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# Investigation of Seventh Grade Students van Hiele Geometric Thinking Levels in Circle Subject

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## Abstract

The purpose of the study was to determine the association between van Hiele geometric thinking levels and circular achievement exam results for middle school students in 7th grade. The design of the study was a survey method. 157 middle school students were participated into study from three different schools in Kayseri in the spring semester of the 2017-2018 academic year. Van Hiele geometry thinking test which was developed by Usiskin and adapted into Turkish by Duatepe (2000) and a circle achievement test that was developed by the researcher were used to collect the data. Pearson The circle achievement test and van Hiele geometry thinking levels were correlated to find their link. Assumptions were met, hence independent groups t-test and ANOVA were performed. The results revealed that Grade 7. Students van Hiele geometric thinking levels were lower than expected levels. A moderate correlation was found between the circle achievement test and the van Hiele geometric thinking test. The results obtained from the van Hiele geometry thinking test showed statistically significant mean difference across schools 1 and 2. Moreover, there is no statistically significant mean difference across schools at the circle achievement test. Boy and girl middle school students' geometry thinking levels and circle achievement test scores were not statistically significantly different from each other.

**Keywords:** Circle achievement test; gender; middle school; spatial ability; van Hiele, geometric thinking

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## Introduction

Mathematics is a branch of science that analyses numbers, shape, space, size, and their relationship. It is a universal language with its' own symbol and shape. Mathematics involves processing, estimating, analysing, problem-solving, and producing by using its language. Mathematics provides individuals with reasoning, creative thinking skills, and the development of aesthetic sense (Ministry of National Education-MoNE, 2009). All other sciences receive the help of mathematics when they reach a level that can reach numerical correlations. Therefore, mathematics is compulsory in learning all sciences. Basic and social sciences gain the identity of being a science in terms of mathematical expression (Göker, 1997, p. 24-25).

This universal language and culture transform from civilization to civilization without distinguishing language, religion, race, and country. Mathematics is a science that makes silent reforms (Göker, 1997, p. 22) and an important tool for solving problems in our daily lives. Therefore, mathematics-related objectives are encountered throughout our entire learning life (Baki, 2006, p. 46). Nowadays, we use mathematics in many places, starting from setting up the clock of a to set the time and continuing with the solution of the problems that will require calculation of the four operations at home and work during the day (Umay, 1996) The main purpose in mathematics is not to learn the rules, but to solve the problems of numbers and shapes in daily life (Binbasioglu, 1991). We can list the practical benefits of mathematics, its effects on people and the characteristics of people who know good mathematics with the followings:

- Provide the right decision and reasoning.
- Learn and apply scientific thinking ways and adopt positive thinking principles.
- Improve various and critical thinking, allow to think in different ways in any subject.
- Allow you to compare the gaze of others about a subject with your own ideas and find the right one. It can reach the level of independent thinking without being dependent on anyone's thinking.
- Provide new thoughts.
- Open wide horizons to people and enables the human to be activated with the wealth they give to themselves.
- Improve systematic and logical thinking, gain habits such as analysis, research, and criticism.
- Provide a great benefit to the personality of the person by producing practical solutions to the events in our daily lives and by making the right decisions.

- Being very careful when solving mathematical problems and finding hidden relationships in problems give people joy and excitement. It arouses a desire to find new things in people.

Geometry is one of the most important branches of mathematics. Geometric shapes can be useful and can be performed to the maximum function in everyday life (Pesen, 2008). Students are introduced to these geometric shapes at an early age. According to van Hiele, geometry teaching should be started at an early age with games. Comprehensive teaching methods can be created with games such as puzzles, pattern blocks, or pattern creation. Thus, children begin to recognize geometric shapes and features (Hiele, 1986). It is very important to associate teaching geometry with daily life. Students gain skills such as generalization, comparison, criticism, scheming, learning, analyzing, synthesizing, expressing thoughts clearly, being careful, patient, and tidy with geometry (Baykul, 1998, p. 267). To acquire these skills from geometry, Pierre van Hiele and Dina van Hiele, who said that teaching should be provided in accordance with their learning and development levels, formed a geometric thinking model consisting of five levels progressively progressing from level 0 to 4, in accordance with the children's learning and development levels in the 1950s (Swafford et al, 1977). The van Hiele couple realized that students at the same level thought the same way asked the decisive questions of each level and correlated the transition between the levels. However, this transition is progressively progressing. Each step of levels 0-1-2-3-4 is passed in sequence. No level is skipped. Progression between levels does not occur according to age or mental development. High school and elementary school students of very different ages may be at the same level (Terzi, 2010).

The concept of the circle in geometry is shown in the sub-learning area of geometry in mathematics classes and starts from the second grade of elementary school until the seventh grade in middle school in Turkey. The basic features of the circle, basic elements, angle, circumference, and area are taught in the spiral structure of the mathematics curriculum in elementary grades in Turkey. In addition, it is aimed to solve the daily life problems related to the circle.

**van Hiele Geometric Thinking Test Levels**

The van Hiele geometric thinking model examined how a student perceives geometry at the geometric thinking levels. Dina van Hiele and Pierre van Hiele designed this model together. They observed what difficulties students have encountered while learning geometry, they investigated the causes of the difficulties and how they could be solved (Duatepe Paksu, 2016). The most important characteristic of the van Hiele geometric thinking model is the classification of geometry in five levels (0 to 4) that progresses hierarchically. Each of the five levels clarifies the geometric thinking processes. The more we know about levels, the more we understand the different types of ideas in terms of geometry and how we think about geometry. The main difference between one level and the other that follows it is the object. In other words, we can think geometrically (van de Walle, Karp, and Bay-Williams, 2016). Van Hiele geometric thinking levels have the following features:

- While learning geometry, van Hiele geometric thinking levels are passed, respectively. Subsequent levels cannot be reached before passing one level.
- Thinking products at all levels are the objects of thinking of the next level.
- The transition between levels does not occur according to age. A third-grade student and a high school student may be at level 0.
- The basic point that affects the transition between the levels is the geometry experiences. Students should do research, talk about their research, and interact with the content of the next level by increasing their geometry experience.
- If the instruction and the language used higher than the student's level of understanding, there can be a communication gap between the student and the instructor. For example, if the expression is a square or rectangle then it is taught without telling the relationship, the student can memorize this expression (Van de Walle et al., 2016).

Van Hiele geometric thinking levels were arranged as 1-5 by Clements and Battista (1992). There is no meaningful difference between the two enumerations within them and the meaning of integrity. In addition to these levels, Clements and Battista (1992, p. 429) have been defined “pre-recognition” as a level. At this level, children recognize shapes but because of their insufficient perception, they only know some of the visual features of the shapes. To illustrate, they can distinguish between round and angular shapes such as circles and squares but cannot distinguish between two angular shapes such as squares and rectangles (Oflaz, 2010).

Van Hiele geometric thinking levels were analyzed as levels 1-5 in this study. Students who were not at any level would be referred to as pre-recognition, level 0, as expressed by Clements and Battista (1992). Van Hiele geometry thinking levels are Level 1: Visualization, Level 2: Analysis, Level 3: Informal Inference, Level 4: The Inference, Level 5: Systematic Thinking.

**Visualization: Level 1**

Shapes and appearance of the shapes were considered at the level of visualization. Students are interested in and understand the general appearance of shapes. For example, they define a frame as a frame because it looks like a square. Focusing on the appearance of shapes at this level prevents us from recognizing the features of shapes. When a frame is rotated 45 degrees, it is now viewed as a rhombus. At this level, the students look at the appearance of the shapes and classify them as if they are similar.

The products of Level 1 are classes and similar forms (Van de Walle et al., 2016).

Students recognize the rectangular shape, i.e., the appearance of the rectangular shape, which is previously shown in parallel with the opposite sides and not equal to 90 degrees at this level. For example, if the concept of a rectangle is introduced to a student as in Figure 1 (a), the student would respond to the rectangle in Figure 1 (b) "thin rectangular" and to the rectangle in Figure 1 (c) "It is not a rectangle" (Duatepe Paksu, 2016).

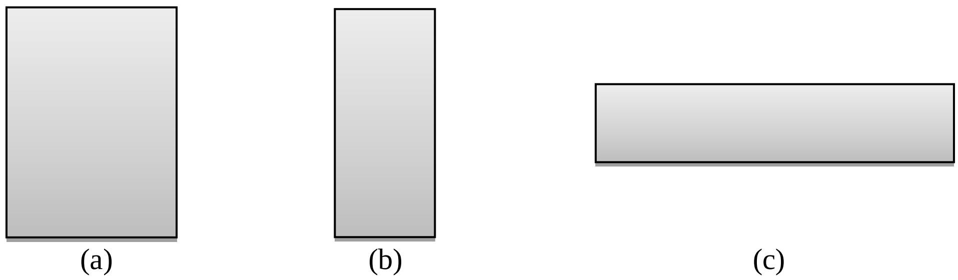

Figure 1. *Different views of rectangular figures*

Even students at this level keep the A4 paper vertically and show that the figure is rectangular when it is displayed and that the shape of the A4 paper is not rectangular when it is placed horizontally in front of their eyes (Duatepe Paksu, 2016). Suitable instructional activities for students in Level 1:

- It should include classification and separation activities that include different and similar aspects of the figures. The more the students encounter, the higher gaining relationship between the shapes.
- Students should be able to draw two and three-dimensional shapes to draw them into pieces. These activities should include distinctive features of shapes. Thus, students develop an understanding of geometrical properties (Van de Walle et al., 2016).

Students need to gain a variety of geometric experiences to change their minds and progress among the levels at the visualization level. Students should be asked questions that are compelling and examine to move from visualization level to analysis level. For example, "Can this be right for other rectangles?" or "Can you draw a triangle with no right angle?". Asking these questions that will force students to think and realize that the characteristics they observe in a particular shape will be appropriate for other shapes (van de Walle, et.al., 2016).

**Analysis: Level 2**

At the level of analysis, what is considered is the group of shapes, not the shape itself. Students think about all the forms of the shapes together with the figures shown to them. They do not just talk about a specific rectangle; they can generalize for all rectangles. They talk about the properties of the rectangular class as having parallel and opposed four sides, four right angles, and identical diameters. The size or shape of the shape which is not related to the features of the figures is pushed into the background and the students' group according to the characteristics. If a shape is a cube, it must bear all the geometric characteristics of the cube, to illustrate, it must consist of six square faces. In Level 2, students can list all the features of that shape when defining the shapes. The products of Level 2 consist of the features of the shapes (Van de Walle et al., 2016).

Students classify square, rectangular, and parallelogram with respect to their properties at the analysis level. On the other hand, they do not understand that they are a subset of each other. They cannot see that the square is in fact a special rectangle, or it is a special parallelogram. They also do not recognize the relationships between the features of shapes. For example, in the parallelogram, the opposite sides are identical and parallel, but they do not recognize the relationship between identical and parallel characteristics. In other words, if a convex quadrilateral equals all the opposite sides, they cannot think that they should be parallel (Duatepe Paksu, 2016).

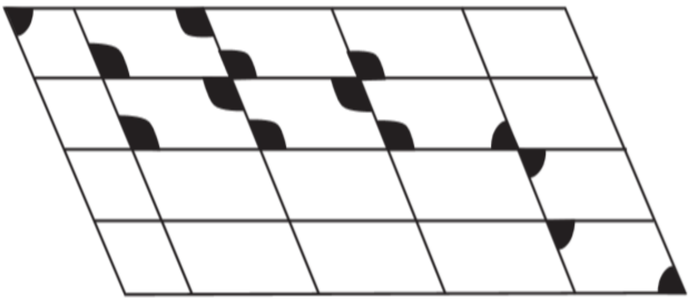

Figure 2. *An activity indicating that the parallel angles of the parallelogram are equal (adapted from Fuys, Geddes, Tischler (1988) cited in Duatepe Paksu, 2016)*

At this level, students can be able to perform other activities such as angle, edge, diagonal measurement, identification, shape or fragmentation, and other shapes and classification activities. In Figure 2, the activity was performed by painting the angles on the grid pattern consisting of parallelogram shapes. Thus, the student will realize that the parallel angles are equal. This activity can be shown in the rectangular, square, and rhombus quadrants, which is the subclass of the parallelogram, and in the other quadrilaterals of the parallelogram. This way, the relationship between the parallelogram and the lower class is established (Duatepe Paksu, 2016). Appropriate instructional activities for students in Level 2:

- It should be concentrated on the properties rather than the appearance of the figures. As the new geometric concepts become familiar, these features can be expanded and increased.
- Ideas should be applied to all classes (all rectangles and triangles), not shapes, but classes. For example, how do you divide all triangles into groups? Can you explain the triangular types with these groups? (Van de Walle et al., 2016).

We must force the students to think with questions such as why? and other questions that they can reason for moving from analysis level to informal inference level. For example, "If we have a quadrilateral with all sides, can we call it a square?" and "Can you find a contrasting example? (Van de Walle et al., 2016).

**Informal Inference: Level 3**

What is thought at the level of informal inference is the characteristics of shapes. Students notice the properties of shapes and realize the relationship between other forms and their properties at the informal inference level. For example, if there are four right angles, this figure is rectangular and if the shape is square, all the angles are vertical, then the square also recognizes the square and rectangular relationship as a rectangle. The geometric shapes can be separated by a small number of classifications, as in the example, the if-if reason is considered by reasoning. The products of Level 3 are the relations of shapes with each other (Van de Walle et al., 2016). Students also realize the relationship between their characteristics and the shape. In a convex quadrilateral, the opposite sides can understand that the equal work will be parallel, and the opposite sides will be parallel and vice versa. Also, if the opposite sides are parallel in a quadrilateral, they also see an equal relationship (Duatepe Paksu, 2016). Moreover, students understand the relationships between shapes and list the features that should be in the definition of a concept. Instead of sorting out the features of a figure with a long list, they are short and clear with enough and necessary features. They can make more than one definition of a concept. For example, for the parallelogram, definitions as "a quadrilateral with parallel sides" "quadrilateral with opposite sides, parallel and inner angles are equal to 360 degrees" or "quadrilateral with opposite angles parallel" can be made. Students at this level will be able to identify all the features of the parallelogram based on the definition of the relation between these definitions and the definition of the parallel sides in the first definition (Duatepe Paksu, 2016).

In Level 3, students can follow and understand logical arguments by making informal inferences with the features of shapes and properties of shapes. The proofs they make can be intuitive rather than achieved step by step, but they also understand that logical arguments give more accurate results. However, the known structure of the system connected to formal deduction works in the background (Van de Walle et al., 2016).

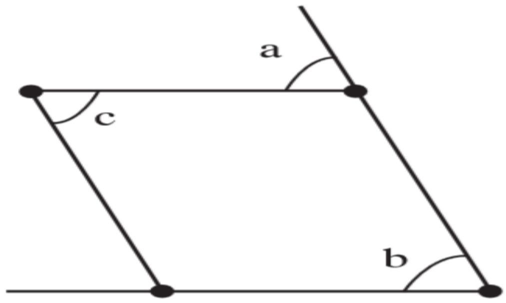


Figure 3. *An activity that explains why mutual angles are identical in parallelogram (Fuys, Geddes, Tischler (1988). Adapted from the study: Duatepe Paksu, 2016)*

Students at this level may be asked to indicate why the angles of the parallelogram are shown in Figure 3. The activities that would comment on the features of these shapes with different shapes, produce ideas, and discuss them would help students to think about the relationship of shapes with each other (Duatepe Paksu, 2016). Suitable instructional activities for students in Level 3:

- It should include questions that lead to a hypothesis and testing the hypothesis. To illustrate, “Is this always, right?”, “Is this right for all the triangles”, or “Is it right for the only equilateral triangle?”.
- It should include studies on which features are enough and necessary for different shapes. As, “What diagonal diagrams tell us that the figure is the exact square?”
- It should include informal inference language. All, ever, if, like, some, then, what happens, etc.
- Students should be encouraged to make an informal inference. Students may be asked to explain the logic of an informal proof by other students or teachers (Van de Walle et al., 2016).

**Inference: Level 4**

What is considered at the inference level is the relationship between the properties of geometric shapes. “Are the assumptions they made in relation to the features of the shapes at the previous thought level?”. The question is considered. As the informal inferences are analyzed, a system consists of axiom, definition, and theorem. Then, the results of the theorem develop, and these are the tools used to obtain the geometry lines. At this level, students think about abstract propositions about geometric properties and make logical inferences that are not intuitive (Van de Walle et al., 2016).

Students are aware that the diagonals of the rectangle are centered on each other at the inference level. On the other hand, they understand that this must be proven in relation to inferential propositions. The difference between Level 3 from the previous is that students at Level 3 follow logical arguments but think that there is no need for more. The products of thought are axiomatic systems based on deduction geometry at level 4 (Van de Walle et al., 2016).

Students can prove other theorems through deduction using the proven theorems. They can also comprehend the stages of reasoning through induction. At this level, students begin to understand the role of axiom, theorem, and proof to make inferences. At this level, students can prove that the opposite sides of the convex quadrilateral are equal, or that the opposite sides are parallel or vice versa (Duatepe Paksu, 2016).

Students who reach Level 4 can understand the relationship between undefined terms, theorems, axioms, and postulates in Euclidean Geometry. However, they do not understand the non-Euclidean geometry because they understand the axiom and its' definitions are not variable but fixed (Duatepe Paksu, 2016). Level 4 is the level of high school geometry that can distinguish one who thinks. Students who learn geometry at the high school level try to put geometry on the inferential system. They construct the theorems on the axiom list and definitions. Logical reasoning is used to prove the theorems of geometry (Van de Walle et al., 2016).

**Systematic Thinking: Level 5**

The inferential axiomatic systems of geometry are considered at the level of systematic thinking (Level 5). Axiomatic inferential systems of the van Hiele, the highest level, is not the inferences within one system, but the objects considered in the systematic thinking. Similar and different aspects between different axiomatic systems are recognized at this level. For example, the spherical geometry is based on the lines drawn on another sphere rather than the lines drawn in space or plane. It is based on geometry, axioms, and theorems. Level 5 is the university level. Learners could think about similarities and differences between different axiomatic systems in Level 5 (Van de Walle et al., 2016).

Students can understand that axioms and definitions are variable and can create definitions and theorems within various axiomatic systems at the systematic thinking level. Thus, students begin to recognize and interpret the non-Euclidean geometry and formulate the definitions and theorems outside the Euclidean geometry. For example, instead of the 5th postulate with which Euclid "can draw only a parallel line from a point outside a line", Riemann said, "It cannot be drawn parallel from a point outside of a line" or from Lobatchevski's point of view "Many parallel lines can be drawn from a point outside of a line?". Students reaching this level, while the sum of the interior angles of a triangle in Euclidean geometry 180 degrees and can understand the cause of the Riemann geometry in Lobatchevski to be different from 180 degrees. They also understand that geometry systems have more than one of them, which means that only one of them should be correct, and which of these systems should be used (Duatepe Paksu, 2016).

**Transition Between Levels**

The transition between van Hiele's geometry thinking levels depends on the age and the education that is not due to maturity. Van Hiele designed a teaching plan that included five levels so that the students could progress through the levels of geometric thinking. Teachers support the development of students' levels of geometry thinking if they carry out teaching according to these levels by taking into consideration the students' geometry learning levels. Van Hiele's teaching plan consists of five levels: research, orientation, netting, self-study, and integration, respectively (Baykul, 2000; Crowley, 1987; Hiele, 1986; Hoffer, 1983).

- Research: The teacher understands their level by asking questions and presenting the concepts and symbols appropriate to their level. It makes the distinction of how geometric shapes are and tries to attract students' attention with materials. During the teaching phase, emphasis is placed on the teaching of concepts and the discovery of materials by the students.
- Orientation: At the previous level, students' answers are evaluated, and students are directed and assigned to do research. With these assignments, students discover and learn new structures. In addition, activities such as games and puzzle-solving can also help students find and feel shapes. The orientation level is the phase in which the basic structural properties of the geometric shapes begin to be noticed.
- Netting: At the netting level, students discuss and determine their own views based on their experience in the previous research and orientation phases. At this stage, the teachers encourage their students to use mathematical language correctly. It also allows students to wonder about the new topic and creates a discussion environment.
- Self-study: At the level of self-study, students try to solve a multi-step problem by using different solutions. At this level, the students find their own solutions and gain experience. They recognize and disclose the different forms and the relationships between them.

- Integration: With the level of integration, the students transfer information between their own studies and the studies that have been done until then. Thus, students absorb the information they have learned by creating a new scheme in their minds. Teachers are asking questions to see what level the student is at, while the students at this level summarize the information in their minds and answer the questions.

The purpose of the study was to determine the association between van Hiele geometric thinking levels and circular achievement exam results for middle school students in 7th grade. The research problem is that what is the level of van Hiele geometric thinking levels and circle achievement of the 7th grade middle school students. The sub research problems are as follows:

- How do the seventh-grade students distribute among the van Hiele geometric thinking levels?
- Is there a significant relationship between the seventh-grade students' circle achievement scores and van Hiele geometric thinking levels scores?
- Is there a statistically significant mean difference between the circle achievement test and van Hiele geometric thinking levels of seventh-grade students among the schools?
- Is there a statistically significant mean difference between van Hiele geometric thinking levels test scores and circle achievement test scores between the seventh-grade boy and girl students?

**Methods**

A survey model, one of the quantitative research models, was used to investigate the relationship of the van Hiele geometric thinking levels and circle achievement levels of seventh-grade students in this study. The survey model is a type of research that describes a situation that has existed in the past or is currently in existence. In this model, the research topic is depicted as it is. No attempt is made to change or be affected (Karasar, 2014, p.77).

**Sample**

In this research, convenient sampling was used. This is the type of sampling where the researcher is easy to reach to the participants, where appropriate and voluntary individuals are selected to participate in the research (Gravetter & Forzano, 2012). This research has been done in the spring semester of the 2017 - 2018 academic year by studying seventh-grade students in three middle schools in Kayseri. A total of 157 (74 girl and 83 boy students) seventh-grade students participated in this study. The class selection was used in School 2 by using simple random sampling and randomly selected and selected tests were applied to selected students.

## Instruments

The circle achievement test that was developed by the first author of the article and van Hiele geometric thinking test were administrated to 7th-grade middle school students for collecting data.

### *Van Hiele Geometry Thinking (VHGT) Test*

The van Hiele geometry thinking test was developed by Usiskin (1982). Duatepe (2000) translated and adopted it into Turkish. It was used to determine the geometry thinking levels of 7th-grade students in the study. VHGT test, which has 25 multiple choice questions, consists of five levels with five questions at each level. The first five questions consists of level 1, the second block of five questions is level 2, the third block of five questions is level 3, the fourth block of five questions is level 4, and the fifth block of five questions is level 5 (Usiskin, 1982). The reliability of the first 15 questions of the VHGT test made using the first three levels of the Cronbach Alpha reliability coefficients were found as 0.82, 0.51, and 0.70, respectively at the Turkish sample (Duatepe, 2000).

The classification of VHGT test levels can vary according to research. In some of the studies, it was classified as 0-IV and in others as I-V. To define students who do not belong to any level as level 0, IV classification is more useful (Senk, 1983). In this study, the levels were classified as 1- V and the students who did not belong to any level were accepted as level 1.

The determination of VHGT levels of students differs according to various criteria. These criteria are those students who answer at least 3 or 4 of the five problems at each level correctly. Which of the criteria is selected depends on the type of error to be controlled? If the VHGT levels they were students in the study to be avoided by leaving the upper level to answer at least four of the five issues involved at all levels right, if one wants to prevent the emergence at a lower level than the level, they found the five problem areas in each level at least three prompted the correct answer to questions (Usiskin, 1982). The researchers decided that it was required to have answered at least three of the five problems in each level correctly. As the levels are hierarchically advanced, the students are not able to pass to a higher level if at least three questions of a level are not answered correctly.

As the research was applied to 7th-grade middle school students, the VHGT levels of them were measured using the first 15 questions which were appropriate to the VHGT test. The features of the questions in the VHGT test are given in Table 1.

Table 1

*Features of the questions in the VHGT Levels (Altun, 2018, p.163)*

| Levels | Range of Questions | Features of questions at each level |
|---|---|---|
| Level 1 | 1-5 | Questions about visual shapes determine whether the students recognize the shape from the appearance of the shape. |
| Level 2 | 6-10 | Determines whether students who recognize visual shapes know the properties of shapes. |
| Level 3 | 11-15 | Determines whether students can notice the relationship between other shapes and their properties while thinking about the properties of shapes. Students who correctly answer the questions at this level have been able to identify and have knowledge about the axioms. |
| Level 4 | 16-20 | Geometry has questions that can be judged and logical inference. It is determined whether the students have proof and writing levels. |
| Level 5 | 21-25 | Reasons in Euclidean and non-Euclidean geometries. |

***Circle Achievement Test***

A 20-item multiple-choice circle achievement test was developed to measure students' geometry skills over circles by the first researcher. Items were prepared using the objectives of the circle unit which is determined by the Turkish Ministry of National Education from the beginning of the 2018 academic year. Although the objectives of the 7th grades' circle unit were intense, the 6th grades' objectives were included in the test. Objectives of the circle achievement test are as follows:

- Draw a circle and determine its center, radius, and diameter.
- Explain the definition of a circle.
- Determine the ratio of the circumference of a circle to its' diameter as a fixed value.
- Calculate the length of a given circle with its' diameter or radius.
- The center angles in the circle determine the relations between the arcs and angle measurements.
- Calculate the length of the circle and circle part.
- Calculate the area of a circle and circle slice.

Since the content of the objectives was different in intensity, each objective was tested with a different number of questions when preparing the circle achievement test. Objectives are not combined, respectively. Three experts and three mathematics teachers were consulted during the preparation of the items. These people stated that the items were about the circle.

## Data Collection Process

In the study, the circle achievement test which was developed by the first researcher and the VHGT test were administered to the 7th-grade middle school students. Ethical permissions were obtained to administer these tests at the schools. The tests were carried out in different class hours of the same class during the same day. At the end of the one class hour, 45 minutes, tests were collected.

## Analysis of Data

The circle achievement test and VHGT test were coded as 1 for the right and 0 for the wrong and empty answers for the analysis. SPSS was used for analysis. In the descriptive analysis, mean, standard deviation, percentage, and frequency tables were inspected.

To find out students' level which takes place in the VHGT levels, three of the five questions at the levels were required to answer correctly. As they progress in the levels is hierarchical, the student must have passed the previous levels successfully to belong to a level. Students who do not belong to any level are accepted as level 0. After the VHGT levels of the students were determined, the data were analysed by using SPSS. Percentage and frequency tables are used to determine which geometric thinking level students have. To determine whether the distribution of the VHGT test and the circle achievement test is normal, the skewness coefficient was calculated by dividing its' standard error. The circle achievement test has a coefficient of skewness of 0.113 and a standard error of 0.194. The standardized value is 0.113 / 0.194 = 0.058. VHGT test has a coefficient of skewness of 0.297 and a standard error of 0.194. The standardized value is 0.297 / 0.194=1.53. The obtained values were compared with the values in the z table and it was decided whether they are normal or not. The values which are between +1.96 and -1.96 at a 5% significance level according to the z table show normal distribution (Karasar, 2017). In both tests, the standardized values are in this range, so the distributions in the tests are normal. Normal distribution means that parametric tests can be used. In this study, the Pearson Correlation Test, which is one of the parametric tests, was used to find the statistically significant relationship between two scores of the students. Independent samples *t*-test run to define whether there ais statistically significant mean difference between VHGT test scores and circle achievement test scores of the girl and boy 7th-grade middle school students and one-way analysis of variance (ANOVA) were run to estimate school differences.

**Results**

**Distribution of VHGT Test Levels Across Schools**

The first research problem of the study, “what is the level of VHGT test levels of 7th-grade students?”, is analysed. The distribution of students’ frequencies among VHGT levels is interpreted in Table 2 across schools and sample distributions across VHGT levels.

Table 2
*Frequencies of VHGT Test Levels across Schools*

| | School | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| Levels | f | % | f | % | f | % |
| Level 0 - Not belonging to any level | 3 | 21.4 | 5 | 12.5 | 32 | 31.1 |
| Level 1 - Visualization | 8 | 57.1 | 21 | 52.5 | 52 | 50.5 |
| Level 2 - Analysis | 2 | 14.3 | 9 | 22.5 | 11 | 10.7 |
| Level 3 – Informal Inference | 1 | 7.1 | 5 | 12.5 | 8 | 7.8 |

According to Table 2 at school 1, 21.4% (three students) of the students was at level 0, 57. The 1% (eight students) was at level 1 (visualization), 14.3% (two students) was at level 2 (analysis), 7.1% (one student) was at level 3 (informal inference). The majority of the students was at the visualization level of VHGT levels at School 1. The students at School 1 are below level 3 (informal deduction) which is thought to be the middle school students were mostly below this level. At School 2, 12.5% (five students) of the students was at level 0 (not belonging to any level), 52.5% of the students (21 students) was at level 1 (visualization), 22.5% of the students (nine students) was at level 2 (analysis), 12.5% of the students (five students) was at level 3 (informal inference). Students were at the level of visualization level of VHGT levels. The students at School 2 were below level 3 (informal deduction) which is supposed to be reached the middle school students. 31.1% of the students in School 3 (32 students) was at Level 0, 50.5% of the students (52 students) was at level 1 (visualization), 10.7% of the students (11 students) was at level 2 (analysis), 7.8% of the students was at (eight students) level 3 (informal inference). Most of the students were at the level of visualization level of VHGT levels. The students at School 3 are below the level 2 (Analysis), which is thought to be middle school students. 25.5% of the sample of the students (five students) was at level 0 (not belonging to any level), 52.5% of the students (21 students) was at level 1 (visualization), 22.5% of the students (nine students) was at level 2 (analysis), 12.5% of the students (five students) was at level 3 (informal inference). Students were at the level of visualization level of VHGT levels. The students at School 2 were below level 3 (informal deduction) which is supposed to be reached the middle school students. 31.1% of the students in School 3 (32 students) was at Level 0, 50.5% of the students (52 students) was at level 1 (visualization), 10.7% of the students (11 students) was at level 2 (analysis), 7.8% of the students was at (eight students) level 3 (informal inference). Most of the students were at the level of visualization level of VHGT levels. The students at School 3 are below the level 2 (analysis), which is thought to be middle school students of 157 students in our sample was at level 0, 51.6% of the sample (81 students) was at level 1 (visualization), 14% of the sample (22 students) was at level 2 (analysis) and 8.9% of the sample (14 students) was at level 3 (informal inference). Most of the students were at the visualization level of VHGT levels. The number of 7th-grade middle school students who was at level 3 (informal inference), was only 14. Thus, the 7th-grade middle school students were below the expected level.

**Correlation between the Circle Achievement Test and VHGT Test**

The second research problem, “Is there a significant relationship between the 7th-grade students' VHGT levels and their score from the circle achievement test?” is analysed. The relationship between the circle achievement test and the VHGT test was found by calculating the Pearson correlation coefficient. The correlation coefficient has a low level in the range .00-.30, a moderate level in the range of .30-.70, and a high level of correlation in the range of .70 - 1.00 (Büyüköztürk, 2007). The correlation coefficient between circle achievement test and VHGT test was $r = .573$, $p = .001 < .05$. A correlation coefficient greater than 0 indicates a positive correlation between the circle achievement test and the VHGT test. The relationship between these two tests is moderate and the scores of the circle achievement test would increase as the VHGT test scores increase.

**The difference among Schools in the Circle Achievement Test and VHGT Test**

The third research problem of the study, “Is there a statistically significant mean difference between the schools in terms of VHGT levels and circle achievement test of 7th-grade students?” is analysed. Descriptive statistics of the circle achievement test and the VHGT test among schools (mean, sample across schools, minimum score, and maximum score) are given in Table 3.

Table 3
*Descriptives of the VHGT Test Scores and the Circle Achievement Test Scores across Schools*

| School | Circle Achievement Test | | | | | | VHGT Test | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | N | M | SD | SE | Min | Max | M | SD | SE | Min | Max |
| School 1 | 14 | 10.36 | 4.483 | 1.198 | 5 | 20 | 6 | 2.77 | 0.74 | 2 | 12 |
| School 2 | 40 | 13.48 | 5.223 | 0.826 | 3 | 20 | 7.47 | 2.28 | 0.36 | 1 | 12 |
| School 3 | 103 | 11.72 | 4.702 | 0.463 | 3 | 20 | 6.05 | 2.47 | 0.24 | 0 | 15 |

The mean of the VHGT test scores of the grade 7. middle-grade students was 6, 7.47, and 6.05 in Schools 1, 2, and 3, respectively. The minimum score of VHGT test was zero across schools. The highest number of the right answers of the VHGT test was 15 right questions at School 3. As interpreted in Table 3, the mean of the circle achievement test of the students at School 1 was 10.36, at School 2 was 13.48 and 11.72 at School 3. The minimum number of right answers was three at the circle achievement test. The highest number of right answers and the highest right score of the circle achievement test was 20 in three schools.

Normality assumption is satisfied for both the VHGT test and the circle achievement test. Then, the Levene statistics run to determine homogeneity of variances. Homogeneity of variances assumption was satisfied for circle achievement test scores and the VHGT test scores across schools since the level of significance was .341 ($p$>.05) for the circle achievement test and the level of significance was .888 ($p$>.05) for the VHGT test. ANOVA was run to verify whether there is a statistically significant mean difference across schools in terms of the circle achievement test. The results of ANOVA verified that the mean of the circle achievement test is not statistically significant across schools ($F$ (2, 154) = 2.853, $p$=.061 >.05). On the other hand, ANOVA was analysed that whether there is a statistically significant mean difference across the schools in terms of the VHGT test scores. There was a statistically significant mean difference across schools in terms of the VHGT scores ($F$ (2, 154) = 3.125, $p$=.047 <.05). Scheffe test was used as a post-hoc to determine the origin of differentiation between schools. The results of the Scheffe test are given in Table 4. There was no statistically significant mean difference between School 2 and 1 ($p$ = .578> .05) with the mean difference between the two schools -.279, between School 1 and 3 ($p$ = .886> .05), the mean difference between these two schools with 0.120. A statistically significant mean difference was found between School 2 and 3 on the side of School 2 ($p$ = .047 <.05).

Table 4

*Test of Scheffe for the VHGT Test across Schools*

| Schools | (MD) Mean Difference | SE | Sig. (*p*) |
|---|---|---|---|
| School 1- 2 | -.279 | .266 | .578 |
| School 1- 3 | .120 | .244 | .886 |
| School 2- 3 | .399 | .159 | .047 |

**The VHGT Test and Circle Achievement Test with respect to Gender**

The fourth research problem, “Is there a statistically significant mean difference between 7th-grade middle school boys and girls at the VHGT test and circle achievement test?” is analysed. The distribution of grade 7. middle school boy and girl students in the VHGT test levels are given in Table 5.

Table 5

*VHGT Levels Scores Distribution across Gender*

| Levels of the VHGT | Boys | Girls |
|---|---|---|
| Not belonging to any level – 0 Level | 20 | 20 |
| Visualization – 1 Level | 45 | 36 |
| Analysis – 2 Level | 10 | 12 |
| Informal Inference – 3 Level | 8 | 6 |

When Table 5 is analysed, 20 of the grade 7. girl middle school students were at the not belonging to any level (0 Level). 36 of the 7th-grade girl middle school students were at visualization level (level 1). Then, 12 of the 7th-grade girl middle school students were at analysis (level 2). Then, six of the 7th-grade girl middle school students were at informal inference (level 3). 20 of the 7th-grade boy middle school students were at the not belonging to any level (level 0). 45 of the 7th-grade boy middle school students were at visualization level (level 1). 10 of the 7th-grade boy middle school students were at analysis (level 2). Then, eight of the 7th-grade boy middle school students were at informal inference (level 3). The mean of the circle achievement test of 7th-grade boy middle school students was 10.9 with SD=4.307. The mean circle achievement test scores for girl students was 13.32 with SD=5.185. Independent samples t-test was run to analyse whether there was a statistically significant mean difference between the circle achievement test scores of 7th-grade girl and boy middle school students. There was a statistically significant mean difference between 7th grade boy and girl middle school students in favor of 7th-grade girl middle school students according to analysis results of the circle achievement test scores ($t$ (155) = 3.194, $p$ = .009 <.05). Moreover, independent groups t-test was run to analyse whether there was a statistically significant mean difference between the VHGT levels between 7th-grade boy and girl middle school students. The mean of VHGT test scores of 7th-grade girl middle school students was 6.48 with SD= 2.53, while the mean of VHGT test of 7th-grade boy middle-grade students was 6.34 with SD = 0.27. There was no statistically significant mean difference was found between the VHGT test scores of the 7th-grade boy and girl middle school students ($t$ (155) =.340, $p$ = .993> .05).

## Discussion and Conclusion

The study aimed to analyse the results of the circle achievement test and VHGT test scores of 157 7th grade students at three middle schools at Kayseri in the 2017-2018 academic year. According to the findings of the first research problem of the study, the VHGT levels of students were lower than the expected level. Only 14 (8.9%) of the 7th-grade middle school students have access to level 3 (informal inference), which were middle school students being reached. The remaining students, 81 of the 7th-grade middle school students (51.6%) were at level 1 (visualization) and 22 of the 7th-grade middle school students were at (14 %) was at level 2 (analysis). The majority of them were at level 1 (visualization) of the VHGT test. The remaining 40 students (25.5%) were at level 0, so those students were not at the levels. Students can only distinguish rounded objects with angular objects at Level 0. Kindergarten and elementary school students are expected to be at Level 0 of the VHGT test (NCTM, 2000; Altun, 2005; Karapınar, 2017). It is very difficult for those students at this level to reach the goals of the 7th-grade objectives at the circle unit. In the formation of these results, students may have learned missing or wrong concepts in their previous learning grades.

Karapınar (2017) examined 8th-grade students' geometrical objects in terms of their VHGT level and concluded that 6 (3.8%) of 161 students reached level 3. The 33 (20.5%) of the other students were level 2, 105 (65.2%) were level 1 and 17 (10.5%) were at level 0. In this study, the VHGT level of the students remained below the expected level. The findings obtained in this study were parallel with the findings obtained in this study. Bayrak (2015) studied with $8^{th}$-grade middle school students, Hurma (2011) studied with the 9th-grade high school students, Altun (2018) studied with 11th-grade high school students, Duatepe Paksu (2013) studied with preservice elementary teachers, Bayrak (2015) studied with computer, classroom, and science in-service teachers found that VHGT levels of students and prospective teachers were below the expected level. Moreover, Usiskin (1982) stated that the majority of the high school students remained at levels 1 and 2 and they did not reach the expected level in the United States. The findings obtained in these studies were in parallel with the findings obtained in this study.

Correlation coefficient between the circle achievement test and VHGT test was $r$ = .573 with a significance level $p$ = .001 <.05. A correlation coefficient greater than 0 indicates a positive correlation between the circle achievement test and the VHGT test. The relationship between these two tests is moderate and the score of the achievement test increases as the VHGT test score increases.

The mean of the 7th-grade middle school students' answers to the VHGT test was found as 6.41. When the mean of VHGT test scores of the schools analysed, the mean of school 1 was 6.00, school 2 was 7.47, and school 3 was 6.05. When the ANOVA results were examined in terms of VHGT levels between schools, a statistically significant mean difference was found between School 2 and School 3 in favor of School 2. No statistically significant mean difference was found between other school pairs.

The mean of the 7th graders' responses to the circle achievement test was found to be 12.04. When the schools were analysed in detail, the means of the circle achievement test were 10.36, 13.48, and 11.72, in School 1, School 2, and School 3, respectively. When the ANOVA results were examined in terms of the circle achievement test, no statistically significant mean difference was found between schools. School 2 was the most successful school among them according to the mean VHGT and mean of circle achievement test. These differences can be caused by many reasons such as the environment where the students are located, the economic and cultural level of the families, the structure of the school, and the mathematics teachers.

Lastly, students’ VHGT levels were examined with respect to their gender. 20 (27.02%) of the girls were at not belonging to any level (Level 0), 36 girls (48.64%) were at visualization level (Level 1), 12 girls (16.21%) were at analysis level (Level 2), and six girls (8.1%) were at informal inference level (Level 3). 20 of the boys (24.09%) were at not belonging to any level (Level 0), 45 boys (54.21%) were at visualization level (Level 1), 10 boys (12.04%) were at analysis level (Level 2) and eight boys (9.63%) were at informal inference level (Level 3). The mean of girl students at VHGT test was 6.48, the mean of boy students VHGT test scores was 6.34, and girl and boy students' VHGT test scores mean were close to each other. There was no statistically significant mean difference of the boy and girl students between the VHGT test scores. According to these results, gender was not important in students' geometry achievements. Studies found no statistically significant relation between VHGT levels and gender (Oflaz, 2010; Bal, 2011; Oral, İlhan, & Kınay, 2013; Bayrak, 2015; Çadırlı, 2017; Ersoy, İlhan & Sevgi, 2019). However, there are studies showing that there is a statistically significant relationship between VHGT levels and gender (Duatepe, 2000; Şahin, 2008; Fidan & Türnüklü, 2010; Karapınar, 2017). 7th-grade students’ scores gained from the circle achievement test were analysed according to their gender. The mean difference between boys’ and girls’ circle achievement scores is statistically significant. This mean difference was in favor of girl students ($p = .009 < .05$). The mean of 7th-grade girl students on the circle achievement test was 13.32, the mean of 7th-grade boy students on the circle achievement test was 10.90. According to means, girl 7th-grade students were more successful than boy 7th-grade students.

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