Entrepreneurship and Small Business Review
ISSN: 2789-6374
Volume 2, pp: 94-112

ENTREPRENEURSHIP AFTER RETIREMENT: A FUZZY MULTI-ATTRIBUTE DECISION-MAKING MODEL WITH MACHINE LEARNING

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Abstract

This paper introduces a novel decision-making algorithm tailored for newly retired government servants aspiring to venture into entrepreneurship. The proposed algorithm integrates multi-attribute decision-making (MADM) with intuitionistic fuzzy sets, aggregation operators and employs polynomial regression within a machine learning framework to guide users in selecting optimal business opportunities. This approach addresses the challenge faced by retirees who possess limited or no business experience, providing them with a data-driven method to evaluate potential ventures. Furthermore, the algorithm offers predictive insights on product pricing through the analysis of historical price data, enhancing decision-making accuracy. The effectiveness of the algorithm is demonstrated through a series of case studies, highlighting its potential to significantly impact post-retirement business planning. The study contributes to the field of entrepreneurial decision support systems and extends the application of green finance principles by promoting sustainable and informed business choices among senior entrepreneurs. By incorporating sustainability criteria into the decision-making process, the algorithm encourages investment in environmentally responsible ventures, considers long-term economic viability, and supports business models aligned with green finance principles.
In the advanced countries number of researches has been conducted related to IWB and TL. This study has contribution to the existing literature on SMEs with in the developing contribution with special focus on the Pakistan. it gives the significant insights regarding the influence of the leadership styles on innovation.

Keywords: Entrepreneurship, Retired employees, Entrepreneurial decision making model, Fuzzy multi-attribute decision making model, machine learning.

Submission 04-Nov-24; Revision 27-Dec-24; Accepted 27-Jan-25; Published 30-Jan-25

Introduction

Starting an entrepreneurial setup right after getting retirement is a dream to many employees around the world. Many employees wish to earn income even after their retirement and use their energies in a most productive manner, while others may have concerns for their survival and social status (Gray, 2007). In many of the Asian countries, government employees who are termed as permanent employees get a decent remuneration in form of pensions. However, there are certain countries in Asia like Taiwan, Malaysia, Singapore and Japan that may also provide substantial support to workers who were not government employees (Holzmann, MacArthur, & Sin, 2000). However, studies indicate that soon after retirement they employees become dependent on their children financial support. In a study conducted in Malaysia this support is calculated around 67% from children (Hariati, 2007). Parents who have always remained financial supporters and contributors get mentally and emotionally disturbed due to these financial dependences. The good thing is that studies provide evidences that elderly employees are found more creditable, possess special skills and superior problem solving abilities.

There is a strong need to help retirees in decision making process in their entrepreneurial ventures. This study purports a fuzzy multi-attribute decision making model (MDMA) with machine learning (ML).

Multi-attribute decision-making (MADM) plays a crucial role in business contexts, especially in supplier selection, where various attributes such as quality, technical capability, and financial soundness need to be considered (Jayapriya, Selvakumari, & Kavitha, 2024). MADM methods aid in evaluating decisions with multiple co-attribute options effectively, ranking them from most to least suitable, and are commonly used to identify the best solutions (Shoba, Selvakumari, & Kavitha, 2024). It is observed that in the domain of cybersecurity, the MADM techniques are used to choose the best solutions out of a large number of alternatives to enrich the security of computer and network system (Alamleh et al., 2023). Furthermore, deploying information systems incorporating fuzzy pattern recognition has enhanced the decision-making process in business; specifically, through the measurement of optimal solutions according to attribute weights to make it easier to choose and calculate proximity (Sun & Kong, 2023). Moreover, MADM methods are applied in the aerospace field in order to evaluate the technology readiness levels and to determine the development schedules for missions which enables decision makers for mission planners (Chen, Dastoor, Castro, Balchanos, & Mavris, 2022).

More and more businesses are being transformed by machine learning (ML), specifically more towards small and medium-sized enterprises (SMEs) in particular, who are faced with challenges attributed to lack of ML knowledge and resource and can be alleviated using partnerships and necessary tools (Aruna et al., 2023a). ML applications in marketing have grown significantly, utilizing various techniques like supervised, unsupervised, and reinforcement learning algorithms to solve problems related to consumer behavior, recommender systems, and text analysis (Aruna et al., 2023b; Ngai & Wu, 2022). The role of ML in Business Intelligence (BI) is central and involves the efficient data collection by filters through which business can make a better decision, as well as ML models such as those used to predict something, customer segmentation, anomaly detections, using NLP for sentiment analysis, recommendation systems for personalized suggestion, and so on, which lead to an increase in accuracy, improvements in decision making and improved customer experience for the organization (Duarte et al., 2022).

Over the years, Intuitionistic fuzzy sets have received much attentions in business applications; this happens since intuitionistic fuzzy sets are capable to process the uncertainty and vagueness. It offers a more nuanced representation of information based on membership and non membership functions and hesitation margins (Imanov & Aliyev, 2023). There has been a use of Atanassov's intuitionistic fuzzy sets to express arguments and judgments in full – with the pros, cons and hesitations – which is necessary for veracious ranking of alternatives (Szmidt, Kacprzyk, & Bujnowski 2022a) (Szmidt, Kacprzyk, & Bujnowski 2022b). In the field of artificial intelligence applications, the vague, inconsistent, and imprecise knowledge has been addressed by employ a combination of rough set theory and intuitionistic fuzzy sets due to benefits of intuitionistic fuzzy sets in a complex information characterization (Singh & Som, 2022). In addition to this, intuitionistic fuzzy set theory has proven to be useful in solving such type of uncertain mathematical problems and to conform to such uncertainty it provides new operations and also new notions which enables to increase the efficiency of decision making in the case of real life problems (Kumari & Kumar, 2022).

The significance of the aggregation operators in numerous business decision making processes can not be underrated. They are used for aggregating individual preferences, opinions or judgments on group members in a collective decision (Petrovi´c, Mihajlovi´c, Markovi´c, Hashemkhani Zolfani, & Madi´c, 2023). In 1988, Yager introduced the concept of an ordered weighted averaging (OWA) operator which gave birth to a family of aggregation functions which have been drawing considerable attraction among researchers to look into its properties and applications (Mahmood, Ali, & Albaity, 2022). These aggregation methods are of key importance for the assessment of problems of multi attribute decision making especially in the case of fuzzy sets and of complex linguistic fuzzy sets (Csiszar, 2021).

Bonferroni mean operator has received an extensive study in the fields of multicriteria decision making, support systems and intuitionistic fuzzy set theory (J. Xu, Ma, & Xu, 2023).

The proposed approach has been further extended to incorporate its variations like geometric Bonferroni mean that exhibits the property of geometric Bonferroni mean and also geometric mean (Kumar & Jain, 2023). Besides, the Bonferroni mean has been introduced in new aggregation operators such as BON-OWAAC and BON-OWAIMAM operators with distance measures and norms for continuous aggregations and multiple comparisons in the entrepreneurial contexts (Li, Zeng, & Li, 2016). In addition to this, the mean of the linear operators, like the mean resolvent operator in flow analysis, is important for predicting linear response to forcing and optimizing input output dynamics in flow control applications (Chabbabi et al., 2019; Blanco-Mesa & Merig´o, 2016).

Motivation for Research

This research is motivated by the fact that the number of government employees that go into retirement with a desire to start a business, but little business experience is increasing. Since this demographic shift is unique, there is a challenge we face how to enable retirees with tools for structured decision making opportunities that fits in with their skills and resources in a way that helps minimize the risks. Current decision making approaches basically are not able to account for the uncertainty and multi-faceted nature of entrepreneurial choices faced by this group. This thesis attempts to fill this gap by integrating multi attribute decision making (MADM) techniques and intuitionistic fuzzy sets with machine learning. Intuitionistic fuzzy sets offer a more refined modeling of uncertainty since the membership and nonmembership degrees of uncertainty is modeled together with hesitation. This provides the retirees with a comprehensive framework for assessment that takes into account the real world complexities. Moreover, including polynomial regression provides forecast ability for enhancing decision making accuracy, especially in finance and strategic analysis.

The need to fuel retirement capital – beyond what a logical rate of return may offer – and enable retirees to make educated, data driven entrepreneurial decisions that align with their interests, experiences as well as their means for sustainable financial health, drives research. Hence, the study endeavor to help in broad terms to smoothen the pathway to post retirement entrepreneurship and in general terms to increase economic participation and well being of senior citizens.

Preliminaries

An Intuitionistic fuzzy set (IFS) is an extension of fuzzy sets as added by Krassimir Atanassov (Takeuti & Titani, 1984) with an extra layer of uncertainty.

In contrast to classical fuzzy sets in which each element has a single membership degree (μ) determining its relationship to a set, an intuitionistic fuzzy set can be associated with each element two membership degrees (μ and ν) and an hesitation margin (π). It is defined for each element x in the universe of discourse X as:

- **Membership Degree (μ)**: Represents how much the element x belongs to the set, with $\mu(x) \in [0,1]$.
- **Non-membership Degree (ν)**: Represents how much the element x does not belong to the set, with $\nu(x) \in [0,1]$.
- **Hesitation Margin (π)**: Calculated as $\pi(x) = 1-\mu(x)-\nu(x)$, this parameter quantifies the uncertainty or indecision regarding the membership of x.

So an intuitionistic fuzzy set I can be written as $I = \{< x, \mu(x), \nu(x) > | x \in X\}$. The **intuitionistic fuzzy number** IFN α_x is defined as $\forall x \in X$ $\alpha_x = (\mu(x), \nu(x))$. The **score function** s of any intuitionistic fuzzy number α_x is defined as $s(\alpha_x) = \mu(x) - \nu(x)$. **Some important operations on IFN:**

$\alpha 1 \oplus \alpha 2 = (\mu\alpha 1 + \mu\alpha 2 - \mu\alpha 1\mu\alpha 2, \nu\alpha 1\nu\alpha 2)$ $\alpha 1 \otimes \alpha 2 = (\mu\alpha_1\mu\alpha_2, \nu\alpha_1 + \nu\alpha_2 - \nu\alpha_1\nu\alpha_2)$

intuitionistic fuzzy Bonferroni mean

For any p,q ≥ 0 , if

$$\Omega^{p,q}(\alpha_1, \alpha_2, \alpha_3 \ldots, \alpha_n) = \left(\frac{1}{n(n-1)}\left(\bigoplus_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\alpha_i^p \bigotimes \alpha_j^q\right)\right)\right)^{\frac{1}{p+q}}$$

then $\Omega^{p,q}$ is called intuitionistic fuzzy Bonferroni mean (Z. Xu & Yager, 2010) of the intuitionistic fuzzy numbers $\alpha_1, \alpha_2, \alpha_3 \ldots, \alpha_n$

Problem Statement

A long serving government college lab attendant, Bilal Manzoor, recently retired and is now ready to spend the rest of his life into business. Bilal's excitement, however, is hampered by lack of experience in how to deal with or even decide on a business venture. Ultimately, this study is dedicated to building a decision making framework that addresses challenges such as Bilal facing when he decides to venture into entrepreneurship. To execute the context of the proposed problem, the framework relies on a novel integration of multi-attribute decision making, fuzzy sets and machine learning techniques to assess four distinguished business options: retail stores, online businesses, franchises and consulting services. Each option is evaluated against some basic characteristics such as market potential, financial viability, coherence with personal interests and skills and risk factors to each with their positive and negative reservations. Bilal and other retirees will now have a structured approach that will enable them to make data driven decisions regarding what will be the best fit for their aspirations, capabilities and financial objectives.

This study assesses four business alternatives which retirees like Bilal Manzoor would be interested in, which are unique because of their varying nature and characteristics that would allow them to be present in an active element of business. Here are briefs of each:

First alternative retail store

As being interested in photography throughout his life, Bilal thinks of opening a small retail store dealing with photography equipment and accessories. This business would be able to tap into the community, let his passion shine, and give personalized advice to customers from his years of experience in photography. Furthermore, contacting them with the products and customers would increase the tactile pleasure of the retail environment which would also improve his post-retirement life.

Second alternative online business

Also, Bilal is thinking of putting his technical skills that he developed while running the college lab to use when he starts an online business. He has come up with the idea of selling science kits and educational tools online for students, schools and hobbyists. It was also an option that would allow him to work out of his home, at his own pace, and perhaps reach a wider market without having to have a physical storefront.

Third alternative franchise

Bilal has a stable financial reserve and wants to buy a business with a tested model, so he starts exploring purchasing a franchise in the educational toys sector. It is ideal because it would serve as a structured system and support network for him, reducing the risks that come with starting up a new company from scratch; yet, allowing him to contribute to educational development.

Fourth alternative services

Within his years of experience in the technical educational environment over the years Bilal considers providing consulting services to the educational institutions in terms of setting up and efficiently managing the science labs. However, he would use this venture to offer value to institutions searching for ways to improve the scientific teaching facilities offered in their institutions.

Each of these alternatives provides Bilal with pathways to continue being active in work, work using his expertise, and do entrepreneurship in retirement. Next, we will identify the crucial set of attributes and sub attributes for evaluation of whole set of business alternatives for Bilal. These will enable to prove a successful and suitable track for every venture whether in positive or negative aspects.

First attribute market potential

This attribute measures the overall attractiveness and viability of the market for a new business.

Positive sub-attribute ”high demand”

Assesses the current and projected demand for the business’s offerings, indicating potential for growth and profitability.

Negative sub-attribute “high competition”

Evaluates the density of competitors within the market, which can influence market entry strategy and potential for market share.

Second attribute financial feasibility

Evaluates the economic viability of the business from start-up to established operations.
Positive sub-attribute “low initial investment”
Considers the required capital to start the business, critical for retirees like Bilal who may have limited funds to risk.

Negative sub-attribute “high operational costs”

Looks at the ongoing expenses associated with running the business, including utilities, inventory, and employee salaries, which affect the long-term financial sustainability.
Third attribute personal interest and skills
Focuses on the alignment of the business with Bilal’s personal interests and existing skills.
Positive sub-attribute “relevant experience”
Reflects how Bilal’s previous work and life experiences can contribute to the success of the new business.

Negative sub-attribute “learning curve

Indicates the effort and time required for Bilal to acquire any additional skills or knowledge needed to effectively manage and grow the business.

Fourth attribute risk and stability

Assesses the potential risks and overall stability of the business, crucial for ensuring that Bilal's investment remains secure.

Positive sub-attribute "low business risk"

Measures the likelihood of significant problems arising that could jeopardize the business, important for providing peace of mind to retirees.

Negative sub-attribute "economic sensitivity"

Gauges how fluctuations in the economy could impact the business, important for planning in uncertain economic times.
Each of these attributes and sub-attributes plays a pivotal role in guiding Bilal through the decision-making process, ensuring that his transition into entrepreneurship is as informed and strategic as possible.

First algorithm multi-attribute decision-making

To develop a multi-attribute decision-making (MADM) framework employing intuitionistic fuzzy numbers and the Bonferroni mean operator, the following steps delineate the mathematical structure and necessary procedures for the algorithm:

Step 1: Collection of Judgment Scores

Let $\alpha_i\ 0 \geq i \leq n$ represent the set of alternatives, and each attribute $\beta_j\ 0 \geq i \leq m$ is analyzed based on two sub-attributes: a positive μ and a negative v. Define $\mu_{i,j}$ and $v_{i,j}$ as the judgment scores provided by the decision-maker for the positive and negative sub-attributes, respectively, of the i-th alternative and j-th attribute. These scores satisfy the following conditions:

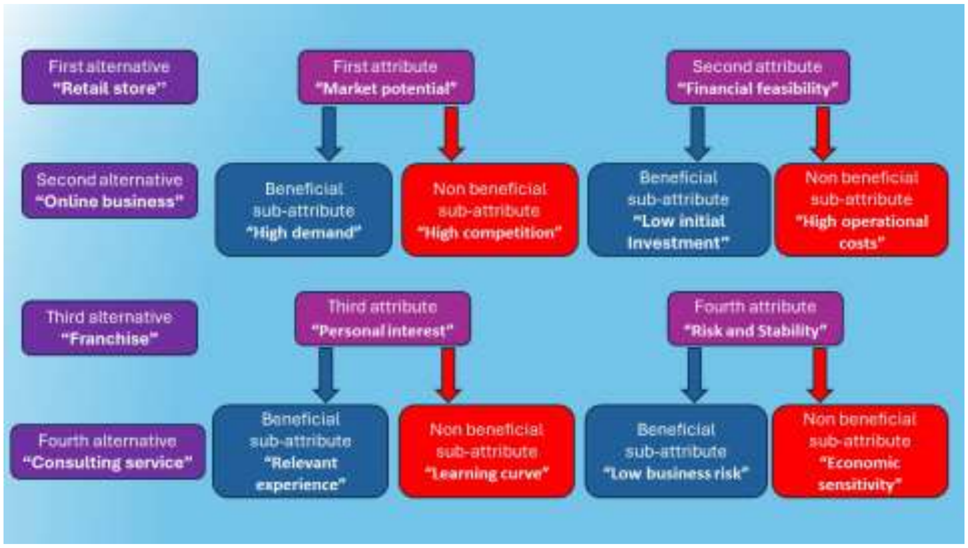

Figure 1: Description of alternatives, attributes and sub-attributes

- $\mu_{i,j} \geq 0$ and $\mu_{i,j} \leq 10$
- $v_{i,j} \geq 0$ and $v_{i,j} \leq 10$
- $\mu_{i,j} + v_{i,j} \geq 0$ and $\mu_{i,j} + v_{i,j} \leq 10$

Step 2: Conversion to Intuitionistic Fuzzy Decision Matrix

Once the judgment scores are collected, the second step will be to induce these scores to intuitionistic fuzzy decision matrix. It enables the addition of the dual nature of intuitionistic fuzzy numbers that cover the degree of membership and non membership.

Intuitionistic Fuzzy Decision Matrix:

Let $\gamma i, j$ be the intuitionistic fuzzy number for the judgment score of the ith alternative and $j - th$ attribute. In each intuitionistic fuzzy decision matrix, we have a representation of each entry as $\gamma i, j = (\mu i, j/10, \nu i, j/10)$.

where:

- $\mu_{i,j}$ is the score indicating the degree of satisfaction (membership) of the *i*-th alternative with respect to the *j*-th attribute.
- $v_{i,j}$ is the score indicating the degree of non-satisfaction (non-membership) of the *i*-th alternative with respect to the *j*-th attribute.

Using these pairs for each alternative relative to each attribute, intuitionistic fuzzy decision matrix is constructed. The entries of this matrix provide a good overall evaluation of each considered alternative in respect to positive and negative assessments:

$$\Gamma = \begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,m} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n,1} & \gamma_{n,2} & \cdots & \gamma_{n,m} \end{bmatrix}$$

This matrix Γ now serves as the basis for the subsequent aggregation and decision-making processes, encapsulating the fuzzy evaluations in a structured format.

Step 3: Aggregation Using the Bonferroni Mean Operator

After that, the intuitionistic fuzzy decision matrix achieved from the transformation of the decision scores has to be aggregated for each alternative to synthesize the overall evaluation. Bonferroni mean operator is going to be used for the aggregation.

Application of the Bonferroni Mean:

Applying the Bonferroni mean operator for each row of the intuitionistic fuzzy decision matrix Γ that represents each choice of the alternatives for each year, it aggregates the intuitionistic fuzzy numbers.ζi = Bonferroni mean of $(\gamma i, 1, \gamma i, 2, \ldots, \gamma i, m)$:

Additionally, this aggregation process forms a single score per alternative, making it quite convenient for straightforward comparison between alternatives.

$$\zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_n \end{bmatrix}$$

Finally, this final vector ζ will be used to form the final ranking and decision of each alternative, which will be a comprehensive aggregation of the performance of each alternative, which is across the attributes according to the Bonferroni mean operator specified earlier.

Step 4: Conversion of Intuitionistic Fuzzy Scores to Crisp Scores

With the intuitionistic fuzzy numbers reduced into a single vector ζ, the next step would be to convert the fuzzy numbers into crisp scores, so that there could be a clear ranking and comparison for the objective.

Application of the Intuitionistic Fuzzy Score Function

Therefore, the intuitionistic fuzzy score function is defined as: To convert each aggregated intuitionistic fuzzy number ζi into a crisp score:

$$S(\zeta i) = \mu(\zeta i) - \nu(\zeta i)$$

Where $\mu(\zeta i), \nu(\zeta i)$ are the membership and non membership values of the intuitionistic fuzzy number ζi. Hence, this function calculates the score through the subtraction of non-membership degree from membership degree and produces a scalar value that represents the overall preference for each alternative.

$$\mathfrak{S} = \begin{bmatrix} \mathfrak{S}(\zeta_1) \\ \mathfrak{S}(\zeta_2) \\ \vdots \\ \mathfrak{S}(\zeta_n) \end{bmatrix}$$

This vector S provides a straightforward mechanism to rank the alternatives, with higher scores indicating preferable options.

Final ranking and decision making:

The alternatives are ranked based on the scores in S. The alternative associated with the highest score in S is selected as the best option, concluding the decision-making process.

Application of the Multi-Attribute Decision-Making Algorithm

First step judgment matrix construction

In the application of our decision making algorithm, the judgment matrix is composed of four alternatives and four attributes to evaluate each alternative. Two sub attributes μ and ν are analyzed with each attribute on the basis of these. For clarity, the matrix is tabulated with μ and ν scores in separate columns. The matrix 1 displays the judgment scores for each alternative across each attribute. Here, μ represents the satisfaction level, and ν represents the dissatisfaction level.

Table 1: Judgment matrix provided by the decision maker

	μ_{β_1}	ν_{β_1}	μ_{β_2}	ν_{β_2}	μ_{β_3}	ν_{β_3}	μ_{β_4}	ν_{β_4}
α_1	7	2	6	3	8	1	5	4
α_2	6	3	6	2	5	4	5	3
α_3	8	1	5	4	5	4	6	3
α_4	9	0	8	1	6	3	7	2

Each row in this matrix corresponds to an alternative, and each pair of columns under the attributes $\beta_1,\beta_2,\beta_3,\beta_4$ shows the respective μ and ν scores.

Second step: Conversion to intuitionistic fuzzy decision matrix

The judgment matrix is converted to intuitionistic fuzzy decision matrix whose each entry is a pair (μ,ν) that represents the membership and non-membership value of each alternative and attribute. This is transformed as shown in the table below:

Third step: Application of the Bonferroni mean operator

In Step 2, we have obtained the intuitionistic fuzzy decision matrix and then applied the Bonferroni mean operator to aggregate the membership (μ), and nonmembership (ν) score of each alternative. It provides this aggregation method that considers the interdependence among attributes.

Table 2: Intuitionistic Fuzzy Decision Matrix

	β_1	β_2	β_3	β_4
α_1	(0.7, 0.2)	(0.6, 0.3)	(0.8, 0.1)	(0.5, 0.4)
α_2	(0.6, 0.3)	(0.6, 0.2)	(0.5, 0.4)	(0.5, 0.3)
α_3	(0.8, 0.1)	(0.5, 0.4)	(0.5, 0.4)	(0.6, 0.3)
α_4	(0.9, 0)	(0.8, 0.1)	(0.6, 0.3)	(0.7, 0.2)

A balanced approach to synthesizing multiple criteria into a single intuitionistic fuzzy number for each alternative.

The results of applying the Bonferroni mean operator are as follows:

- The aggregated intuitionistic fuzzy number for first alternative ζ_1 is (0.6517, 0.245).

- The aggregated intuitionistic fuzzy number for second alternative ζ_2 is (0.5499, 0.2989).

- The aggregated intuitionistic fuzzy number for third alternative ζ_3 is (0.5999, 0.2974). • The aggregated intuitionistic fuzzy number for fourth alternative ζ_4 is (0.755, 0.1367).

These numbers represent the comprehensive evaluation of each alternative, reflecting both positive (membership) and negative (non-membership) assessments. The results are crucial for the final decision-making process, where these numbers will be converted into crisp scores to determine the most preferred alternative.

Fourth step: Conversion of intuitionistic fuzzy scores to crisp scores

Given the intuitionistic fuzzy numbers obtained from applying the Bonferroni mean operator, we now proceed to convert these fuzzy scores into crisp scores using the intuitionistic fuzzy score function, defined as:

$$S(\zeta_i) = \mu(\zeta_i) - \nu(\zeta_i)$$

For the obtained results:

$$S(\zeta_1) = 0.6517 - 0.245 = 0.4067$$
$$S(\zeta_2) = 0.5499 - 0.2989 = 0.251$$
$$S(\zeta_3) = 0.5999 - 0.2974 = 0.3025$$
$$S(\zeta_4) = 0.755 - 0.1367 = 0.6183$$

These crisp scores also presented in Figure 2 represent the final decision scores for each alternative, enabling a clear ranking and selection process.

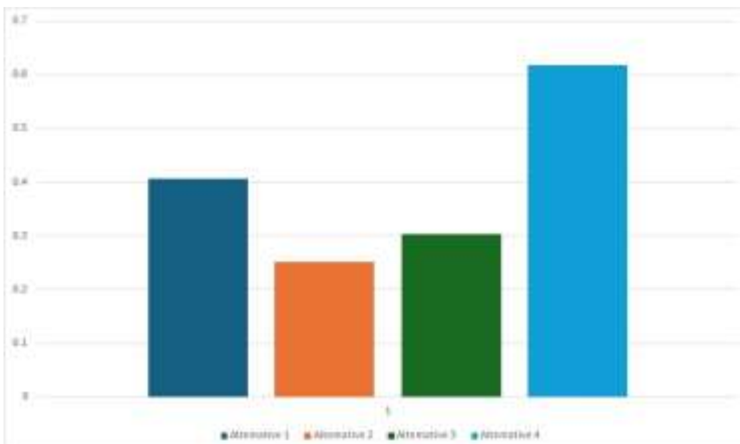

Figure 2: Final ranking of Alternatives

Price Prediction Using Polynomial Regression

Polynomial regression models find diverse applications in business research, offering robust alternatives for tasks like cost prediction (Satheesh & Kumar, 2022). However, these models often face challenges due to multicollinearity, potentially leading to incorrect conclusions about the significance of model terms (Oransirikul & Takada, 2019). However, polynomial regression is invaluable in many fields of business including finance, marketing, human resource management and operation, for the purposes of credit scoring, risk analysis, demand forecasting and green supply chain management (Isaac, Adedeji & Ismail, 2012). Moreover, polynomial regression is used to predict the number of waiting passengers at bus stations by using passive monitoring of Wi-Fi signals from mobile devices for practical usage in the transportation field (Chatterjee & Greenwood, 1990). On the basis of this literature review, it is evident, that the polynomial regression has a great opportunity to be a versatile and valuable tool for solving various business problem and challenges.

Objective

The design task is to find a price for the new children's T-shirt that will compete with the best market price. This will enable the setting of a price commensurate with the area market value and enable compete in the business.

Data Collection

We are given the data on the prices of children's T-shirts from a shop from the competitor, during a span of the last ten years. The following table (table 3) will demonstrate the products price in the last 10 years and these prices are as per the market reaction which we will use as input for our polynomial regression model.

Methodology: Polynomial Regression

Polynomial regression is utilized to model the historical price data. This method is particularly suited for this task as it can fit data that shows fluctuations and non-linear trends typical in Table 3.

Table 3: Historical Price of T-shirts from 2014 to 2023

Year	Price	Year	Price
2014	11	2019	60
2015	20	2020	72
2016	31	2021	82
2017	42	2022	90
2018	50	2023	101

market-driven pricing. The model will be of the form:

$$P(t) = a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^n$$

where P(t) represents the price in year t, and a0,a1,...,an are coefficients that will be determined based on the data. We applied polynomial regression of order 2 on the data given in table 3 and get the polynomial given in equation 6 bellow.

$$P(t) = -\frac{6550690388046567\,t^2}{576460752303423488} + \frac{7867971958111081\,t}{140737488355328} - \frac{571140600573991}{8589934592}$$

the predicted price for year 2024 is 110.8167 the error analysis of polynomial genrated by polynomial regression is given in the figure 3

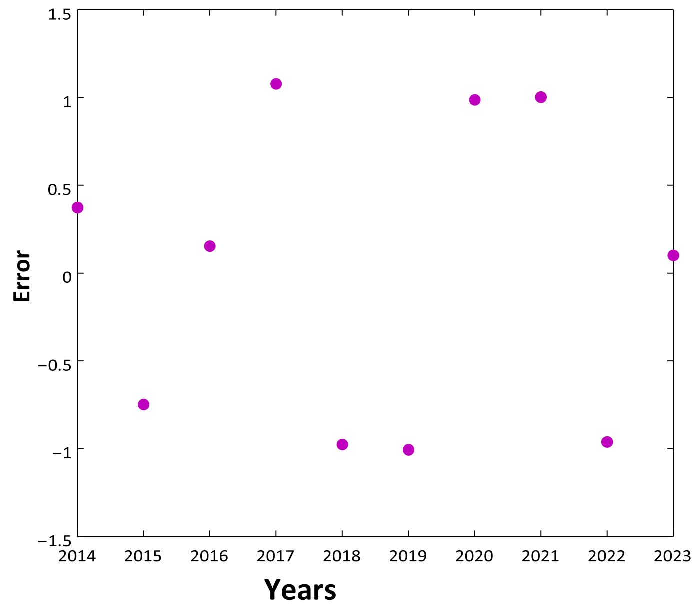

Figure 3: Error analysis of prices genrated by polynomial regression

Managerial Implications

The proposed decision-making algorithm presents several managerial implications, particularly for those overseeing post-retirement transition programs and entrepreneurial support services.

Key insights include:

- **Structured Decision Framework**: Managers can leverage the presented algorithm to design comprehensive decision-support tools that aid retirees in assessing various business opportunities. This structured approach helps in balancing multiple criteria, ensuring that decisions are not only data-driven but also considerate of personal skills and interests.

- **Risk Management**: The algorithm includes attributes of economic sensitivity, and operational costs to aid in the identification and mitigation of risks to new business ventures. As a result, resources are better allocated more efficiently and have better risk adjusted decision-making.

- **Enhanced Customization**: By using intuitionistic fuzzy sets, the flexibility in modeling the uncertainty and hesitation that generally exist in the real decision making becomes possible. By this, managers can adjust weights and attributes of the algorithm in accordance to different retiree profiles and therefore, improve the relevance and precision of the recommendations.

- **Predictive Pricing Strategies**: Moreover, polynomial regression for analysis of price can further be used for wider market predictions and financial planning. This method can be applied by managers to forecast trends and retirees can thereby use this information to set their pricing strategies based on current market situations to position themselves better in the competitive market.

- **Sustainability Considerations**: In fact, focus on attributes concerning market potential and personal alignment tends to foster sustainable business practices. This framework can be used by managers to encourage ventures as the ventures should align with green principles of finance and responsible entrepreneurship from senior entrepreneurs.

- **Training and Development**: By making the learning curve a sub attribute it provides an instrument to the algorithm to inform the training program of retirees about what kind of areas needs to be added to a given training plan to maximize his performance. Managers can plan out workshops and mentorship programs that target their specific skill gaps and make smoother transits into being an entrepreneur.

Conclusion and Future Research Direction

In this study, a complete decision making algorithm is introduced that takes advantage of the multi-attribute decision making (MADM) methods with intuitionistic fuzzy sets and their aggregation operators and polynomial regression with machine learning technologies. The algorithm was therefore designed to allow newly retired persons to balance between both the positive and negative attributes of entrepreneurial opportunities. Intuitionistic fuzzy sets offered an effective way to deal with the uncertainty in the construct models, while the Bonferroni mean operator allowed the multi-attributes to be taken into account and aggregated.

Practical case studies demonstrated the efficacy of the algorithm and its potential for guiding retirees on how to make informed and data driven entrepreneurial decisions. Furthermore, the framework emphasized the significance of integrating sustainability and interests of the person themselves to the process of making decisions of whether to proceed on a business venture, driving individuals towards responsible and customized business ventures.

Future Research Direction: Even though this algorithm is of high value, there is much future research to be done to improve upon it. Instead of a complete transformation, these potential enhancements include refining the existing machine learning framework using more advanced yet interpretable models, improving attribute weighting mechanisms, and enhancing uncertainty handling in decision making. Deep learning models and real time data streams seem promising, however, their integration should be evaluated in terms of feasibility and relevance with the aim to avoid complexity that may dilute the decision making process. Additional research should also include validating the algorithm with larger datasets and through other case studies to test the algorithm's ability to generalize in different business contexts.

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