Fundamentals of Heat Transfer

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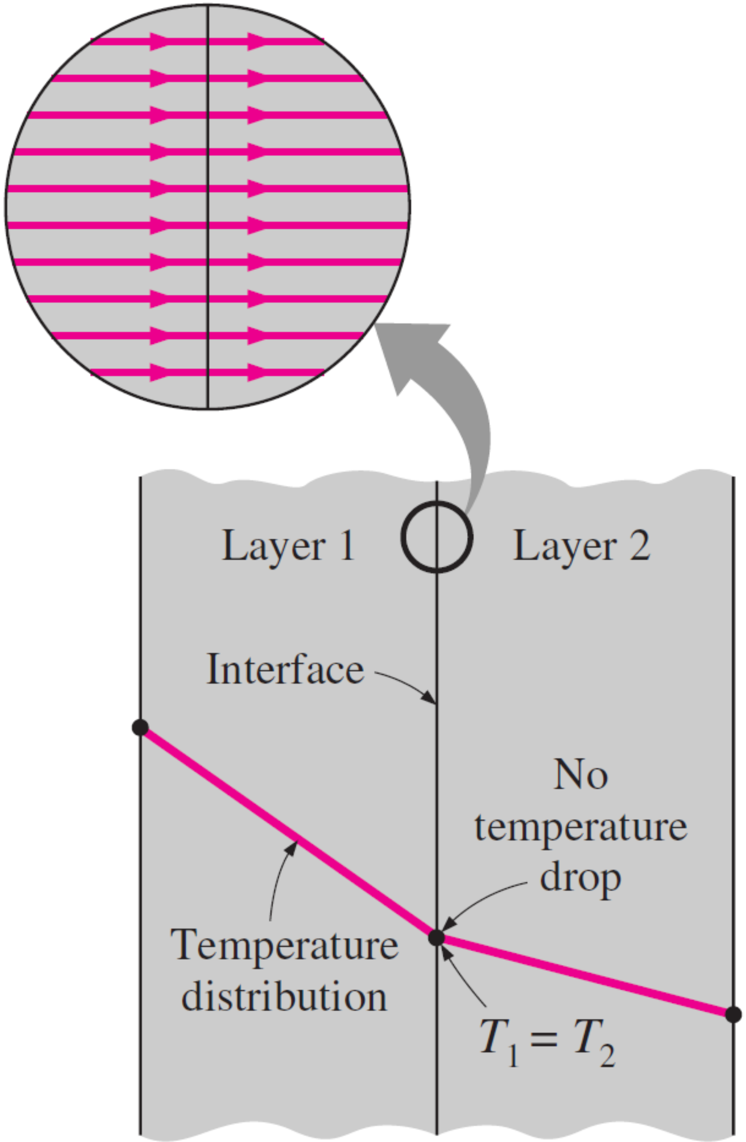
Figure taken from: http://heatexchanger-design.com/2011/10/06/heat-exchangers-6/ Dated: 17-Jan-2012

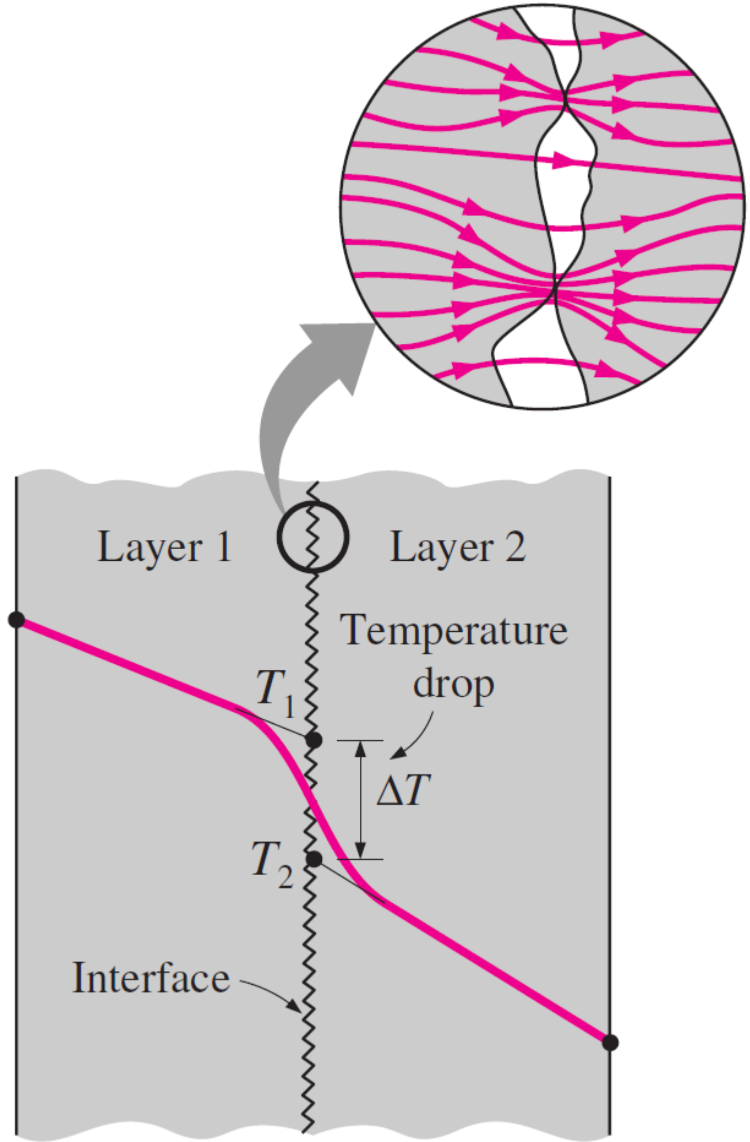
Course contents mid term

Introduction to transfer processes. Definition, applications, and various units of heat transfer. Modes of heat transfer: Conduction, convection, and radiation heat transfer. Fourier's law of heat conduction. Thermal conductivity of gasses, liquids, and solids. Units of thermal conductivity. Effect of temperature, pressure, and composition on thermal conductivity of materials. Estimation of thermal conductivity of gases, liquids, and solids. Introduction to steady-state heat transfer. Heat conduction through plane wall, hollow cylinder, and hollow sphere. Numerical problems related to heat conduction through plane wall, hollow cylinder, and hollow sphere. Thermal resistances in series: Composite plane wall, composite hollow cylinder, and composite hollow sphere. Numerical problems related to heat conduction through composite plane wall, composite hollow cylinder, and composite hollow sphere. Free and forced convection. Rate equation for convective heat transfer coefficient. Brief description of hydrodynamic boundary layer and heat transfer coefficient. Units of heat transfer coefficient. Individual and overall heat transfer coefficients: plane wall and hollow cylinder. Numerical problems regarding overall heat transfer coefficient. Determination of heat transfer coefficient. Description of various heat transfer correlations. Heat transfer in coiled and jacketed agitated vessels.

Thermal contact resistance [3]



(*a*) Ideal (perfect) thermal contact



(*b*) Actual (imperfect) thermal contact

Thermal contact resistance [3]

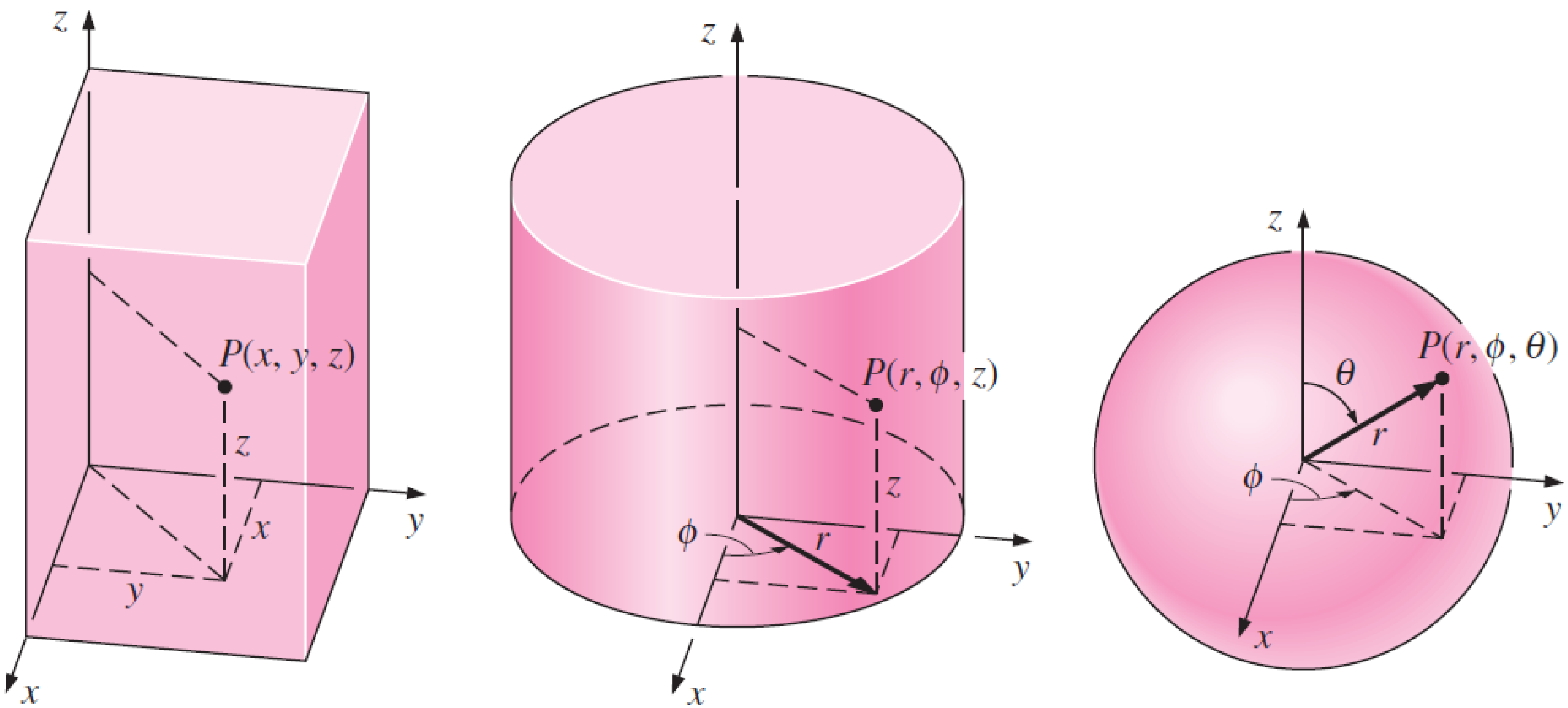
Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm (from Fried, Ref. 5)

Fluid at the Interface	Contact Conductance, h_c, $W/m^2 \cdot °C$
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

General heat conduction equation

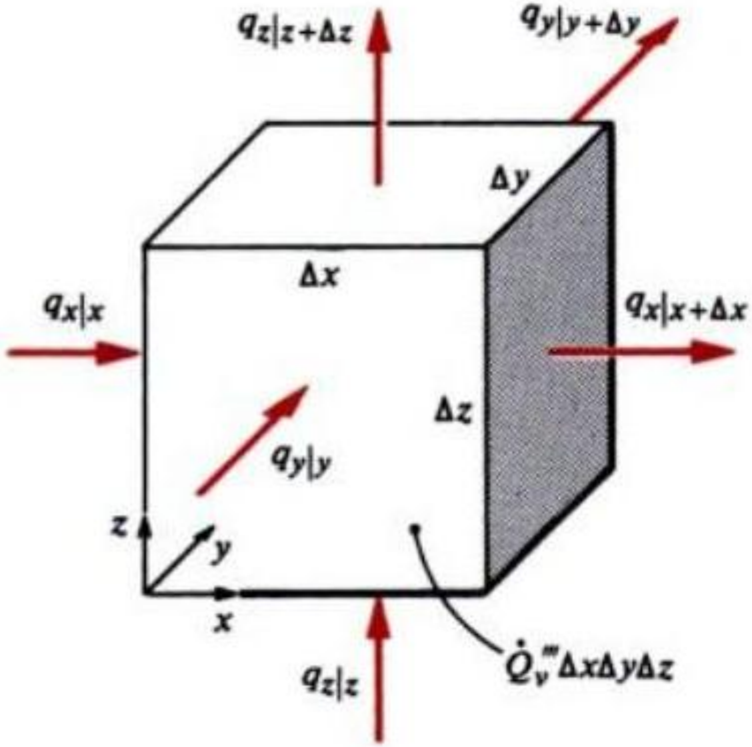
It takes into account, unsteady-state condition, all the three dimensions, and any heat generation within the material.

Coordinate systems [3]



(a) Rectangular coordinates (b) Cylindrical coordinates (c) Spherical coordinates

General conductivity equation in rectangular coordinates [10]



General conductivity equation in rectangular coordinates (Errors and omissions are expected)

$$(Rate\ of\ heat\ in) - (Rate\ of\ heat\ out) + (Rate\ of\ heat\ generation)$$
$$-(Rate\ of\ heat\ consumption) = (Rate\ of\ heat\ accumulation)$$

$$\left(q_x\big|_x + q_y\big|_y + q_z\big|_z\right) - \left(q_x\big|_{x+\Delta x} + q_y\big|_{y+\Delta y} + q_z\big|_{z+\Delta z}\right) + q' \times (\Delta x \Delta y \Delta z) - 0 = \rho c_p (\Delta x \Delta y \Delta z) \frac{\partial T}{\partial t}$$

Applying Fourier's law of heat conduction, it may shown that

$$\left(-k_x \Delta y \Delta z \frac{\partial T}{\partial x}\bigg|_x - k_y \Delta x \Delta z \frac{\partial T}{\partial y}\bigg|_y - k_z \Delta x \Delta y \frac{\partial T}{\partial z}\bigg|_z\right)$$
$$-\left(-k_x \Delta y \Delta z \frac{\partial T}{\partial x}\bigg|_{x+\Delta x} - k_y \Delta x \Delta z \frac{\partial T}{\partial y}\bigg|_{y+\Delta y} - k_z \Delta x \Delta y \frac{\partial T}{\partial z}\bigg|_{z+\Delta z}\right) + q' \times (\Delta x \Delta y \Delta z) = \rho c_p (\Delta x \Delta y \Delta z) \frac{\partial T}{\partial t}$$

General conductivity equation in rectangular coordinates (Errors and omissions are expected)

$$k_x \frac{\left(\left. \frac{\partial T}{\partial x} \right|_{x+\Delta x} - \left. \frac{\partial T}{\partial x} \right|_{x} \right)}{\Delta x} + k_y \frac{\left(\left. \frac{\partial T}{\partial x} \right|_{y+\Delta y} - \left. \frac{\partial T}{\partial x} \right|_{y} \right)}{\Delta y} + k_z \frac{\left(\left. \frac{\partial T}{\partial x} \right|_{z+\Delta z} - \left. \frac{\partial T}{\partial x} \right|_{z} \right)}{\Delta z} + q' = \rho c_p \frac{\partial T}{\partial t}$$

By the definition of derivative (differentiation), it may be shown that

$$\frac{\partial}{\partial x}\left(k_x \cdot \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k_y \cdot \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k_z \cdot \frac{\partial T}{\partial z} \right) + q' = \rho c_p \frac{\partial T}{\partial t}$$

Taking $k_x = k_y = k_z = k$ and assuming k, ρ, and c_p as independent of position and temperature, it may be shown that:

$$k \cdot \left(\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z} \right) \right) + q' = \rho c_p \frac{\partial T}{\partial t}$$

General conductivity equation in rectangular coordinates (Errors and omissions are expected)

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+q'=\rho c_p\frac{\partial T}{\partial t}$$

Defining $\alpha=\dfrac{k}{\rho c_p}=\textit{thermal diffusivity}$, it may shown that

$$\frac{\partial T}{\partial t}=\alpha\cdot\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{q'}{\rho c_p}$$

A high value of thermal diffusivity $\left(\alpha=\dfrac{k}{\rho c_p}\right)$ could result either from a high value of the thermal conductivity, which would indicate a rapid energy transfer rate, or from a low value of the thermal heat capacity (ρc_p). A low value of the thermal heat capacity would mean that less of the energy moving through the material would be absorbed thus more will transfer.

General conductivity equation in rectangular coordinates (Errors and omissions are expected)

Prove that the temperature profile through a plane wall having constant thermal conductivity is

$$T = T_1 + \frac{T_2 - T_1}{r_2} r$$

Starting from the general heat conduction equation (constant α)

$$\frac{\partial T}{\partial t} = \alpha \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{q'}{\rho c_p}$$

For the case of steady-state, one dimensional (x-direction), without heat generation heat conduction problem, the general heat conduction equation for constant α reduces to

$$\frac{d^2 T}{dx^2} = 0$$

$$\frac{d}{dx}\left(\frac{dT}{dx} \right) = 0$$

General conductivity equation in rectangular coordinates
(Errors and omissions are expected)

Integrating the above equation, it may be shown that

$$\frac{dT}{dx} = C_1$$

Integrating again, it may shown that

$$T = C_1 \cdot x + C_2 \qquad (1)$$

Applying boundary conditions

B.C. 1: At $x = 0$, $T = T_1$

B.C. 2: At $x = \Delta x$, $T = T_2$

General conductivity equation in rectangular coordinates (Errors and omissions are expected)

From Eq. 1 and using B.Cs., it may shown that

$$T_1 = C_2$$

$$T_2 = C_1 \cdot \Delta x + C_2 \Rightarrow C_1 = \frac{T_2 - T_1}{\Delta x}$$

Inserting values of C_1 and C_2 in Eq. 1, it may shown that

$$T = \frac{T_2 - T_1}{\Delta x} \cdot x + T_1$$

$$T = T_1 + \frac{T_2 - T_1}{\Delta x} \cdot x$$

General conductivity equation in rectangular coordinates (constant k, c_p and ρ)

$$\frac{\partial T}{\partial t} = \frac{k}{\rho \cdot c_p} \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{q'}{\rho \cdot c_p}$$

$$\frac{\partial T}{\partial t} = \alpha \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{q'}{\rho \cdot c_p}$$

For derivation see class notes.

General conductivity equation in rectangular coordinates (constant k, c_p and ρ)

In case no heat generation, we have the following equation. This form of equation is called as Fourier's second law of heat conduction or heat diffusion equation as the temperature variation are a function only of thermal diffusivity.

$$\frac{\partial T}{\partial t} = \alpha \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

General conductivity equation in rectangular coordinates (constant k, c_p and ρ)

For steady-state condition, the general heat conduction equation can be written as the following. This form of equation is called as Poisson's equation for heat transfer.

$$\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=-\frac{q'}{k}$$

General conductivity equation in rectangular coordinates (constant k, c_p and ρ)

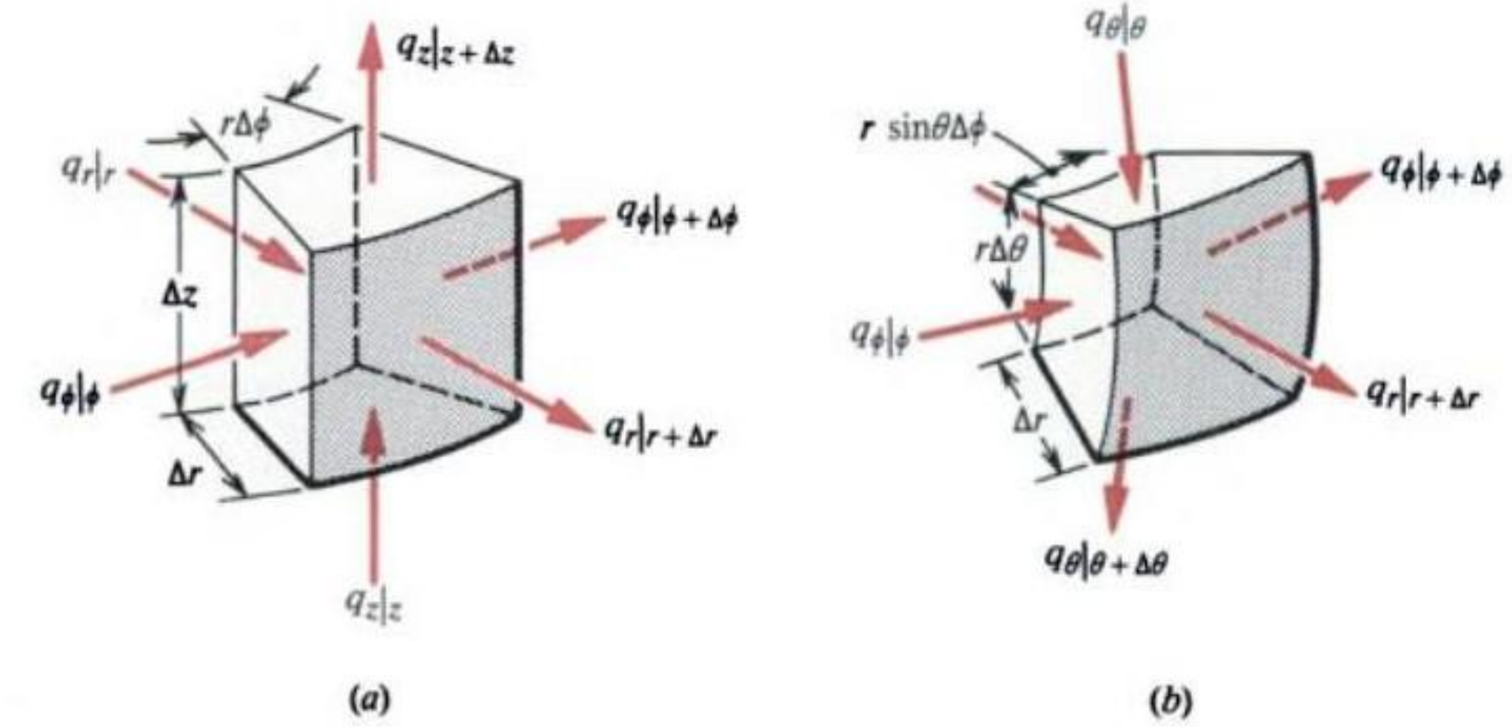
For steady-state condition and no heat generation situation, the general heat conduction equation can be written as the following. This form of equation is called as Laplace equation for heat transfer.

$$\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=0$$

Using the Laplacian operator, it may be shown that

$$\nabla^2 T = 0$$

General conductivity equation in other coordinates [10]



(a) Cylindrical coordinates (b) Spherical coordinates

General conductivity equation in other coordinates (constant k, c_p and ρ)

In cylindrical coordinates:

$$\frac{\partial T}{\partial t} = \alpha \cdot \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{q'}{\rho \cdot c_p}$$

In spherical coordinates:

$$\frac{\partial T}{\partial t} = \alpha \cdot \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin\theta}\frac{\partial^2 T}{\partial \phi^2} \right) + \frac{q'}{\rho \cdot c_p}$$

Homework problems

Workout the temperature profiles:

a) For a case of steady-state, one dimensional (r-direction), and without heat generation heat conduction problem in cylindrical coordinates

b) For a case of steady-state, one dimensional (r-direction), and without heat generation heat conduction problem in spherical coordinates

Conduction with internal heat generation (Not for exams)

Using general conductivity equation, derive an expression for internal heat generation in a plane wall with constant thermal conductivity where heat transfer occurs only in the x-direction.

References

1. Geankoplis, C.J. (2003). Transport processes and separation process principles: includes unit operations. 4th ed. Prentice-Hall International, Inc.
2. Holman, J.P. (2010). Heat transfer. 10th ed. McGraw-Hill Higher Education, Singapore.
3. Cengel, Y.A. (2003). Heat transfer: A practical approach. 2nd ed. McGraw-Hill.
4. Incropera, F.P.; DeWitt, D.P.; Bergman, T.L.; Lavine. A.S. (2007) Fundamentals of heat and mass transfer. 6th ed. John Wiley & Sons, Inc.
5. Kern, D.Q. (1965). Process heat transfer. McGraw-Hill International Book Co., Singapore.
6. McCabe, W.L.; Smith, J.C.; Harriott, P. (1993). Unit operations of chemical engineering. 5th ed. McGraw-Hill, Inc., Singapore.
7. Coulson, J.M.; Richardson, J.F.; Backhurst, J.R.; Harker, J.H. (1999). Coulson and Richardson’s Chemical engineering: Fluid flow, heat transfer and mass transfer. vol. 1. 6th ed. Butterwoth-Heinemann, Oxford.
8. Staff of Research and Education Association. (1984). The heat transfer problem solver. Research and Education Association, New Jersey.
9. Kreith, F.; Manglik, R.M.; Bohn, M.S. (2011). Principles of heat transfer, 7th ed., Cengage learning.
10. Mills A.F. (1995). Heat and mass transfer. Richard D. Irwin, Inc.