > b - a$

(b) $\int_{a+\lambda}^b f dx = M, 0 < \lambda < b - a$

(c) $\int_{a+\lambda}^b f dx > M, 0 < \lambda < b - a$

(d) $\int_{a+\lambda}^b f dx < M, 0 < \lambda < b - a$

7. A sequence of functions $\{f_n\}, n = 1, 2, 3, ...$, converges uniformly on E to a function f, if for every $\epsilon > 0$, there is an integer N such that $n \geq N$ implies (1 mark)

(a) $|f_n(x) - f(x)| \leq \epsilon$

(b) $|f_n(x) - f(x)| > \epsilon$

(c) none of the above

(d) both (a) and (b)

8. The sequence of functions $\{f_n\}, n = 1, 2, 3, ...$, defined on E, converges uniformly on E if and only if for every $\epsilon > 0$, there exists an integer N such that $m \geq N$, $n \geq N$, $x \in E$ implies (1 mark)

(a) $|f_n(x) - f_m(x)| \leq \epsilon$

(b) $|f_n(x) - f_m(x)| > \epsilon$

(c) none of the above

(d) both (a) and (b)

9. Suppose $\{f_n\}, n = 1, 2, 3, ...$,is a sequence of functions defined on E, and suppose $|f_n(x)| \leq M_n$ $(x \in E)$. Then Σf_n converges uniformly on E if (1 mark)

(a) ΣM_n diverges

(b) ΣM_n converges

(c) none of the above

(d) both (a) and (b)

10. Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$, for $n = 1, 2, 3, ...$, and suppose $f_n \to f$ uniformly on $[a, b]$, then (1 mark)

(a) f_n does not belong to $\mathcal{R}(\alpha)$ on $[a, b]$

(b) $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$

(c) none of the above

(d) both (a) and (b)

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Real Analysis-II | **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-307 Part – II | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION II-Questions with Short Answers

1. Show that the sequence $\{(\sin x)^{1/n}\}$ converges but not uniformly on $[0, \pi]$. (4 marks)

2. Evaluate $\int_0^3 [x] d(e^x)$. (4 marks)

3. Test the convergence of $\int_0^{\pi/2} \frac{\sin x dx}{x^p}$. (4 marks)

4. If f and g are functions of bounded variations in $[a, b]$, then $f + g$ is a function of bounded variation in $[a, b]$. (4 marks)

5. If f is monotonic on $[a, b]$, and if α is continuous on $[a, b]$, then $f \in \mathcal{R}(\alpha)$. (4 marks)

SECTION III-Questions with Brief Answers

6. Evaluate $\int_0^3 f(x) d([x] + x)$, where (6 marks)

$$f(x) = \begin{cases} [x], & 0 \leq x < 3/2 \\ e^x, & 3/2 \leq x \leq 3. \end{cases}$$

7. Show that $\int_0^1 x^{m-1}(1-x)^{n-1} dx$ exists if and only if m, n are both positive. (6 marks)

8. A function α increases on $[a, b]$ and is continuous at x' where $a \leq x' \leq b$. Another function f is such that $f(x') = 1$, and $f(x) = 0$, for $x \neq x'$. Prove that $f \in \mathcal{R}(\alpha)$ over $[a, b]$, and $\int_a^b f d\alpha = 0$. (6 marks)

9. Show that the sequence $\{f_n\}$, where

$$f_n(x) = \begin{cases} n^2 x, & 0 \leq x \leq 1/n \\ -n^2 x + 2n, & 1/n \leq x \leq 2/n \\ 0, & 2/n \leq x \leq 1. \end{cases}$$

is not uniformly convergent on the interval $[0, 1]$. (6 marks)

10. Show a polynomial $p(x)$ is a function of bounded variation in each closed interval $[a, b]$. Describe a method of finding the total variation of $p(x)$ on $[a, b]$ from the knowledge of zeros of the derivative $p'(x)$. (6 marks)

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Rings and Vector Spaces **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-308 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q. 2

(i) Show that the homomorphic image of a ring is sub-ring. (4)

(ii) Prove that eigen values of a symmetric matrix are all real. (4)

(iii) Define similar matrices and prove that eigen values of the similar matrices are same. (4)

(iv) Dene the terms: Change of basis, Orthogonal basis, Orhonormal basis, characteristic polynomial (4)

(v) Prove that one-to-one linear transformation preserves the basis and dimension. (4)

Q.3 Let R be a commutative ring with identity. The ideal P is prime ideal *iff* the quotient ring R/P is an integral domain. (6)

Q.4 Let X and Y be vectors spaces over the field F with dimensions m and n respectively, then $Hom(X,Y)$ is of dimension mn over F. (6)

Q.5 Distinguish between integral domain and division ring. (6)

Q.6 Prove that quotient ring is a ring. (6)

Q.7 Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (6)

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Rings and Vector Spaces **TIME ALLOWED: 15 Mints.**

Course Code: MATH-308 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q. 1 MCQs (1 Mark each) **(1x10=10)**

(i) nZ is a maximal ideal of a ring Z if and only if n is ....

a) Prime number b) Composite number c) natural number d) None of these

(ii) If R & R' be arbitrary ring $\phi : R \to R'$ is ring homomorphism such that $\varphi(a) = a \ \forall \ a \in R$ then $Ker\phi =$--------------

(a) R' (b) $\{0\}$ (c) R (d) None of these

(iii) Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ is

a) Linear b) Not Linear c) Rational d) None of these

(iv) What are Zero divisors in the Ring of integers modulo 5

a) $\bar{2}$ b) $\bar{2}$ and $\bar{3}$ c) no zero divisor d) None of these

(v) The number of proper ideals of R is............................

(a) 0 (b) 1 (c) 2 (d) 3

(vi) Which of the following is vector space

(a) $Q(Q)$ (b) $Q(R)$ (c) $R(C)$ (d) $C(Z)$

(vii) The dimension of $C(R)$ is ....

(a) 0 (b) 1 (c) 2 (d) 3

(viii) The dimension of $\operatorname{Im} T$ is called ....

(a) Rank (b)Nullity (c) basis (d) none of these

(ix) The vectors $(1,2)$ and $(1,-2)$ are...................

(a) linearly independent (b) linearly dependent (c) parallel (d) perpendicular

(x) An indexed set of vectors $\{v_1, v_2, \ldots, v_p\}$ in R^n is said to be ....if the vector equation $x_1v_1 + x_2v_2 + \cdots + x_pv_p = 0$ has only the trivial solution.

(a) Linearly independent (b) Linearly dependent (c) Basis (d) None of these

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Complex Analysis-II **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-309 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Short Questions

Q2. Solve the following short questions **(4x5=20)**

1. Evaluate $\oint_C \frac{2z+6}{z^2+4}dz$ where C is the circle $|z-i|=2$.
2. Evaluate $\int_0^\infty \frac{x\sin x}{x^2+9}dx$
3. Prove that the sum of residues at the poles in a cell of an elliptic function is zero.
4. Define analytic continuation with example.

SECTION II (Long Questions) **(3x10=30)**

Q3. Prove that for an elliptic function the number of zeros in a cell is equal to the number of poles in a cell.

Q4. State and Prove Mittag-Leffler's Expansion theorem.

Q5. Show that the function $f(z)=\frac{1}{a}+\frac{z}{a^2}+\frac{z^2}{a^3}+\ldots$ can be continued analytically.

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Complex Analysis-II **TIME ALLOWED: 15 Mints.**

Course Code: MATH-309 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q1. Encircle the correct choice of the following questions **(1x10=10)**

1. The branch point $z = 0$ is a .... singularity of Ln z.
   a) isolated b) nonisolated c) removable
   d) None of these

2. The series representation of $\frac{\sin z}{z^4}$ is valid for ....
   a) $0<|z|<\infty$ b) $0<|z-1|<\infty$ c) $0<|z-2|<1$
   d) None of these

3. .... is a simple pole of $f(z) = \frac{\sin z}{z^2}$.
   a) z=0 b) $z=\pi$ c) z=1,2 d) none of these

4. If the principal part in the Laurent series is zero, then the singularity is called a ....
   b) Isolated singularity b) nonisolated singularity c) essential singularity
   d) removable singularity

5. Res($e^{3/z}$,0)=................
   a) 2 b) 3 c)1 d)-11

6. Sum of Residues of $f(z) = \frac{z^3}{(z-1)(z-2)}$ is ....
   a) 0 b) -1 c) 7 d=-7

7. $f(z) = \frac{z^2+1}{z(z-1)}$ is...........function.
   a) Entire b)meromorphic c) elliptic d) None of these

8. For an elliptic function P(z) which of the following is also elliptic ....
   a) $\int P(z)dz$ b) $P'(z)$ c) $\frac{P'(z)}{z}$ d) None of these

9. Sum of residues of an elliptic function is ....
   a) -1 b) 0 c) π d) None of these

10. The analytic function $f(z) = z\sin z^2$ has a zero at z=0 of order..................
    a) 1 b) 0 c)2 d) 3

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years

Roll No. ....

PAPER: Mechanics | **TIME ALLOWED: 2 Hrs. & 45 Min.**

Course Code: MATH-310 Part – II | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Questions with Short Answers. **(2x10=20)**

(a) (2 marks) What is principal axis and principal M.I.

(b) (2 marks) A 5kg sphere of radius 0.2m has time period 0.7 sec. Find angular momentum.

(c) (2 marks) What is apparent velocity and true velocity.

(d) (2 marks) What is the M.I of rod of mass M and length $2L$ about a line through mid point and perpendicular to rod?

(e) (2 marks) Write down the formula of M.I of uniform mass distribution about y, z-axis.

(f) (2 marks) Prove that linear velocity $v = \omega \times r$

(g) (2 marks) Define Equimomental Syatem.

(h) (2 marks) Expression for acceleration in moving system.

(i) (2 marks) If a rigid body rotates about a fixed axes with angular velocity ω, prove that the kinetic energy of rotation is $T = \frac{1}{2}I\omega^2$ where I is the moment of Inertia about the axis.

(j) (2 marks) Write down the equilibrium conditions for a rigid body.

Questions with Brief Answers: **(3x10=30)**

Question 1 .... *10 marks*

(a) (5 marks) Find the moment of inertia of solid cone about its diameter.

(b) (5 marks) Find the principal moments of Inertia of a square plate about a corner.

Question 2 .... *10 marks*

(a) (5 marks) A system consist of three particles, each of unit mass, with positions and velocities as follows:

$$\begin{aligned} r_1 &= i + j\ , \quad v_1 = 2i \\ r_2 &= j + k\ , \quad v_2 = j \\ r_3 &= k\ , \quad v_3 = i + j + k \end{aligned}$$

Find the position and velocity of the center of mass. Find also the linear momentum of the system. (*a*) Find the kinetic energy of the above system.

(b) (5 marks) Find the angular momentum about the origin.

Question 3 .... *10 marks*

(a) (5 marks) Show that two equal and opposite rotations of a rigid body about two distinct parallel axis are equivalent to a translation of body.

(b) (5 marks) State and prove parallel axis theorem.

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Mechanics **TIME ALLOWED: 15 Min.**

Course Code: MATH-310 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Question 1 .... *10 marks*

(a) (1 mark) For rotating coordinate system Centrifugal force is given by

A. $m(\vec{\omega}\times(\vec{\omega}\times\vec{r}))$ B. $m((\vec{\omega}\times\vec{\omega})\times\vec{r})$ C. $m(\vec{\omega}\times\vec{r})$ D. none of these

(b) (1 mark) For uniform mass distribution, product of inertia I_{31} = – – – – – –
A. $\int\int\int \rho xydV$ B. $-\int\int\int \rho yzdV$ C. $\int\int\int \rho xydV$ D. $-\int\int\int \rho xzdV$

(c) (1 mark) Angular Velocity is also called – – – – – –
A. Angular Displacement B. Rotation C. Spin D. Angle

(d) (1 mark) Moment of inertia of hoop of mass m and radius b is – – – – – –
A. $I=b^2$ B. $I=mb$ C. $I=mb^2$ D. $I=m^2b^2$

(e) (1 mark) Radius of Gyration: r_g= – – – – – –
A. I/m B. I/m^2 C. $(I/m)^{\frac{1}{2}}$ D. $(I/m)^{-1}$

(f) (1 mark) The number of Co-ordinates required to specify the position of the system of one or more particles, called
A. Degree of freedom B. Position of particles C. Rigid body D. Translation position

(g) (1 mark) When a body is in rest position or moving with constant velocity, then force required to change the state of motion is called
A. Centripetal force B. Inertia C. Equimomental force D. Angular Momentum

(h) (1 mark) Equation of Momental Ellipsoid is– – – – – –
A. $I_{11}+I_{22}+I_{33}=0$ B. $x^2+y^2+z^2=2$ C. $I_{11}x^2+I_{22}y^2+I_{33}z^2=1$
D. $I_{11}x^2+I_{22}y^2+I_{33}z^2=3$

(i) (1 mark) $\omega\times\vec{r}$ is called
A. Coriolis acceleration B. Apparent acceleration C. Transverse acceleration D. Angular acceleration

(j) (1 mark) The moment of inertia of a body is always minimum with respect to its
A. Base B. Centroidal axis C. Vertical axis D. Horizontal axis

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Functional Analysis-I **TIME ALLOWED: 2 Hrs. & 45 Mints.**
Course Code: MATH-311 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q.2 (i) Prove that every closed ball $\bar{B}(a;r)$ in a metric space (X,d) is closed. (4)

(ii) Discuss the $\mathbb{N}$ of natural numbers as a subset of real line $\mathbb{R}$ is of the first category. (4)

(iii) Let N be a Normed space and F be a field. Then show that (4)

(i) The function $f: N \times N \to N$ defined by $f(x,y) = x + y$ is uniformly continuous.

(ii) The function $g: F \times N \to N$ defined by $g(x,y) = cx, c$ is constant, is continuous.

(iv) Show that for any subset A of a Hilbert space H, $A \subseteq A^{\perp\perp}$. Also show that $A^{\perp}$ is a closed subspace of H. (4)

(v) Prove that the space $C[0, \pi/2]$ is a normed space but not inner product space. (4)

SECTION-III

Q.3 Prove that the space l^{∞} is a Banach space. (6)

Q.4 Let $T: N \to M$ be a surjective linear operator. Then prove that (6)

(i) T^{-1} exists if and only if $Tx = 0$ implies $x = 0$

(ii) If T T is bijective and $\dim N = n$, then show that M also has dimension n.

Q.5 For any $a = (a_1, a_2, \ldots a_n) \in R^n$ define $f_a : R^n \to R$ by $f_a(x) = \sum_{i=1}^{n} a_i x_i, x \in R^n$ then prove that (i) f_a is linear functional (ii) f_a is bounded (ii) $\|f_a\| = \|a\|$. (6)

Q.6 Let M be a subset of a finite dimensional normed space N. Then M is compact if and only if M is closed and bounded. (6)

Q.7 Let A be non-empty complete convex subset of an inner product space V, and $x \in V \setminus A$. Then there is a unique $y \in A$ such that $\|x - y\| = \inf_{y' \in A} \|x - y'\|$. (6)

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Functional Analysis-I **TIME ALLOWED: 15 Mints.**

Course Code: MATH-311 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q. 1 **MCQs (1 Mark each)**

(i) $d(ax, by) =$

(a) $abd(x, y)$ (b) $ad(x, y)$ (c) $bd(x, y)$ (d) None of these

(ii) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2, \forall x \in \mathbb{R}$, then f ---------

(a) not continuous

(b) continuous but not uniformly continuous

(c) both continuous and uniformly continuous

(d) None of these

(iii) The subset $A = \{2, 5, 8, 11, ...\}$ of $\mathbb{R}$ is --------- in $\mathbb{R}$ with usual metric on $\mathbb{R}$

(a) Neither open nor closed (b) clopen (c) Closed (d) None of these

(iv) In $(\mathbb{R}, d)$ with usual metric d on $\mathbb{R}$ the boundary of the set $A = \{\sqrt{2}, \sqrt{13}, \sqrt{37}\}$ is ---------

(a) A (b) $\mathbb{R}$ (c) $\mathbb{R} \setminus A$ (d) None of these

(v) In $(\mathbb{R}, d)$ with usual metric d on $\mathbb{R}$ the closure of $(1,3) \setminus \{2\}$ is ---------

(a) $(1,2) \cup (2,3)$ (b) $[1,2) \cup (2,3]$

(c) $[1,3]$ (d) None of these

(vi) Let $A = (0,1) \subseteq \mathbb{R}$ and d be usual metric on $\mathbb{R}$. Then the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ ---------

(a) convergent sequence in A

(b) convergent sequence but not Cauchy

(c) Cauchy sequence but not converges in A

(d) None of these

(vii) Let N be a normed space and $f : N \to F$ be a linear functional then $Ker f$ = ---------

(a) $\{x \in N : f(x) \neq 0\}$ (b) $\{x \in N : f(x) = -1\}$

(c) $\{x \in N : f(x) = 0\}$ (d) None of these

(viii) A subset A of a linear space N is convex if for any $x, y \in A$ and $\alpha + \beta = 1, \alpha \geq 0, \beta \geq 0$

(a) $\alpha x + \beta x \in A$ (b) $\alpha x + \beta y \in A$

(c) $\alpha x + \beta \in A$ (d) None of these

(ix) The norm of the linear functional f on $C[-1,1]$ defined by $f(x) = \int_{-1}^{0} x(t)dt - \int_{0}^{1} x(t)dt$

(a) 1 (b) 0 (c) 2 (d) None of these

(x) For any x, y in a complex inner product space V then $\langle x, y \rangle =$ ---------

(a) $\frac{1}{4}\left[\left(\|x+y\|^2 - \|x-y\|^2\right) + \left(\|x+iy\|^2 - \|x-iy\|^2\right)\right]$

(b) $\frac{1}{4}\left[\left(\|x+y\|^2 - \|x-y\|^2\right) + i\left(\|x+iy\|^2 - i\|x-iy\|^2\right)\right]$

(c) $\left[\left(\|x+y\|^2 - \|x-y\|^2\right) + i\left(\|x+iy\|^2 - i\|x-iy\|^2\right)\right]$

(d) None of these

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Ordinary Differential Equations **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: MATH-312 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q2. Solve the following short questions **(4x5=20)**

1. Solve the given differential equation $\frac{dy}{dx} = \sec^2 x - (\tan x)y + y^2$; $y_1 = \tan x$ be the known solution of the given differential equation.
2. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \ln x$.
3. Prove the identity $\frac{d}{dx} P_n(-1) = \frac{(-1)^{n+1}}{2} n(n+1)$
4. Define self-adjoint operator and prove that the SL operator is self-adjoint.

(Long Questions) **(3x10=30)**

Q3. Find the eigenvalues and eigenfunctions of the system

$$\frac{d^2y}{dx^2} + \lambda \frac{dy}{dx} = 0, \qquad y(0) + y'(0) = 0, \quad y'(1) = 0$$

Q4. . Show that $\int_{-1}^{1} (1-x^2)^n (1-2xt+t^2)^{-n-1/2} dx = \frac{2^{2n+1}(n!)^2}{(2n+1)!}$

Q5. Prove that $P_n(x) = (-1)^n F\left[-n,\ n+1;\ 1;\ \frac{1+x}{2}\right]$

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Ordinary Differential Equations **TIME ALLOWED: 15 Mints.**

Course Code: MATH-312 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q1. Encircle the correct choice of the following questions (10)

1. The general solution of $\frac{d^2y}{dx^2} - 4y = 0$ is $y = \ldots\ldots\ldots$

a) $Ae^{x} + Be^{-x}$ b) $Ae^{2x} + Be^{-2x}$ c) $Ae^{2x} + B$ d) $Ae^{-2x} + B$

2. The singular points of the differential equation $(x^2+1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 6y = 0$ are........

a) ± 1 b) $\pm i$ c) 0 d) no singular point

3. Eigen values of S-L system are always

b) Real b) complex c) rational d) integral

4. The differential operator that annihilates $4e^{2x} - 10xe^{2x}$ is ....

a) D^4 b) $(D-2)^2$ c) $D+2$ d) $D-2$

5. For a gamma function, the values of $\Gamma[\frac{1}{2}] = \ldots\ldots\ldots$

a) $\sqrt{\pi}$ b) $\frac{\pi}{2}$ c) 1 d) none of these

6. The solutions of S-L equation are called the

a) S-L functions b) particular solutions c) eigen functions d)None of these

7. $\frac{dy}{dx}[x^n J_n(x)] = \ldots\ldots\ldots\ldots$

a) $nx^{n-1}J_n(x)$ b) $-x^{-n}J_{n+1}(x)$ c) $x^n J_{n-1}(x)$ d) $J_{n-1}(x)$

8. $J_{-1/2}(x) = \ldots\ldots\ldots\ldots$

a) $\sqrt{\frac{2}{\pi x}}\sin x$ b) $\sqrt{\frac{2}{\pi x}}\cos x$ c) $\sqrt{\frac{2}{\pi x}}(\frac{\sin x}{x} - \cos x)$ d) $-\sqrt{\frac{2}{\pi x}}(\frac{\cos x}{x} + \sin x)$

9. $P_2(x) = \ldots\ldots\ldots\ldots$

a) $\frac{1}{2}(x^2-3)$ b) $\frac{1}{2}(3x^2-2)$ c) $\frac{1}{2}(3x^2-1)$ d) $\frac{1}{2}(3x^2-x)$

10. The function $y = Ae^{-2x} + Be^{3x}$ gives a differential equation of degree

a) 1 b) 2 c) 3 d) 4

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2018
Examination: B.S. 4 Years Programme

PAPER: Partial Differential Equations **TIME ALLOWED: 30 mins.**
Course Code: MATH-402 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

NOTE: Attempt all questions from each section.

SECTION I

1. $c^2\nabla^2 u = \frac{\partial^2 u}{\partial t^2}$ is called (1 mark)

   (a) heat equation for u.

   (b) Laplace equation for u.

   (c) wave equation for u.

   (d) none of the above

2. If ζ_1 and ζ_2 are two linearly independent solutions of the second order partial differential equation, then which one of the following is also a solution? (1 mark)

   (a) $\zeta_1 + \zeta_2$

   (b) $\zeta_1 - \zeta_2$

   (c) $\zeta_1\zeta_2$

   (d) none of the above

3. To convert $u_{xx} - 5u_{xy} + 6u_{yy} = 0$ into canonical form, we use (1 mark)

   (a) $\xi = 2x + y, \eta = 3x + y$

   (b) $\xi = x + y, \eta = x$

   (c) $\xi = x - y, \eta = y$

   (d) none of the above

4. The heat equation is a ——— partial differential equation (1 mark)

   (a) hyperbolic

   (b) parabolic

   (c) elliptic

   (d) none of the above

5. $\frac{\partial^2 z}{\partial x \partial y}$ = ——— (in usual notation) (1 mark)

   (a) p

   (b) q

   (c) r

   (d) s

P.T.O.

6. An equation relating partial derivatives is called a (1 mark)

(a) ordinary differential equation

(b) partial differential equation

(c) none of the above

(d) all of the above

7. The equation $\left(\frac{\partial \theta}{\partial x}\right)^2 + \frac{\partial \theta}{\partial t} = 0$ is a (1 mark)

(a) first order equation in four variables

(b) second order equation in four variables

(c) first order equation in two variables

(d) second order equation in four variables

8. $2z_{xx} - 4z_{xy} - 6z_{yy} + z_x = 0$ is (1 mark)

(a) parabolic

(b) hyperbolic

(c) elliptic

(d) none of the above

9. For $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$, $x \in (0, a)$, $t > 0$, $u(x, 0) = k$, where k is a constant, is called (1 mark)

(a) boundary condition

(b) initial condition

(c) stationary condition

(d) none of the above

10. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ is called (1 mark)

(a) heat equation

(b) Laplace equation

(c) wave equation

(d) none of the above

UNIVERSITY OF THE PUNJAB

Seventh Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Numerical Analysis-I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-403 **MAX. MARKS: 50**

(Subjective)

Attempt this Paper on Separate Answer Sheet provided.

Q.2. Answer the following short questions: $4 \times 5 = 20$

(i). What is the ill-conditioned linear system? Explain with example.

(ii). Apply Crout's Decomposition Method to solve the system of equations:

$x + 4y - z = 3, \quad x + 2y + z = 4, \quad 2x + y + 3z = 5$

(iii). Prove Lagrange's Interpolation formula for unequal intervals.

(iv). Find Eigen values and Eigen Vectors of: $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

(v). Use the Lagrange's interpolating formula to approximate $f(3)$ for the following data:

x	1	2	5
$f(x)$	3	12	147

LONG QUESTIONS $6 \times 5 = 30$

Q.3. Find an iterative formula to find, $(N)^{1/3}$ where N is a positive number and hence, find $(7)^{1/3}$ correct to four decimal places.

Q.4. Solve the following system of equations by Gauss's Seidel Method:

$$10x_1 + x_2 + x_3 = 15$$

$$x_1 + 10x_2 + x_3 = 24$$

$$x_1 + x_2 + 10x_3 = 33$$

Q.5. Prove that $\Delta^r y_k = \nabla^r y_{k+r}$

Q.6. Find the solution of $e^x - x^2 - 5 = 0$ by Newton Raphson's method.

Q.7. Find the 2nd degree polynomial which passes through the points (0, 1), (1, 3), (2, 7) and (3, 13).

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Ring Theory **TIME ALLOWED: 30 mins.**

Course Code: MATH-407 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1 MCQs (1 Mark each)

(i) The set $Z_4 = \{0,1,2,3\}$ under addition and multiplication modulo 4 forms

(a) Division Ring (b) Integral domain (c) Commutative Ring (d) Field

(ii) The maximal ideal ring in the ring Z of integers is

(a) Z (b) $7Z$ (c) $4Z$ (d) $6Z$

(iii) Which of the following is not a prime ideal of the ring Z of integers?

(a) $2Z$ (b) $3Z$ (c) $7Z$ (d) $4Z$

(iv) Units of $Z(i)$ are

(a) ± 1 (b) $\pm i$ (c) $\pm 1, \pm i$ (d) None of these

(v) Every .... is irreducible in an integral domain.

(a) Integer (b) Prime (c) Real number (d) none of these

(vi) A ring which is commutative with identity element and having no zero divisor is called

(a) Division Ring (b) Integral domain

(c) Prime Ring (d) nilpotent ring

(vii) If R & R' be arbitrary ring $\phi : R \to R'$ is ring homomorphism such that

$\varphi(a) = a \ \forall \ a \in R$ then $Ker\phi =$ --------------

(a) R' (b) $\{0\}$ (c) R (d) None of these

(viii) 2π is algebraic over

(a) Q (b) R (c) Z (d) None of these

(ix) If C is finite extension of R, then $[C : R] =$ ------------

(a) 2 (b) 3 (c) 4 (d) 5

(x) A ring with non zero characteristic is

(a) Z (b) Q (c) Z_3 (d) $Z \times Z$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Number Theory-I | **TIME ALLOWED: 30 mins.**
Course Code: MATH-408 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1	MCQs (1x10 = 10 Marks) Time: 20 min
(i)	641 divides a) F_5 b) F_3 c) F_2 d) F_0
(ii)	The least common multiple of 60 and N is 1260. Which of the following could be the prime factorization of N? a) $2.3^2.5.7$ b) $2^3.5.7$ c) $3^2.5.7$ d) $2.5.7^2$ e) Not Given
(iii)	What is the remainder when 3^{15} is divided by 11? a) 0 b) 1 c) 2 d) Not Given
(iv)	The sum of positive divisors of 100 is a) 217 b) 317 c) 417 d) Not Given
(v)	The number of primitive root mod 31 are (a) 5 (b) 6 (c) 7 (d) 8 e) Not Given
(vi)	If 15x+7y = 210, then a) x=2, y=5 b) x=7, y=15 c) x=2, y=15 d) x=7, y=16
(vii)	If 2 has exponent 3 mod 7, then 2^6 has exponent (a) 1 (b) 3 (c) 5 (d) 7 e) Not Given
(viii)	If $\sigma(n) = 2n$, then n is called (a) Composite (b) Perfect (c) Prime (d) Mersenn
(ix)	If p is a prime number and d is a factor of p-1 then the number of solutions of the congruence $x^{d-1} \equiv 0 \pmod{p}$ is a) p-1 b) p c) d-1 d) d
(x)	$\tau(75) =$ (a) 3 (b) 4 (c) 5 (d) 6 e) Not Given

UNIVERSITY OF THE PUNJAB

Seventh Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Number Theory-I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-408 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2	**Short Questions (5x4 = 20 Marks) Time: 40 min**	
(i)	Find $E_{100}(13)$. That is, exponent of 13 in 100!	(4)
(ii)	Prove that there are infinitely many primes.	(4)
(iii)	Prove that $\phi(p^k) = p^k - p^{k-1}$, where ϕ is Euler's phi ffunction	(4)
(iv)	By means of theory of exponent, Prove that ${}^n p_r$ and ${}^n c_r$ both are integers.	(4)
(v)	Using indices, Solve the following congruence $7x^3 \equiv 3 \pmod{11}$	(4)

Section-III

	Long Questions (6x5 = 30 Marks) Time: 90 min	
Q.3	State and prove Lagrange's Theorem for the solution of polynomial congruences modulo a prime number.	(5)
Q.4	Define primitive root of an integer. Prove that if a is a primitive root modulo m then a^k is a primitive root modulo m if and only if $\gcd(k, \varphi(m)) = 1$.	(5)
Q.5	State and prove Wilson Theorem. Apply it to find remainder of 35! when it is divided by 37.	(3+2)
Q.6	Prove that $\sum_{d\mid n} \frac{\varphi(d)}{d} = \prod_{i=1}^{r} [1 + k_i \frac{p_i - 1}{p_i}]$, where φ is the Euler's phi function.	(5)
Q.7	Let a be primitive root modulo n and b, c be integers, then show that $$Ind\ bc \equiv Ind\, b + Ind\, c \pmod{\varphi(m)}.$$	(5)
Q.8	Prove that $n = \sum_{d\mid n} \phi(d),\ n \geq 1.$	(5)

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Theory of Approximation & Splines -I **TIME ALLOWED: 30 mins.**

Course Code: MATH-413 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE

Q1. Encircle the correct answer (1x10=10)

1. $\Delta^n y_k =$ ....
   a) $\Delta^{n-1} y_{k+1} - \Delta^{n-1} y_k$ b) $\Delta^{n-1} y_{k-1} - \Delta^{n-1} y_k$ c) $\Delta^{n-1} y_{k+1} - \Delta^{n-1} y_{k-1}$ d) $\Delta^{n-1} y_{k+1}$
2. .... is the method of finding value inside the given data points
   a) Interpolation b) Approximation c) Extrapolation
   d) curve fitting
3. Every isometry is -------------------------
   a) One-one b) Onto c) Into d) none of these
4. The value of y when $x = 10$ obtain from the given data points (5,12), (6,13), (9,14), (11,16) is ....
   a) 14.66667 b) 10.1235 c) 0 d) none of these
5. The operator used in the Gauss's Forward interpolation formula is
   a) E b) δ c) ∇ d) Δ
6. The matrix of rotation is
   a) $\begin{bmatrix} \cos 2t & \sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$ b) $\begin{bmatrix} \cos 2t & \sin 2t \\ \sin 2t & -\cos 2t \end{bmatrix}$ c) $\begin{bmatrix} -\cos 2t & \sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$
   d) $\begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}$

7 ∇y_3=-------------------
   a) $y_2 - y_3$ b) $y_2 + y_3$ c) $y_3 - y_2$ d)None of these

8 $\mu =$ ----------------
   a) E+1 b) $\Delta + \nabla$ c) $\frac{E^{1/2} + E^{-1/2}}{2}$ d) $\frac{E^{1/2} - E^{-1/2}}{2}$
   d) None of these

9. The composition of reflection and rotation is
   a)Rotation b) Reflection c) Translation d) None of these
10. If y_x is a polynomial of *nth* degree, then its *nth* differences are..........
    a) one b) zero c) constant d) n

UNIVERSITY OF THE PUNJAB

Seventh Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Theory of Approximation & Splines -I **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: MATH-413 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Q2. Solve the following short questions (4x5=20)

1. Let A and B be two points on a circle, and let the tangents to the circle at A and B meet at P. Prove that AP=BP.

2. Find the missing term in the following table

x	1	2	3	4	5	6	7
$f(x)$	2	4	8	--------	32	64	128

3. Drive the normal equation for finding the least-squares power fit $y = Ax^2$.

4. Find the least-squares parabola for the four points (-3, 3), (0, 1), (2, 1) and (4, 3).

Q3. Solve the following **Long Questions.**

1. Prove that Euclidean transformation is an equivalence relation. (08)

2. Determine the image of circle $x^2 + y^2 = 16$, under the transformation of stretching along along y-axis by factor 3. (07)

3. For the given set of data, find the least-squares curve for $f(x) = Cx^A$ (08)

x_k	y_k
1	0.6
2	1.9
3	4.3
4	7.6
5	12.6

4. Define interpolation. Derive Gauss's forward interpolation formula. (07)

UNIVERSITY OF THE PUNJAB

Seventh Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Fluid Mechanics-I | **TIME ALLOWED: 2 hr. 30 min**
Course Code: MATH-415 | **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Section – I (Short Questions)

Q.2 Solve the following Short Questions. **(5 × 4= 20)**

(i) Define Newton's law of viscosity and explain it briefly.

(ii) What is Equation of Continuity in Cartesian coordinates for incompressible fluid? Convert the same in cylindrical coordinates.

(iii) Vorticity vector is defined as $\vec{\zeta} = \nabla \times \vec{V}$. Derive vorticity vector in cylindrical coordinates for a two dimensional motion (Don't deduce it from three dimensional motion).

(iv) Show that the equipotential lines and the streamlines are orthogonal to each other.

(v) The specific weight of water at ordinary pressure and temperature is 9810 N/m3. The specific gravity of mercury is 13.55. Compute the density of water, specific weight and density of mercury.

Section – II (Long Questions) (3 × 10 = 30)

Q.3 The velocity potential for a certain two dimensional incompressible irrotational fluid flow is $\varphi = x + \frac{1}{2}\ln(x^2 + y^2)$, by converting it in polar coordinates, evaluate the following:

(i) Stream function (ii) Equipotential lines and Streamlines

(iii) Speed and Stagnation points (iv) Complex velocity potential

Q.4 Explain the flow of a two dimensional vortex in a uniform stream.

Q.5 Derive the Bernoulli's equation for steady inviscid flow under conservative forces.

UNIVERSITY OF THE PUNJAB

Roll No. ....

Seventh Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Fluid Mechanics-I | **TIME ALLOWED: 30 mins.**
Course Code: MATH-415 | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION – I (Objective)

Q.1 Tick the correct option. (1×10 = 10)

(i)	In material derivative $\frac{DH}{Dt} = \frac{\partial H}{\partial t} + \vec{V} \cdot \nabla H$, the term $\frac{\partial H}{\partial t}$ is known as ____________ rate of change. (a) Local (b) Convective (c) Stokes (d) Substantial
(ii)	The ____________ of a two dimensional source is defined to be the volume of fluid which emits from it in unit time. (a) Mass (b) Specific volume (c) Strength (d) Velocity
(iii)	A ____________ represents the type of flow in which the fluid particles move in circular paths about a central point. (a) Doublet (b) Source (c) Sink (d) Vortex
(iv)	$\frac{dx}{dt} = u, \frac{dy}{dt} = v, \frac{dz}{dt} = w$ represents the differential equation for the ____________ lines. (a) Stream (b) Streak (c) Path (d) All of these
(v)	The velocity potential function and the stream function are ____________ functions. (a) Continuous (b) Orthogonal (c) Conjugate (d) All of these
(vi)	For describing the motion in fluid mechanics, the ____________ method is commonly used. (a) Eulerian (b) Lagrangian (c) Newtonian (d) Archimedes
(vii)	Euler's equation of motion refers to conservation of ____________. (a) Momentum (b) Mass (c) Newton's law (d) Force
(viii)	The Vorticity vector is ____________. (a) Rotational (b) Irrotational (c) Orthogonal (d) Solenoidal
(ix)	The reciprocal of density is known as specific ____________. (a) Gravity (b) Weight (c) Mass (d) Volume
(x)	Pascal-second is the unit of ____________ viscosity. (a) Kinematic (b) Dynamic (c) Both (a) & (b) (d) None of these