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ON FUZZY d-ALGEBRAS

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Abstract

In this paper we introduce the notions of fuzzy subalgebras and d-ideals in d-algebras, and investigate some of their results.

1. INTRODUCTION

Y. Imai and K. Iseki [1, 2] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the notion of BCI-algebras is a generalization of BCK-algebras. J. Neggers and H. S. Kim [8] introduced the class of d-algebras which is another generalization of BCK-algebras, and investigated relations between d-algebras and BCK-algebras.
L. A. Zadeh [6] introduced the notion of fuzzy sets, and A. Rosenfeld [11] introduced the notion of fuzzy group. Following the idea of fuzzy groups, O.

G. Xi [5] introduced the notion of fuzzy BCK-algebras. After that, Y. B. Jun and J. Meng [12] studied fuzzy BCK-algebras. B. Ahmad [10] fuzzified BCI-algebras. In this paper we fuzzify d-algebras.

2. PRELIMINARIES

In this section we cite the fundamental definitions that will be used in the sequel:

Definition 2.1

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCK-algebra* if it satisfies the following conditions:

(1) $((x * y) * (x * z)) * (z * y) = 0$,

(2) $(x * (x * y)) * y = 0$,

(3) $x * x = 0$,

(4) $x * y = 0, y * x = 0 \Rightarrow x = y$,

(5) $0 * x = 0$

for all $x, y, z \in X$.

Definition 2.2

Let X be a BCK-algebra and I be a subset of X, then I is called *an ideal* of X if

(I1) $0 \in I$,

(I2) y and $x * y \in I \Rightarrow x \in I$

for all $x, y \in X$.

Definition 2.3[8]

A nonempty set X with a constant 0 and a binary operation $*$ is called a *d-algebra* , if it satisfies the following axioms:

(d1) $x * x = 0$,

(d2) $0 * x = 0$,

(d3) $x * y = 0$ and $y * x = 0 \Rightarrow x = y$

for all $x,\ y \in X$.

Definition 2.4[9]

Let S be a non-empty subset of a d-algebra X, then S is called *subalgebra* of X if $x * y \in S$, for all $x, y \in S$.

Definition 2.5[9]

Let X be a d-algebra and I be a subset of X, then I is called *d-ideal* of X if it satisfies following conditions:

(Id1) $0 \in I$,

(Id2) $x * y \in I$ and $y \in I \Rightarrow x \in I$,

(Id3) $x \in I$ and $y \in X \Rightarrow x * y \in I$, i.e., $I \times X \subseteq I$.

Definition 2.6

A mapping $f : X \to Y$ of d-algebras is called *a homomorphism* if $f(x * y) = f(x) * f(y)$, for all $x,\ y \in X$.

Note that if $f : X \to Y$ is homomorphism of d-algebras, then $f(0)= \grave{0}$.

Definition 2.7

Let X be a non-empty set. A *fuzzy (sub)set* μ of the set X is a mapping $\mu : X \to [0, 1]$.

Definition 2.8

Let μ be the fuzzy set of a set X. For a fixed $s \in [0, 1]$, the set $\mu_s = \{x \in X : \mu(x) \geq s\}$ is called an *upper level* of μ.

3. FUZZY SUBALGEBRAS

Definition 3.1

A fuzzy set μ in d-algebra X is called a *fuzzy subalgebra* of X if it satisfies $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x,\ y \in X$.

Example 3.2[8]

Let $X := \{0, 1, 2\}$ be a set given by the following Cayley table:

$*$	0	1	2
0	0	0	0
1	2	0	2
2	1	1	0

Then $(X; *, 0)$ is a d-algebra, but not a BCK-algebra, since $(2 * (2 * 2)) * 2 = (2 * 0) * 2 = 1 * 2 = 2 \neq 0$. We define a fuzzy set $\mu : X \to [0, 1]$ by $\mu(0) = 0.7$, $\mu(x) = 0.02$, where for all $x \neq 0$. It is easy to see that μ is a fuzzy subalgebra of X.

Example 3.3

Let $X = \{0, 1, 2, \cdots\}$ be a set and the operation $*$ be defined as follows:

$$x * y := \begin{cases} 0 & \text{if x} \leq \text{y} \\ x - y & \text{if y} < \text{x} \end{cases}$$

Then $(X; *, 0)$ is an infinite d-algebra. If we define a fuzzy set $\mu : X \to [0, 1]$ by $\mu(0) = t_1$, $\mu(x) = t_2$ for all $x \neq 0$, where $t_1 > t_2$. Then μ is a fuzzy subalgebra of X.

Example 3.4

Let $X = \{0, 1, 2, \cdots\}$ be a set and the operation $*$ be defined as follows:

$$x * y := \begin{cases} 0 & \text{if x} \leq \text{y} \\ 1 & \text{otherwise} \end{cases}$$

Then $(X; *, 0)$ is an infinite d-algebra, but not BCK-algebras, since $(2 * (2 * 0)) * 0 = (2 * 1) * 0 = 1 * 0 = 1 \neq 0$.

Example 3.5

Let $X = [0, a] \subset [0, 1]$, a being a fixed number, and the operation $*$ be defined as follows:

$$x * y = min(x, max(x, y) - min(x, y)), \qquad \forall x, y \in X.$$

Then $(X; *, 0)$ is an infinite d-algebra.

Proposition 3.6

A fuzzy set μ of a d-algebra X is a fuzzy subalgebra if and only if for every $t \in [0, 1]$ the upper level μ_t is either empty or a subalgebra of X.

Proof

Suppose that μ is a fuzzy subalgebra of a d-algebra X and $\mu_t \neq 0$, then for any $x, y \in \mu_t$, we have $\mu(x * y) \geq \min \{\mu(x), \mu(y)\} \geq t$ which implies $x * y \in \mu_t$, and hence μ_t is a subalgebra of X.

Conversely, take t= min $\{\mu(x), \mu(y)\}$, for any x, $y \in X$. Then by assumption, μ_t is a subalgebra of X, which implies $x * y \in \mu_t$, so that $\mu(x * y) \geq$ t= min $\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy subalgebra of X.

Proposition 3.7

Any subalgebra of a d-algebra X can be realized as a level subalgebra of some fuzzy subalgebra of X.

Proof

Let A be a subalgebra of a d-algebra X and μ be a fuzzy set in X defined by

$$\mu(x) := \begin{cases} t, & \text{if x} \in \text{A;} \\ 0, & \text{otherwise.} \end{cases}$$

where $t \in (0, 1)$. It is clear that $\mu_t = A$. Let x, $y \in X$. If x, $y \in A$, then $x * y \in A$. So $\mu(x) = \mu(y) = \mu(x * y) = t$, and $\mu(x * y) \geq$min $\{\mu(x), \mu(y)\}$. If x, $y \notin A$, then $\mu(x) = \mu(y) = 0$. Thus $\mu(x * y) \geq$ min $\{\mu(x), \mu(y)\} = 0$. If at most one of x, $y \in A$, then at least one of $\mu(x)$ and $\mu(y)$ is equal to 0. Therefore, min$\{\mu(x), \mu(y)\}$= 0 and $\mu(x * y) \geq 0$ which completes the proof.

Corollary 3.8

Let A be a subset of X. Then the characteristic function χ_A is a fuzzy subalgebra of X if and only if A is a subalgebra of X.

Lemma 3.9

Let μ be a fuzzy subalgebra with finite image. If $\mu_s = \mu_t$, for some s, $t \in Im(\mu)$, then s=t.

Lemma 3.10

Let μ and λ be two fuzzy subalgebras of X with identical family of level subalgebras. If $Im(\mu) = \{t_1, t_2, \cdots, t_n\}$ and $Im(\lambda)$= $\{s_1, s_2, \cdots, s_m\}$, where $t_1 \geq t_2 \geq \cdots\ t_n$ and $s_1 \geq s_2 \geq \cdots\ s_m$. Then

(1) m=n.

(2) $\mu_{t_i} = \lambda_{s_i}$, for i= 1, 2, $\cdots$, n.

(3) If $\mu(x) = s_i$, then $\lambda(x) = s_i$, for all $x \in X$ and i=1, 2, $\cdots$, n.

Proposition 3.11

Let μ and λ be two fuzzy subalgebras of X with identical family of level subalgebras. Then $\mu = \lambda \Rightarrow Im(\mu) = Im(\lambda)$.

Proof

Let $Im(\mu) = Im(\lambda) = \{s_1, ..., s_n\}$ and $s_1 > ... > s_n$. By lemma 3.10, for any $x \in X$, there exists s_i such that $\mu(x) = s_i = \lambda(x)$. Thus $\mu(x) = \lambda(x), \forall\ x \in X$. This completes the proof.

Proposition 3.12

Two level subalgebras μ_s and μ_t, $(s < t)$ of a fuzzy subalgebra are equal if and only if there is no $x \in X$ such that $s \leq \mu(x) < t$.

Proof

Suppose that $\mu_s = \mu_t$, for some $s < t$. If there exists $x \in X$ such that $s \leq \mu(x) < t$, then μ_t is a proper subset of μ_s, which is contradicting the hypothesis.
Conversely, suppose that there is no $x \in X$ such that $s \leq \mu(x) < t$. If $x \in \mu_s$, then $\mu(x) \geq s$ and so $\mu(x) \geq t$, since $\mu(x)$ does not lie between s and t. Thus $x \in \mu_t$, which gives $\mu_s \subseteq \mu_t$. The converse inclusion is obvious since $s < t$. Therefore, $\mu_s = \mu_t$.

4. FUZZY d - IDEALS

Definition 4.1

A fuzzy set μ in X is called *fuzzy BCK-ideal* of X if it satisfies the following inequalities:

(1) $\mu(0) \geq \mu(x)$,

(2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$, for all x, $y \in X$.

Definition 4.2

A fuzzy set μ in X is called *fuzzy d-ideal* of X if it satisfies the following inequalities:

(Fd1) $\mu(0) \geq \mu(x)$,

(Fd2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$,

(Fd3) $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$

for all x, $y \in X$.

Example 4.3[11]

Let $X := \{0, 1, 2, 3\}$ be a d-algebra with the following Cayley table:

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	0
3	3	3	3	0

Then $(X; *, 0)$ is not BCK-algebra since $\{(1 * 3) * (1 * 2)\} * (2 * 3) = 1 \neq 0$. We define fuzzy set μ in X by $\mu(0) = 0.8$ and $\mu(x) = 0.01$, for all $x \neq 0$ in X. Then it is easy to show that μ is a d-ideal of X.

We can easily observe the following propositions:

- In a d-algebra every fuzzy d-ideal is a fuzzy BCK-ideal, and every fuzzy BCK-ideal is a fuzzy d-subalgebra.
- Every fuzzy d-ideal of a d-algebra is a fuzzy d-subalgebra.

Theorem 4.4

If each non-empty level subset $U(\mu; t)$ of μ is a fuzzy ideal of X then μ is a fuzzy d-ideal of X, where $t \in [0, 1]$.

Proof

Assume that each non-empty level subset $U(\mu; s)$ of μ is a d-ideal. Then it is easy to show that μ satisfies $(Fd_1)$and (Fd_2). Assume that $\mu(x * y) <$ min $\{\mu(x), \mu(y)\}$, for some x, $y \in X$. Take $t_0 := \frac{1}{2}\{\mu(x * y) +$ min $(\mu(x), \mu(y))\}$, then x, $y \in U(\mu; t_0)$. Since μ is a d-ideal of X, $x * y \in U(\mu; t_0)$, therefore, $\mu(x * y) \geq t_0$, a contradiction. Hence assumption is wrong. This completes the proof.

Definition 4.5[4]

Let λ and μ be the fuzzy sets in a set X. The cartesian product $\lambda \times \mu : X \times X \longrightarrow [0, 1]$ is defined by $(\lambda \times \mu)(x, y)=$ min$\{\lambda(x), \mu(x)\}$, $\forall$ x, $y \in X$.

Theorem 4.6

If λ and μ are fuzzy d-ideals of a d-algebra X. Then $\lambda \times \mu$ is a fuzzy d-ideal of $X \times X$.

Proof

For any $(x, y) \in X \times X$, we have

$(\lambda \times \mu)(0,0)=$ min $\{\lambda(0), \mu(0)\} \geq$ min $\{\lambda(x), \mu(y)\} = (\lambda \times \mu)(x, y)$

Let (x_1, x_2) and $(y_1, y_2) \in X \times X$. Then

$(\lambda \times \mu)((x_1, x_2))=$ min $\{\lambda(x_1), \mu(x_2)\}$

$\geq$ min $\{$ min $\{\lambda(x_1 * y_1), \lambda(y_1)\}$, min $\{\mu(x_2 * y_2), \mu(y_2)\}\}$

$=$ min $\{$ min $\{\lambda(x_1 * y_1), \mu(x_2 * y_2)\}$, min $\{\lambda(y_1), \mu(y_2)\}\}$

$=$ min $\{(\lambda \times \mu)((x_1 * y_1, x_2 * y_2)), (\lambda \times \mu)((y_1, y_2))\}$

$=$ min $\{(\lambda \times \mu)((x_1, x_2) * (y_1, y_2)), (\lambda \times \mu)((y_1, y_2))\}$

and $(\lambda \times \mu)((x_1, x_2) * (y_1, y_2))= (\lambda \times \mu)((x_1 * y_1, x_2 * y_2))$
$=$ min $\{\lambda(x_1 * y_1), \mu(x_2 * y_2)\}$

$\geq$ min $\{$ min $\{\lambda(x_1), \lambda(y_1)\}$, min $\{\mu(x_2), \mu(y_2)\}\}$

$=$ min $\{$ min $\{\lambda(x_1), \mu(x_2)\}$, min $\{\lambda(y_1), \mu(y_2)\}\}$

$=$ min $\{(\lambda \times \mu)((x_1, x_2)), (\lambda \times \mu)((y_1, y_2))\}$.

Hence $\lambda \times \mu$ is a fuzzy $d-$ideal of $X \times X$.

Theorem 4.7

Let λ and μ be fuzzy sets in a d-algebra X such that $\lambda \times \mu$ is a fuzzy d-ideal of $X \times X$. Then

(i) either $\lambda(0) \geq \lambda(x)$ or $\mu(0) \geq \mu(x)$, $\forall x \in X$.

(ii) If $\lambda(0) \geq \lambda(x)$, $\forall x \in X$, then either $\mu(0) \geq \lambda(x)$ or $\mu(0) \geq \mu(x)$.

(iii) If $\mu(0) \geq \mu(x)$, $\forall x \in X$, then either $\lambda(0) \geq \lambda(x)$ or $\lambda(0) \geq \mu(x)$.

Proof

(i) We prove it using reductio ad absurdum.

Assume $\lambda(x) > \lambda(0)$ and $\mu(y) > \mu(0)$, for some x, y $\in X$. Then $(\lambda \times \mu)(x, y) = \min \{\lambda(x), \mu(y)\} > \min \{\lambda(0), \mu(0)\} = (\lambda \times \mu)(0, 0)$

$\Rightarrow (\lambda \times \mu)(x, y) > (\lambda \times \mu)(0, 0), \forall\ x,\ y\ \in X.$

which is a contradiction. Hence (i) is proved.

(ii) Again, we use reduction to absurdity.

Assume $\mu(0) < \lambda(x)$ and $\mu(0) < \mu(y),\ \forall\ x,\ y \in X$. Then, $(\lambda \times \mu)(0, 0)= \min \{\lambda(0), \mu(0)\} = \mu(0)$

and $(\lambda \times \mu)(x,\ y) = \min \{\lambda(x),\ \mu(y)\} > \mu(0) = (\lambda \times \mu)(0,\ 0)$

$\Rightarrow (\lambda \times \mu)(x,\ y) > (\lambda \times \mu)(0,\ 0).$

which is a contradiction. Hence (ii) is proved.

(iii) The proof is similar to (ii).

The partial converse of the Theorem 4.6 is the following.

Theorem 4.8

If $\lambda \times \mu$ is a fuzzy d-ideal of $X \times X$, then λ or μ is a fuzzy d-ideal of X.

Proof

By Theorem 4.7(i), without loss of generality we assume that $\mu(0) \geq \mu(x)$, $\forall$ $x \in X$.

It follows from Theorem 4.7(iii) that either $\lambda(0) \geq \lambda(x)$ or $\lambda(0) \geq \mu(x)$. If $\lambda(0) \geq \mu(x)$, $\forall\ x \in X$. Then $(\lambda \times \mu)(0, x)= \min \{\lambda(0), \mu(x)\} = \mu(x) - -(A)$

Since $\lambda \times \mu$ is a fuzzy d-ideal of $X \times X$, therefore,

$(\lambda \times \mu)(x_1, x_2) \geq \min \{(\lambda \times \mu)((x_1, x_2) * (y_1, y_2)), (\lambda \times \mu)(y_1, y_2)\}$

and $(\lambda \times \mu)((x_1, x_2) * (y_1, y_2)) \geq \min \{(\lambda \times \mu)((x_1, x_2), (\lambda \times \mu)((y_1, y_2)\}$

It implies that $(\lambda \times \mu)(x_1, x_2) \geq \min \{(\lambda \times \mu)((x_1 * y_1, x_2 * y_2)), (\lambda \times \mu)(y_1, y_2)\}$.

and $(\lambda \times \mu)((x_1 * y_1, x_2 * y_2)) \geq \min\ \{(\lambda \times \mu)((x_1, x_2), (\lambda \times \mu)((y_1, y_2)\}$.

Putting x_1=y_1=0, we have

$(\lambda \times \mu)(0, x_2) \geq \min\ \{(\lambda \times \mu)((0, x_2 * y_2)), (\lambda \times \mu)(0, y_2)\}$

and $(\lambda \times \mu)((0, x_2 * y_2)) \geq \min\ \{(\lambda \times \mu)((0, x_2), (\lambda \times \mu)((0, y_2)\}$

using equation(A), we have

$\mu(x_2) \geq \min\{\mu(x_2 * y_2), \mu(y_2)\}$

and $\mu(x_2 * y_2) \geq \min\{\mu(x_2), \mu(y_2)\}$.

This proves that μ is a fuzzy d-ideal of X. The second part is similar. This completes the proof.

Definition 4.9[4]

Let A be a fuzzy set in a set S, *the strongest fuzzy relation* on S that is fuzzy relation on A is μ_A given by $\mu_A(x, y)$= $\min\{A(x), A(y)\}$, for all x, $y \in S$.

Theorem 4.10

Let A be a fuzzy set in a d-algebra X and μ_A be the strongest fuzzy relation on X. Then A is a fuzzy d-ideal of X if and only if μ_A is a fuzzy d-ideal of $X \times X$.

Proof

Suppose that A is a fuzzy d-ideal of X. Then

$\mu_A(0, 0)$= $\min\{A(0), A(0)\} \geq \min\ \{A(x), A(y)\}$= $\mu_A(x, y)$, $\forall\ (x, y) \in X \times X$.

For any $x = (x_1, x_2)$ and $y = (y_1, y_2)$$\in X \times X$, we have

$\mu_A(x) = \mu_A(x_1, x_2)$= $\min\ \{A(x_1), A(x_2)\}$

$\geq \min\ \{\ \min\ \{A(x_1 * y_1), A(y_1)\},\ \min\ \{A(x_2 * y_2), A(y_2)\}\}$

$= \min\ \{\ \min\ \{A(x_1 * y_1), A(x_2 * y_2)\},\ \min\ \{A(y_1), A(y_2)\}\}$

$=\min\ \{\mu_A(x_1 * y_1, x_2 * y_2), \mu_A(y_1, y_2)\}$

$= \min\ \{\mu_A((x_1, x_2) * (y_1 * y_2)), \mu_A(y_1, y_2)\}$

$= \min\ \{\mu_A(x * y), \mu_A(y)\}$

and $\mu_A(x * y) = \mu_A((x_1, x_2) * (y_1, y_2))$
$= \mu_A((x_1 * y_1, x_2 * y_2))$
$= \min \{A(x_1 * y_1), A(x_2 * y_2)\}$
$\geq \min \{\min \{A(x_1), A(y_1)\}, \min \{A(x_2), A(y_2)\}\}$
$= \min \{ \min \{A(x_1), A(x_2)\}, \min \{A(y_1), A(y_2)\}\}$
$= \min \{\mu_A((x_1, x_2), \mu_A(y_1, y_2)\}$
$= \min \{\mu_A(x), \mu_A(y)\}$.
Hence μ_A is a fuzzy d-ideal of $X \times X$.
Conversely, suppose that μ_A is a fuzzy d-ideal of $X \times X$. Then
$\min\{A(0), A(0)\} = \mu_A(0, 0) \geq \mu_A(x, y) = \min \{A(x), A(y)\}$, $\forall$ $(x, y) \in X \times X$.
It follows that $A(x) \leq A(0)$, $\forall x \in X$
For any $x = (x_1, x_2)$, $y = (y_1, y_2) \in X \times X$, we have
$\min\{A(x_1), A(x_2)\} = \mu_A(x_1, x_2)$
$\geq \min \{\mu_A((x_1, x_2) * (y_1, y_2)), \mu_A(y_1, y_2)\}$
$= \min \{\mu_A(x_1 * y_1, x_2 * y_2), \mu_A(y_1, y_2)\}$
$= \min \{\min\{A(x_1 * y_1), A(x_2 * y_2)\}, \min\{A(y_1), A(y_2)\}\}$
$= \min \{\min\{A(x_1 * y_1), A(y_1)\}, \min \{A(x_2 * y_2), A(y_2)\}\}$.
Putting x_2=y_2=0, we have
$\mu_A(x_1) \geq \min \{\mu_A(x_1 * y_1), \mu_A(y_1)\}$.
Likewise, $\mu_A(x_1 * y_1) \geq \min \{\mu_A(x_1), \mu_A(y_1)\}$.
Hence A is fuzzy d-ideal of X.

Definition 4.11

Let $f : X \longrightarrow Y$ be a mapping of d-algebras and μ be a fuzzy set of Y. The map μ^f is the *pre-image* of μ under f, if $\mu^f(x) = \mu(f(x))$, $\forall$ $x \in X$.

Theorem 4.12

Let $f : X \longrightarrow Y$ be a homomorphism of d-algebras. If μ is a fuzzy d-ideal of

Y, then μ^f is a fuzzy d-ideal of X.

Proof

For any $x \in X$, we have

$\mu^f(x) = \mu(f(x)) \leq \mu(\acute{0}) = \mu(f(0)) = \mu^f(0)$

Let $x, y \in X$. Then

$\min\{\mu^f(x * y), \mu^f(y)\}$

$= \min\{\mu(f(x * y)), \mu(f(y))\}$

$= \min\{\mu(f(x) * f(y)), \mu(f(y))\}$

$\leq \mu(f(x)) = \mu^f(x)$

and $\min\{\mu^f(x), \mu^f(y)\}$

$= \min\{\mu(f(x)), \mu(f(y))\}$

$= \min\{\mu(f(x)), \mu(f(y))\}$

$\leq \mu(f(x) * f(y)) = \mu(f(x * y)) = \mu^f(x * y)$.

Hence μ^f is a fuzzy d- ideal of X.

Theorem 4.13

Let $f : X \longrightarrow Y$ be an epimorphism of d-algebras. If μ^f is a fuzzy d-ideal of X, then μ is a fuzzy d-ideal of Y.

Proof

Let $y \in Y$, there exists $x \in X$ such that $f(x) = y$. Then

$\mu(y) = \mu(f(x)) = \mu^f(x) \leq \mu^f(0) = \mu(f(0)) = \mu(\acute{0})$

Let $x, y \in Y$. Then there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = y$. It follows that $\mu(x) = \mu(f(a)) = \mu^f(a)$

$\geq \min\{\mu^f(a * b), \mu^f(b)\}$

$= \min\{\mu(f(a * b)), \mu(f(b))\}$

$= \min\{\mu(f(a) * f(b)), \mu(f(b))\}$

$= \min\{\mu(x * y), \mu(y)\}$

and $\mu(x * y) = \mu(f(a) * f(b)) = \mu^f(a * b)$
$\geq \min\ \{\mu^f(a), \mu^f(b)\}$
$= \min\ \{\mu(f(a)), \mu(f(b))\}$
$= \min\ \{\mu(x), \mu(y)\}$.

Hence μ is a fuzzy d- ideal of Y.

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