Fundamentals of Heat Transfer

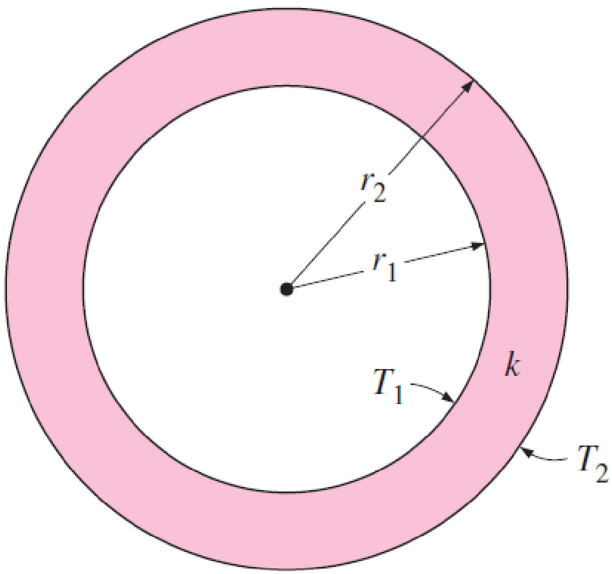
Muhammad Rashid Usman

Institute of Chemical Engineering and Technology

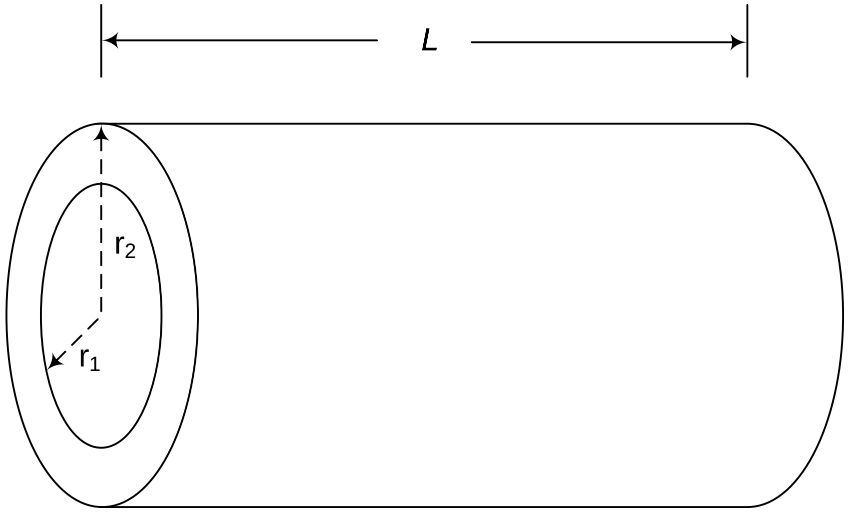
University of the Punjab, Lahore.

Figure taken from: http://heatexchanger-design.com/2011/10/06/heat-exchangers-6/ Dated: 17-Jan-2012

Heat conduction through a hollow cylinder



$$Heat\ transfer\ area = 2 \cdot \pi \cdot r \cdot L$$



Heat conduction through a hollow cylinder

$$\text{Rate of heat transfer} = q = \frac{2 \cdot \pi \cdot k \cdot L}{\ln(r_2 / r_1)} \cdot (T_1 - T_2)$$

$$\text{Rate of heat transfer per unit length} = \frac{q}{L} = \frac{2 \cdot \pi \cdot k}{\ln(r_2 / r_1)} \cdot (T_1 - T_2)$$

$$q = \frac{T_1 - T_2}{\left(\dfrac{\ln(r_2 / r_1)}{2 \cdot \pi \cdot k \cdot L}\right)} = \frac{\textit{Temperature difference}}{\textit{thermal resistance}}$$

Heat conduction through a hollow cylinder: Temperature profile

What about temperature profile in radial direction?

Any idea about temperature profile in axial direction!

The answer is straight forward.

Think!

Heat conduction through a hollow cylinder: Temperature profile

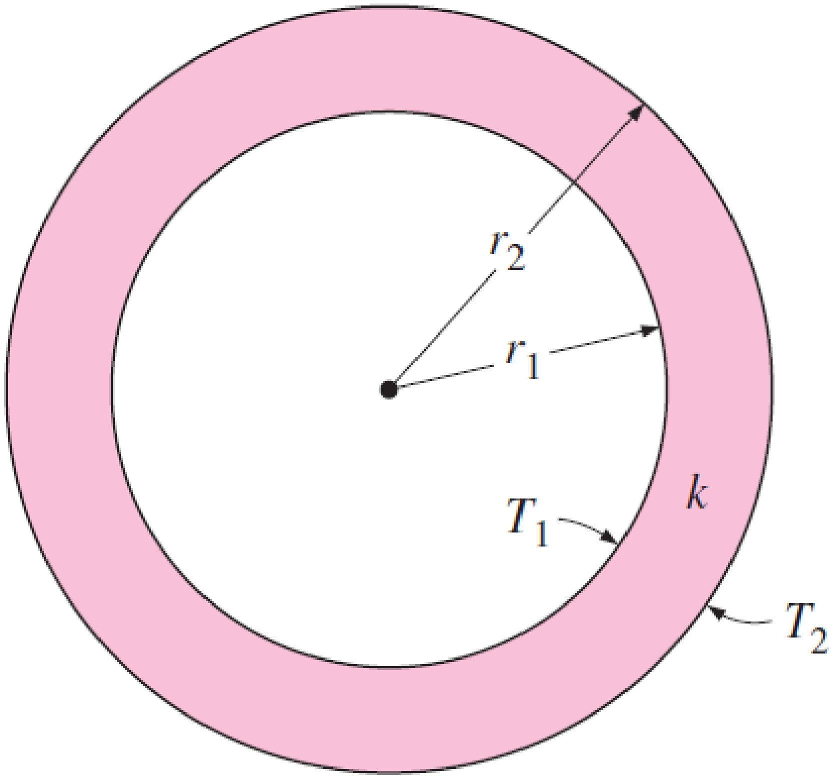
$$T = T_1 + \frac{T_1 - T_2}{\ln(r_1 / r_2)} \cdot \ln \frac{r}{r_1}$$

Heat conduction through a hollow cylinder: Problem-3

A glass pipe has an outside diameter of 6 in, and an inside diameter of 5 in, it will be used to transport a fluid which maintains the inner surface at 200 °F, the outside temperature of the pipe is 175 °F. What will be the rate of heat flow? k = 0.63 Btu/h·ft·°F. [p. 16, 5]

Answer: 542.78 Btu/h·ft

Heat conduction through a hollow sphere



$$Heat\ transfer\ area = 4 \cdot \pi \cdot r^2$$

Heat conduction through a hollow sphere

$$Rate\ of\ heat\ transfer = q = \frac{4 \cdot \pi \cdot k}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \cdot (T_1 - T_2)$$

$$Rate\ of\ heat\ transfer\ per\ unit\ area = \frac{q}{A} = \frac{r_1}{r_2} \cdot k \cdot \frac{T_1 - T_2}{r_1 - r_2}$$

$$q = \frac{T_1 - T_2}{\left(\frac{r_2 - r_1}{4 \cdot \pi \cdot r_1 \cdot r_2 \cdot k}\right)} = \frac{Temperature\ difference}{thermal\ resistance}$$

Heat conduction through a hollow sphere: Temperature profile

What about temperature profile in radial direction?

Heat conduction through a hollow sphere-4: Temperature profile

$$T = T_1 + \frac{T_1 - T_2}{\frac{1}{r_2} - \frac{1}{r_1}} \cdot \left(\frac{1}{r_1} - \frac{1}{r} \right)$$

Heat conduction through a hollow sphere: Problem-4

Calculate the heat loss per m^2 of outside surface area for a heated sphere 15 cm diameter covered with 50 cm insulation with thermal conductivity 0.057 kcal/h·m·°C. The inside and outside temperature of insulation are 315 °C and 80 °C respectively.

Answer: 160.74 kcal/h·m^2

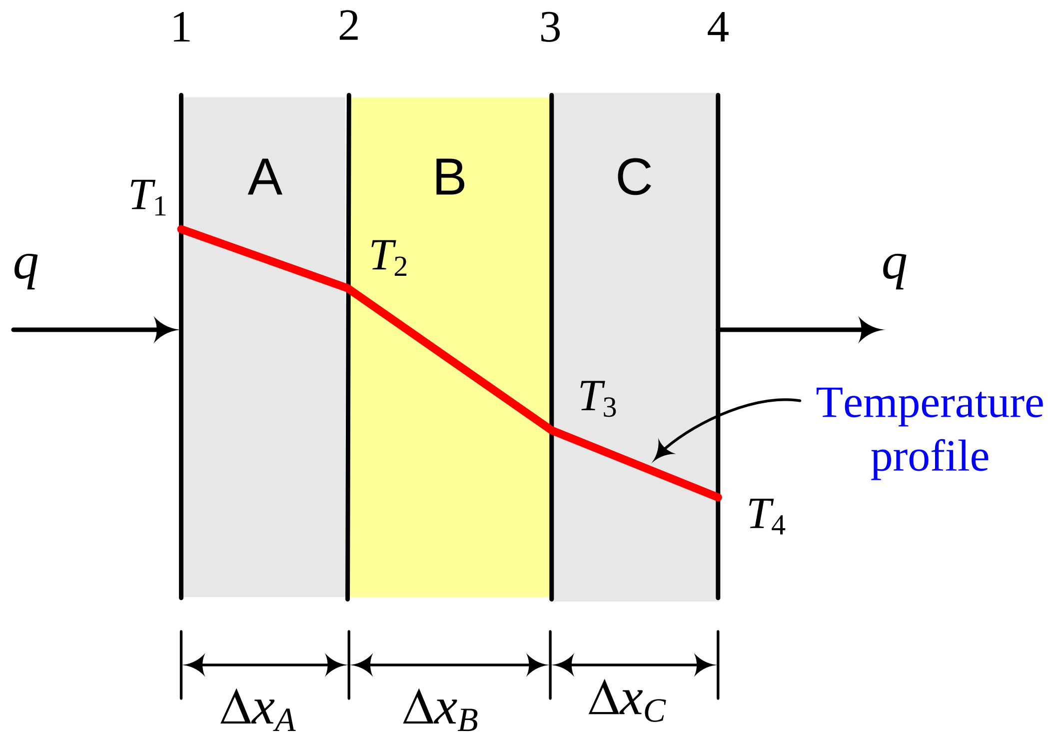
Thermal resistances in series

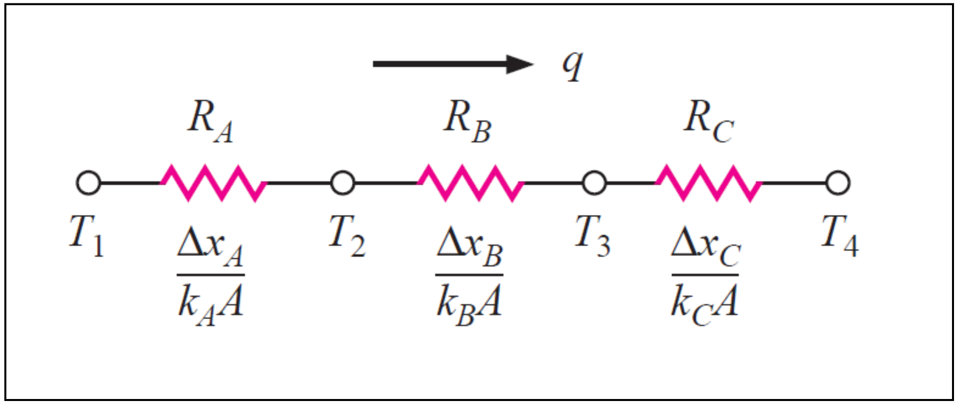
- Heat conduction through a multilayer (composite) plane (flat) wall

- Heat conduction through a multilayer hollow cylinder

- Heat conduction through a multilayer hollow sphere

Heat conduction through a multilayer (composite) plane wall





Heat conduction through a multilayer (composite) plane wall

For layer A:

$$\frac{q}{A} = \frac{T_1 - T_2}{\Delta x_A / k_A}$$

For layer B:

$$\frac{q}{A} = \frac{T_2 - T_3}{\Delta x_B / k_B}$$

For layer C:

$$\frac{q}{A} = \frac{T_3 - T_4}{\Delta x_C / k_C}$$

For steady-state, it may be shown that

$$\frac{q}{A} = \frac{T_1 - T_2}{\Delta x_A / k_A} = \frac{T_2 - T_3}{\Delta x_B / k_B} = \frac{T_3 - T_4}{\Delta x_C / k_C}$$

Heat conduction through a multilayer (composite) plane wall

For the overall change in temperature

$$\frac{q}{A}=\frac{T_1-T_2}{\Delta x_A/k_A}+\frac{T_2-T_3}{\Delta x_B/k_B}+\frac{T_3-T_4}{\Delta x_C/k_C}$$

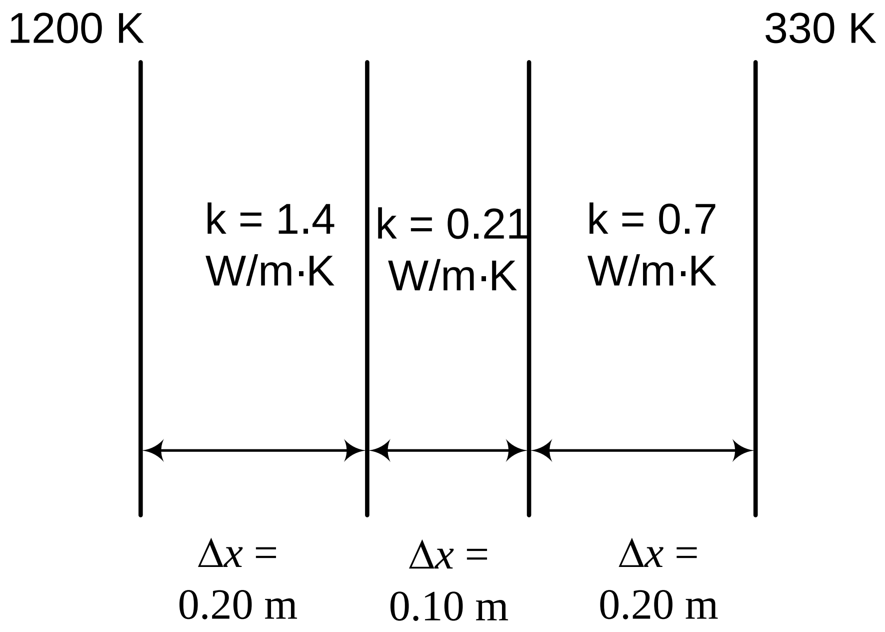
Or

$$Heat\ rate=\frac{Overall\ temperature\ difference}{Sum\ of\ \ all\ the\ resistance\ in\ series}$$

$$q=\frac{(T_1-T_4)}{\dfrac{\Delta x_A}{k_A\cdot A}+\dfrac{\Delta x_B}{k_B\cdot A}+\dfrac{\Delta x_C}{k_C\cdot A}}$$

Heat conduction through a multilayer (composite) plane wall-4: Problem-4 [p. 391, 7]

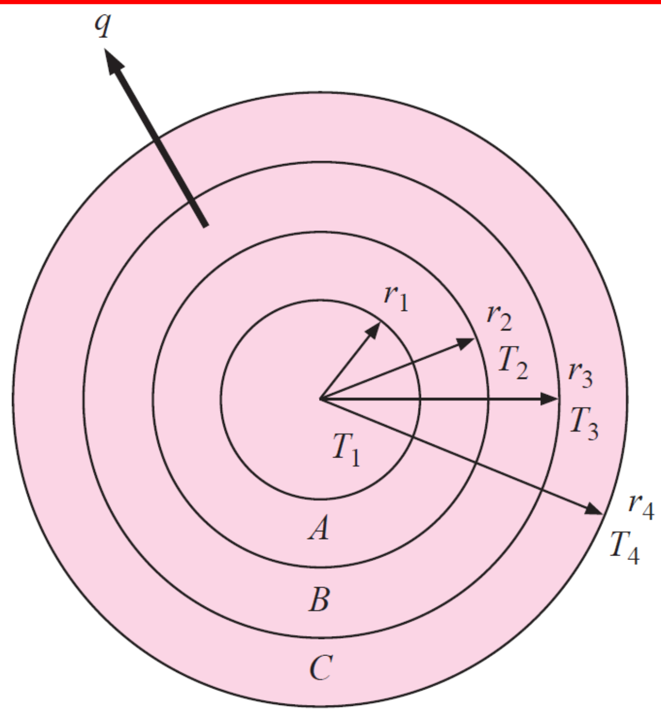
A furnace is constructed with 0.20 m of firebrick, 0.1 m of insulating brick, and 0.20 m of building brick. The inside temperature is 1200 K and the outside temperature is 330 K. Thermal conductivities are of firebrick, insulating brick, and building brick are shown in the Figure. Estimate the heat loss per unit area and the temperature at the junction of the firebrick and the insulating brick.

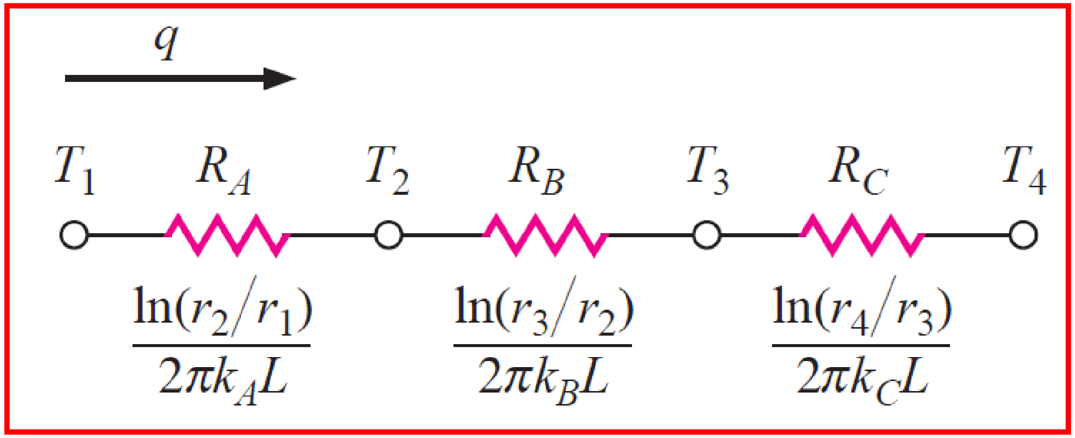


Heat conduction through a multilayer (composite) plane wall-4: Problem-5 [p. 14, 8]

Compute the rate of heat transfer through the walls of a room having inside and outside temperatures 255.4 K and 297.1 K respectively. The walls of the room are made up of 3 in concrete, 4 in cork board, and ½ in wood. The thermal conductivities of the wall materials are k_{concrete} = 0.762 W/m·K, $k_{\text{corkboard}}$ = 0.0433 W/m·K, and k_{wood} = 0.151 W/m·K. Also, find the temperature at the interface between the wood and cork board.

Heat conduction through a multilayer (composite) hollow cylinder-1





Heat conduction through a multilayer (composite) hollow cylinder-2

For material A: $$\frac{q}{L}=\frac{2\cdot\pi\cdot k_A}{\ln\left(r_2/r_1\right)}\cdot\left(T_1-T_2\right)$$

For material B: $$\frac{q}{L}=\frac{2\cdot\pi\cdot k_B}{\ln\left(r_3/r_2\right)}\cdot\left(T_2-T_3\right)$$

For material C: $$\frac{q}{L}=\frac{2\cdot\pi\cdot k_C}{\ln\left(r_4/r_3\right)}\cdot\left(T_3-T_4\right)$$

For steady-state, it may be shown that

$$\frac{q}{L}=\frac{2\cdot\pi\cdot k_A}{\ln\left(r_2/r_1\right)}\cdot\left(T_1-T_2\right)=\frac{2\cdot\pi\cdot k_B}{\ln\left(r_3/r_2\right)}\cdot\left(T_2-T_3\right)=\frac{2\cdot\pi\cdot k_C}{\ln\left(r_4/r_3\right)}\cdot\left(T_3-T_4\right)$$

Heat conduction through a multilayer (composite) hollow cylinder-3

For the overall change in temperature

$$\frac{q}{L} = \frac{(T_1 - T_4)}{\frac{\ln(r_2/r_1)}{2\cdot\pi\cdot k_A} + \frac{\ln(r_3/r_2)}{2\cdot\pi\cdot k_B} + \frac{\ln(r_4/r_3)}{2\cdot\pi\cdot k_C}}$$

Or

$$Heat\ rate = \frac{Overall\ temperature\ difference}{Sum\ of\ all\ the\ resistance\ in\ series}$$

$$q = \frac{(T_1 - T_4)}{\frac{\ln(r_2/r_1)}{2\cdot\pi\cdot k_A\cdot L} + \frac{\ln(r_3/r_2)}{2\cdot\pi\cdot k_B\cdot L} + \frac{\ln(r_4/r_3)}{2\cdot\pi\cdot k_C\cdot L}}$$

Heat conduction through a multilayer (composite) hollow cylinder-4: Problem-6 [p. 298, 6]

A tube 60 mm outer diameter contains two layers of insulation A & B. Thickness of layer A is 50 mm with thermal conductivity 0.055 W/m·°C followed by a 40 mm layer B with thermal conductivity 0.05 W/m·°C. If the temperature of the outer surface of pipe is 150 °C and the temperature of the outer surface of layer B is 30°C. Calculate the heat loss in watts per meter length.

Heat conduction through a multilayer (composite) hollow sphere

Derive an expression for heat conduction through a composite hollow sphere.

References

1. Geankoplis, C.J. (2003). Transport processes and separation process principles: includes unit operations. 4th ed. Prentice-Hall International, Inc.
2. Holman, J.P. (2010). Heat transfer. 10th ed. McGraw-Hill Higher Education, Singapore.
3. Cengel, Y.A. (2003). Heat transfer: A practical approach. 2nd ed. McGraw-Hill.
4. Incropera, F.P.; DeWitt, D.P.; Bergman, T.L.; Lavine. A.S. (2007) Fundamentals of heat and mass transfer. 6th ed. John Wiley & Sons, Inc.
5. Kern, D.Q. (1965). Process heat transfer. McGraw-Hill International Book Co., Singapore.
6. McCabe, W.L.; Smith, J.C.; Harriott, P. (1993). Unit operations of chemical engineering. 5th ed. McGraw-Hill, Inc., Singapore.
7. Coulson, J.M.; Richardson, J.F.; Backhurst, J.R.; Harker, J.H. 1999. Coulson and Richardson's Chemical engineering: Fluid flow, heat trasnfer and mass transfer. vol. 1. 6th ed. Butterwoth-Heinemann, Oxford.
8. Staff of Research and Education Association. (1984). The heat transfer problem solver. Research and Education Association, New Jersey.