Bulletin of Education and Research
April 2022, Vol. 44, No.1 pp. 85-103

Investigation of Mathematics Achievements of Eighth Grade Students on Transformation Geometry and Van Hiele Geometric Thinking Levels

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Abstract

The study aimed to investigate the mathematics achievement of eighth grade students on geometric transformation and geometric thinking levels. This research was carried out in the fall semester of 2018-2019. The sample was 88 students from two different middle schools in an urban area of Erzincan, Turkey. The survey model was used. The geometric transformation achievement test with 20 questions and the first 15 questions of the van Hiele geometric thinking test, which are suitable for the middle school student level, were administered to students at different times. SPSS was used to conduct analysis for descriptive statistics, standard deviation, percentage, mean and frequency, independent samples t test, and Pearson correlation. Pearson correlation was conducted to determine the relationship between the total scores that the students got from the van Hiele geometric thinking test and geometric transformation achievement test. While there was a significant relationship between the van Hiele geometry test and transformation geometry test, the geometric thinking levels of the students were below the expected level. There was no statistically significant mean difference between the boy and girl students across the schools.

Keywords: Geometry, Geometric transformation, Geometric thinking, eighth grade, van Hiele

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Introduction

Through human history, one of the oldest sciences is mathematics. Humans' curiosity opened the doors of science to them. Humans have been trying to understand, explain, recognize, and dominate the events from the day they existed in the world. It is an evident fact that humans, who use this effort, use mathematics as a tool in development of technology direction (Çağlar & Ersoy, 1997, p. 194). Today, the contribution of technology in the continuous advancement of science, updating existing information, and even finding new information is an undeniable fact. Skills, abilities, and knowledge about science are anticipated from students are in this advancing progress. People, who want to live in a more modern environment, should use such widely used technology effectively in the service industry.

Education has a share in this rapid advancement in technology. If countries do not want to be left behind, teachers need to adjust their needs because we can evaluate technological development as a product of the educational process. Technological developments have made it possible to look at education from different perspectives. Education in today's age, where knowledge is seen as equivalent to power; aims to raise individuals who use, strive to present, classify, and share the information produced. For this reason, individuals, who can keep pace with change, are constantly curious and willing to learn, and those who can determine the needs of the age, will be entitled to live comfortably in the future (Öğüt, Altun, Sulak, & Koçer, 2004). The field of mathematics has also taken its share from new technological developments. Rapid developments in educational technologies have changed mathematics, understanding, and making sense over time.

Generally, the knowledge of process, the knowledge of numbers, and figures taught in mathematics are taught as the knowledge of rules with reasoning skills included. Thus, the definitions made recently that mathematics does not only consist of numbers and rules. In short, mathematics, where students create their own mathematical knowledge to new problem situations, the event has evolved into a working solution (Olkun & Toluk, 2003).

National Council of Teachers of Mathematics pointed out that geometry is the most important sub-discipline of mathematics. Geometry of spatial shapes is a sub-area that includes plane, space, line, point, and their relations with them and the dimensions of geometric shapes such as volume, area, and length (Baykul, 2002). In literature, geometry has been defined as the whole of space and shape studies (National Council of Teachers of Mathematics, 2000).

A geometry integrated into daily life is an important tool in preparing people for life. When we look around, we see that many tools are designed geometrically. The main reason behind this is, besides its aesthetic appearance, it is ergonomic and allows it to perform its task better (Pesen, 2003). Rather than we are grown-ups, these shapes attract the attention of children at a young age, so children start to get acquainted with geometric shapes from a young age. van Hiele, on the other hand, emphasized the necessity of teaching children's geometry from a young age. When we look at the present day, we see that most teachers now teach students more permanently with materials and games.

The use of two materials makes it easy to understand the subject in depth, to recognize the shapes, and to learn the features by embodying abstract mathematics and geometry. Geometry is not only in understanding but only in the shape of the objects, buildings and tools around us. Geometry has relationships with other disciplines as well as mathematics. This is a major factor in improving students' perspectives, solving problems and increasing their interpretation power.

It is an obvious fact that we are not at a sufficient level in teaching mathematics and geometry in Turkey. We know that the lowest net average of students who pass middle school to high school, go from high school to university, and even take national exams belong to mathematics. In addition, we wanted to see the level of geometry and mathematics achievement of Turkey by participating in Trends in International Mathematics and Science Study 1999 and Program for International Student Assessment 2003. When we look at the results, we were far below the average and of course the desired achievement level was not reached. According to the Trends in International Mathematics and Science Study 1999, the topics included in the questions are lines, planes, points, triangles, circles, transformations, polygons, some basic sketches, symmetry, and similarity. The most of Turkish students had difficulties in geometry issues. In addition to the exam held in 1999, there was an 8-point increase in algebra in the Trends in International Mathematics and Science Study 2007. Compared to this, there was a 7-point decrease in geometry.

Considering the international and national exams held in Turkey students experience difficulties mostly in mathematics and geometry. The first aim is to determine the level of Turkish students' level so that education and training can be made qualified. For this reason, this study was included in the mathematics curriculum of 2005, Ersoy and Duatepe (2003) stated that students can understand how important mathematics is in daily life, thanks to the transformation geometry, they can find the mathematics in art. They discussed the relationship between the van Hiele geometry levels of the student status in the transformation geometry. This study aims to measure how students know the terms in transformation geometry and whether they understand its' basic features. In addition, van

Hiele aims to make suggestions for identifying and eliminating learning deficiencies by determining the levels of geometric thinking and the relationship between the transformation geometry achievement levels.

Van Hiele Geometric Thinking Levels

The Van Hiele Geometric Thinking (VHGT) model is the result of PhD studies of Dina Van Hiele Gedolf and Pierre van Hiele at Utrecht University. They investigated the reason of the difficulties they faced while teaching geometry with students in the classroom and how to overcome them (Usiskin, 1982). The model was included in the Russian geometry curriculum at first, while it was included in the curriculum of many states of America, in the 1970s (Usiskin, 1982). This has paved the way for much research along this study is of interest (Olkun & Toluk, 2003).

Van Hiele realized that when he was a mathematics teacher, students had difficulties in learning geometry. Although he tried different teaching methods over time, he saw that the problems experienced recurred (van Hiele, 1986). van Hiele determined that the students had a certain level of geometric thinking because of the activities they did in the classroom. In this model, for the student to achieve the desired goals, (s)he must participate in the determined studies and discover the properties of geometric concepts. VHGT model consists of two parts (Gutierrez, 1992):

- Thinking Levels: It is the way of thinking of the student in geometry. The student progresses through several thinking levels in the learning process. The thing to be aware of is the education given to make this progress possible.
- Stages of Learning: There are several stages in the learning of geometric concepts in this model. Explain how the teacher should follow to make the transition between the stages easy.

The most striking feature of VHGT model is that acquisition of spatial ideas can be explained at five levels. In this study, VHGT levels will be examined as IV, and for students who do not belong to any level, they will be expressed in the pre-recognition period, i.e., level 0, specified by Clements and Battista (1992). The geometric thinking levels of van Hiele are as follows:

- Level 1: Visual Term
- Level 2: Analysis
- Level 3: Experiential Inference or Non-Formal Deduction
- Level 4: Making Conclusions or Formal Deductions
- Level 5: Axiomatic Level

Level 1: Visual Term

According to VHGT model, geometric thinking is the first level of visual thinking. Geometric shapes and objects are perceived for students at visual level (Cathcart, Pothier, & Vance, 2000). Based on their appearance, only they can conclude about geometric shapes. They cannot comment on the features of a shape directed at them. For example, we give a shape to a student at this level and their reactions: "This is a rectangle." or in the "square" comment. Student does not know shape's characteristics and cannot make a comparison between them (Hoffer, 1981). According to Fuys (1988 as cited in Yildiz, 2014), the features found in students at visual level are:

- A shape drawn is known from the simple drawings, whether it has a different posture or in a complex shape, according to the exterior.
- The geometric shape can be created, drawn, or copied.
- Names of the geometric shape can be compared and classified by appearance.
- Make verbal explanations about geometric shape.
- Solve problems that do not specify the features of the figure.
- No generalization can be found about the figure.
- Parts of the shape are recognized but cannot infer according to these parts.
- Its' features cannot think that a shape can be descriptive.
- Find a given shape among other shapes according to its' external appearance.

Geometry teaching should be at this level for a student in this level. The teacher should know the features of this level and accordingly set the classroom environment and select the appropriate material. Since the student is in contact with the first environment, (s)he should choose the material in an integrated way with daily life. Briefly, since the appearance of the shapes is important for the student in this period, activities that will make them stand out should be planned and implemented. Students at this level cover the Grade 1, 2, and 3 objectives of primary school (Altun, 2008).

Level 2: Analysis

The second level of VHGT model is analysis. In this period, the student begins to grasp shapes and geometric objects with their features. Not only one shape comes to mind, but it also expresses the properties of all the shapes in that group. For example, instead of a certain rectangle, it reflects and expresses the properties of all rectangles together (4 sides, angles equal and 90 degrees, opposite sides equal). The students at analysis level can also generalize all the features of a shape to the class in which it is included. For example, they can generalize all the properties of a square to all squares, but they cannot think that the square is a sub-group of rectangles. In short, they cannot establish the relationship between the classes of shapes (Baykul, 2009). Students at this level cover the

Grades 4 and 5 (Altun, 2008). According to Fuys (1988, as cited in Yildiz, 2014), the characteristics found in students at the level of analysis are:

- Shapes are classified according to their characteristics,
- A form can be expressed verbally according to its' characteristics,
- Interprets and uses verbal-symbolic expressions of the rules,
- The two geometric shapes are compared according to their features,
- Solve geometry problems using the known features of the shape,
- According to the specifications of the figure it has formally found in non-generalization,
- Cannot explain the relationships between the shape features,
- Shape properties can be discovered and generalized to the class of shapes after the experiment.

The following activities are available for students at this level:

- Make geometric shape from matchstick,
- Create a shape on the spiked board,
- Rotation and symmetry to include field activities,
- Measure the dimensions of the given geometric shapes,
- Classify shapes using the given properties,
- List features related to shapes,
- Compare geometric shapes,
- Ensure that geometric shapes express similar and different aspects,
- Investigate the expansions of three-dimensional geometric objects,
- Figure attention on the most important items (Olkun & Toluk, 2007).

Level 3: Experiential Inference or Non-Formal Deduction

The student at this level is now the level to which (s)he can establish a bond between the shape classes. Classes the shape using its properties, but it cannot be inferred. A geometric proof is observed, but no proof can be made (van de Walle, 2004). For example, student understands that all the squares are a rectangle and a parallelogram. Students at this level cover Grades 6, 7, and 8 (Altun, 2008). According to Fuys (1988, as cited in Yıldız 2014), the characteristics of students at the level of informal deduction are:

- Determine the minimum features that describe a geometric shape,
- Description and formula can be preferred for the class of the figure,
- Formal do not inference,
- Talk about the stages of a proof,

- Explain and prove a statement,
- Explain the difference between a proposition and the inverse informally,
- Draw conclusions from the information provided and verify the inference using logical relationships,
- Know the class properties of geometric shapes and use these features to define shape classes,
- Understand deductive expression and use in problem solving.

The appropriate activity for students at this level is:

- Use a list of models and features,
- Speech activity about inferences,
- Using features to describe a shape or determining the desired shape within the given shapes,
- Activities to determine the necessary and necessary condition for a shape,
- Establish valid contrasts between polygons,
- Use drawings and models to generalize and give opposite examples,
- Activity of establishing and testing hypothesis including figures (Öztürk, 2012).

Level 4: Making Conclusions or Formal Deductions

Having axcess to the proof is the most important difference at this level. In doing this to prove theorems and axioms are benefiting from predefined (Olkun & Toluk, 2007). The goal of the level on thought is the relationship between the properties that exist in geometric objects. They can investigate the fold and fold of the shape features. They can work abstractly on axiom, theorem, and definition (van de Walle, as cited in Terzi, 2010). Students at this level cover the objectives of the secondary education (Altun, 2008). According to Fuys (1988, as cited in Yildiz 2014), the features found in students with inference levels are:

- Determine the relationship between a theorem and the inverse theorem and prove both theorems,
- Explain and compare different proofs related to a theorem,
- Explain the change that will occur in the theorem in definition or postulate change,
- Understand the need for the defined term and postulate,
- Define the properties of a formal definition or explain the value of the definition,
- Decide under which conditions different theorems can be combined.

Level 5: Axiomatic Level

Students realize the differences among axiomatic systems at axiomatic level. They can interpret and apply the axioms, theorems, definitions of Euclidean in non-Euclidean geometries. Students can see these systems as a workable area (Hoffer, 1981). Geometry has now become a science for students. Students at this level correspond to undergraduate or postgraduate period (Altun, 2008). According to Fuys (1988, as cited in Yildiz, 2014), the features found in individuals at the most advanced level are:

- Produce theorems in different axiomatic orders,
- Compare axiomatic orders among themselves,
- Comprehend the adequacy, independence of an axiom and its’ equivalence to a different axioma.
- Find application area for a mathematical theorem.

This study aims to analyze the mathematical achievements of the eighth grade students in transformation geometry and determine the VHGT levels of students. For this purpose, what is van Hiele's geometric thinking levels of eighth grade students on transformation geometry? The sub research questions are:

- What is the level of eighth grade students of the VHGT levels?
- Is there a significant relationship between VHGT levels and transformation geometry test of the eighth grade students?
- Is there a significant mean difference among schools in terms of VHGT levels and transformation geometry test?
- Is there a statistically significant mean difference between eighth grade boy and girl students VHGT levels and transformation geometry test scores?

Method

In this study, which analyzes eighth grade students' VHGT levels and their knowledge of transformation geometry, a survey model from quantitative research was used.

Sample

The research was conducted by eighth grade students in two middle schools affiliated to Turkey Ministry of National Education (MoNE) in the spring term of the 2018-2019 academic year in Erzincan. Convenient sampling techniques was used to select sample. This sample was the school of the first researcher. There is only one middle school with two subject areas as middle school and religious middle school in this area. This research was carried out with the participation of 88 students in three classes of Middle School A in Erzincan, Turkey with a total of 68 students, and a total of 88 students in one branch of

Middle School B. There were 45 girls and 43 boys in sample. The families of students in these schools vary in social, cultural, and economic terms.

Data Collection Tools

The VHGT test and transformation geometry test were used to collect and analyze data.

Van Hiele Geometry Test

Van Hiele Geometry Test developed by Usiskin (1982) is a suitable test to verify the geometric thinking level of eighth grade students. Duatepe (2000) adapted this test to Turkish and conducted reliability and validity studies. van Hiele test contains 25 questions and there are five questions covering each level. Since the research included eighth grade students, the geometric thinking levels of these students were calculated using the only first 15 problems that included their levels in the van Hiele geometry test.

The levels of the test are classified as 0-IV in most studies and IV in most studies. In this study, we would, therefore, classify it as I-IV. Because the level 0 will be accepted for students who have no level. Classification of IV is more useful for such students, that is, what we consider to be level 0 (Senk, 1989, cited in Karapınar, 2017).

It varies according to the criteria in determining the thinking levels of students in the van Hiele geometry test. Five questions are available at each level, it was assumed that the student who answered at least three according to some studies and four according to some studies correctly belonged to that level. Which of these criteria will be used varies according to the type of error desired to be kept under control? In order to prevent this, students are not asked to answer their geometric thinking levels at the lower level of their level, but they are not asked to answer three of the five questions correctly, and to prevent this, they are asked to answer four of the five questions correctly (Usiskin, 1992; cited in İlhan 2011). In this study, there is a condition that at least three of the five question sections have been answered correctly. It has been adhered to the rule that a student who does not correctly answer three of the five questions at a level due to the levels being hierarchical cannot go to the next level. The properties of the questions in the van Hiele geometry test are given in Table 1.

Table 1
Features of the questions in van Hiele geometry test

Levels	Questions	Features of the questions
Level 1	1-5	Includes visual shapes. This level determines whether the students recognize the shape from the image by looking at the shape.
Level 2	6-10	The features of visual shapes are included, and it allows students to understand whether they know these features.
Level 3	11-15	It allows students to determine whether they can recognize the contexts between shapes. Students who answer the questions at this level also have knowledge about axioms and definitions.
Level 4	16-20	There are questions including geometry-specific reasoning and logical implications. It allows students to understand whether they have a level of proof writing and understanding in these questions.
Level 5	21-25	Euclidean and non-Euclidean geometry have been found to be used to determine students' reasoning skills.

Source: Altun, 2018, p.163

Transformation Geometry Test

A multiple-choice achievement test consisting of 20 questions was used to measure the geometric skills of students on transformation geometry. The questions cover the objectives of the transformation geometry of the Ministry of Education 2018-2019 academic year. The objectives of the transformation geometry in the eighth grade mathematics curriculum are:

- “Draws the images of the point, line segment, and other shapes because of the translation,
- Creates a reflection image of a point, line segment, and other shape,
- Creates the image of polygons resulting from translations and reflections.” (MoNE, 2018, p. 74-75).

Because the contents of the objectives have different densities, the transformation geometry has been tested with a different number of questions for each of the objectives. The objectives are lined up, not sequentially. Most of the questions were asked from level 2 as the objectives were particularly related to Level 2. Due to the absence of questions from the 4th and 5th levels of the objectives, questions containing these levels were not included. For the content validity of the test, one expert and four middle school mathematics teachers were consulted and stated that the people whose opinions were taken in this direction were at a level that can be measured. The Pearson correlation coefficient was analyzed between the van Hiele geometry test and the transformation geometry test calculated as .70 to look at the construct validity of the test. According to the result obtained, the transformation geometry test used is suitable for the aim of the study. Regarding the reliability of the test, the Cronbach Alpha coefficient was calculated as .625. If the Cronbach Alpha coefficient is between .61 and .80, transformation geometry test is reliable (Özdamar, 1997).

Data Collection Process

In the study, transformation geometry test and van Hiele geometry test were administered to students. The transformation geometry test consisting of 20 questions and the van Hiele geometry achievement test consisting of 15 questions were applied at different hours on different days to the eighth grade students in the school. The tests were taken from the students the end of 45 minutes which is a class hour.

Data Analysis

The transformation geometry test and van Hiele geometry test were analyzed with SPSS. Analysis was made by coding as 1 blank for questions that students answered correctly and 0 for questions that were answered incorrectly. The descriptive analyses as mean, frequency, standard deviation and percentage are used.

To investigate whether the van Hiele geometry test and the transformation geometry test show normal distribution, the skewness coefficient was calculated by dividing it by its own standard error. Transformation geometry of the skewness coefficient achievement test .442 and standard error of .257 is the standardized value of .442 / .257 = 1.72 is the van Hiele standard error of geometry test .257 and skewness coefficient .104 is the standardized value of .104 / .257 = .40 was obtained. These results were evaluated by looking at z table, whether it is normally distributed or not. Results between -1.96 and +1.96 at 5% significance level according to the z table values show normal distribution since the standardized results of the tests are in this range, data in these tests are in normal distribution. Parametric tests can be applied for normally distributed data.

Results

The VHGT Levels

The first research problem of the research was “what is the level of van Hiele geometric thinking levels?”. In Table 2, the students at the middle school B had the highest visualization level of van Hiele geometric thinking level with 60%. In addition, there are no students with informal inference. Students at Middle School B are unable to reach the informal inference level that middle school students should reach. Students in middle school A to van Hiele levels of geometric thinking level with which they are most clearly seen that visualization level with 58.8%. In addition to Middle School B, there are also a few informal inference students in Middle School A. In this school, many students could not reach the expected level. The highest level of whole students is the visualization level with 59.1%. The least owned level can be said to be the informal inference level with 8%. Only 7 out of 88 students have reached the third level desired to reach middle school students. The students are below the expected level.

Table 2
VHGT levels of Middle Schools A and B

VHGT Levels	Middle Schools A		Middle Schools B	
	f	%	f	%
Level 0 (Not belonging to any level)	2	10	6	8.8
Level 1 (Visualization)	12	60	40	58.8
Level 2 (Analysis)	6	30	15	22
Level 3 (Informal Inference)	0	0	7	10.3

Relation between Transformation Geometry and van Hiele Geometry Test

The second research problem was “Is there a significant relationship between the scores of eighth grade students from the van Hiele geometry test and the transformation geometry test?”. Pearson correlation test is suitable to calculate the relationship between the transformation geometry test and the van Hiele geometry test. The relationship between these two tests was calculated as .70 according to the Pearson correlation. According to the interpretation of the correlation coefficient value, if the result you calculated is between .70 - 1.00, there is a high-level relationship between these two tests (Büyüköztürk, 2007). There is a statistically significant positive relationship between the van Hiele geometry test and transformation geometry test.

Difference among Schools

The third research problem of the research was “Is there a significant mean difference between the levels of van Hiele geometric thinking among schools?”. Firstly, the minimum and maximum numbers and means of the students in two different middle schools of the van Hiele geometry test were estimated, and values are given in Table 2. Students at Middle School B have a mean of van Hiele Geometry Test 5.9 and the students have a minimum of 3 and a maximum of 9. The mean of students at Middle School A is 6.21, and the minimum number of right answers is 2, and the maximum number of right answers is 11. The lowest and highest correct numbers belong to Middle School A.

Table 2
van Hiele Geometry Test Descriptive Statistics of schools

Schools	n	□	SS	SE	Min.	Max.	t	p
Middle School B	20	5.90	1.65	.37	3	9	.589	.558
Middle School A	68	6.21	2.14	.26	2	11		

Levene's test determines whether variances show homogeneous distribution and the values obtained from the analysis. Variances show homogeneous distribution since Levene (1, 86) = 1.86, p = .177 > .05. The independent samples t test was used to see if there was a statistically significant mean difference between schools in terms of van Hiele Geometry Thinking levels. The results obtained from the analysis are given in Table 2. There is no statistically significant mean difference between Middle School A and B in terms of van Hiele Geometry Thinking levels (p = .558 > .05).

The third research problem of the study was "Is there a significant mean difference between schools in terms of the transformation geometry test?". Firstly, the minimum and maximum points and mean of the students in two different middle schools were calculated for the transformation geometry test. Descriptive analysis is given in Table 3. The mean of the transformation geometry test of students at Middle School B is 9.6, and the students have a minimum number of right answers with 5 and a maximum of 16 questions. The mean of the students in Middle School A is 10.3, and the minimum number of right answers is 4, and the maximum number of right answers is 19. The lowest and highest correct numbers belong to Middle School A.

Table 3

Descriptive analysis results of schools' transformation geometry test

School	n	□	SS	SE	Min.	Max	t	p
Middle School B	20	9.6	2.92	0.653	5	16	.771	.443
Middle School A	68	10.3	3.51	0.43	4	19		

To understand whether there is a homogeneous distribution of variance, Levene's the result of the test is applied Levene (1, 86) = .868, p = .868 > .05, and variance appears to show homogeneous distribution. Independent samples t-test was used and the results to see whether there is a statistically significant mean difference in terms of transformation geometry test between schools. According to Table 3, there is no significant mean difference between Middle School A and B in terms of the transformation geometry test (p = .443 > .05).

Gender Differences

The fourth research problem of the study was "Is there a significant mean difference between the van Hiele geometric thinking levels of eighth grade boy and girl students?" Girls and boys of van Hiele distribution results according to geometric thinking levels are given in Table 4. In Table 4, four girl students are in the formal inference level and where 5 students follow the student does not belong to any level with that, with 27 students, we see that the maximum number of students are in the visualization level. We cannot say that the situation is very different for the boy students. The least owned level of the boy students is three levels with both non-level and informal inference, and the highest level of visualization with 25 students.

Table 4
Distribution results of the girl and the boy students according to van Hiele geometric thinking levels

van Hiele Geometric Thinking Levels	Girl f	%	Boy f	%
Level 0 (Not belonging to any level)	5	10.64	3	7.32
Level 1 (Visualization)	27	57.45	25	60.98
Level 2 (Analysis)	11	23.40	10	24.39
Level 3 (Informal Inference)	4	8.51	3	7.32

Table 5 shows whether there is a statistically significant mean difference of van Hiele thinking levels between girls and boys. In Table 5, according to the van Hiele levels, boys are 6.09 for boys and girls are 6.18, while the girl and boy students' geometry test values are close to each other. According to the analysis results, no significant mean difference was found between boy and girl students in terms of the van Hiele levels (p = .846 > .05).

The fourth research problem of the study was "Is there a statistically significant mean difference between the scores of eighth grade girls and boys from the transformation geometry test?". According to Table 5, the mean of the transformation geometry test is 10.07 for girl students and 10.16 for boy students and these two values are close to each other. There is no statistically significant mean difference between boy and girl students in terms of the transformation geometry test (p = .895 > .05).

Table 5
van Hiele geometry and transformation geometry test results of girl and boy students

Test	Gender	N	□	SS	SE	t	p
van Hiele Geometry Test	Girl	45	6.18	2.17	0.32	.194	.846
	Boy	43	6.09	1.91	0.29		
Transformation Geometry Test	Girl	45	10.07	3.41	0.51	-0.133	.895
	Boy	43	10.16	3.39	0.52		

Discussion and Conclusion

The first of the sub research problems of the research, van Hiele geometry thinking level, was below the expected level. In this research, when MoNE (Turkey) objectives are analyzed according to the van Hiele level, 8% of students who should be at the informal inference level are at this level, 9.1% do not belong to any level, 23.9% are at the analysis level, and 59.1% at the level of visualization. Therefore, the most owned level is the visualization level. It seems difficult for eighth grade students at the visualization level to understand the objectives of the middle school geometry. However, it would be quite difficult for eighth grade students at level 0 to achieve their goals in geometry objectives.

In a study conducted with Demir (2019) with seventh grade students on circle achievement, 14 (8.9%) of 157 students, who participated in the study, reached the level of informal inference from van Hiele levels. However, 40 (25.5%) students did not belong to any level. The van Hiele levels remained low compared to other studies. In this study, van Hiele thinking levels were in parallel with the data obtained in the specified study. The levels of thinking were below the desired level in the research with eighth middle school students in Karapınar (2017), in research with seventh grade middle school students in Yiğiter (2019) and Demir (2019), in research with prospective teachers and classroom teachers in Şahin (2008), in research with mathematics pre-service mathematics teachers in İlhan (2011), with pre-service classroom teachers in Duatepe Paksu (2013), and with ninth grade high school students in Hurna (2011).

The Pearson correlation coefficient between the transformation geometry test prepared in line with the objectives of eighth grade and the van Hiele geometry test were found to be .70. A high level of positive correlation was found between the van Hiele geometry test and the transformation geometry test. For this reason, students with high van Hiele level have high results in the transformation geometry test.

The mean of the responses given by the eighth grade students for the van Hiele geometry test from was 6.14. As a result of the detailed examination of the schools, students in Middle School A had a mean of 6.21, and students in Middle School B had a mean of 5.9. No significant mean difference was found between schools. The mean of the responses given by the eighth grade students for the transformation geometry test was 10.1. As a result of the detailed examination of the schools, students in Middle School A had a mean of 10.3, and students in Middle School B had a mean of 9.6. There was no statistically significant differences in means between the schools.

The van Hiele geometry thinking levels of the students were examined according to gender. Among the girls, 5 (11.1%) of the students were at level 0, 25 (55.6%) of the students were at level 1, 11 (24.4%) of the students were at level 2, 4 (8.9%) of the students are at level 3. The boy students were found at the level of 3 (7%) of the students at level 0, 27 (62.8%) of the students at level 1, 10 (23.3%) of the students at level 2, and 3 (7%) of the students at level 3. The van Hiele geometry test achievement mean of the boy students was 6.09, and the mean of the girl students was calculated as 6.18, and the mean of students was found to be 6.14. The boy and girl students' van Hiele geometry test achievement means are close to each other. When compared with boy and girl students, no significant mean difference was found in terms of van Hiele levels ($p = .846 > .05$). Gender did not affect the level of van Hiele's geometric thinking among students. In previous studies, gender was ineffective in van Hiele's geometric thinking levels (Bal, 2011; Oral, İlhan, & Kınay, 2013; Demir, 2019; Çadırlı, 2017; Oflaz, 2010). Contrary to the study, gender has been found in studies that suggest that van Hiele has a thought-provoking relationship with geometric thinking levels (Karapınar, 2017; Fidan & Türnüklü, 2010; Şahin, 2008; Duatepe, 2000). The scores of the students from the transformation geometry test according to gender were analyzed. While the mean of the transformation geometry test was 10.07 for girl students, boy students were 10.16 and these two values were close to each other. According to the findings of analysis, when comparing boy and girl students, there were no thought-provoking differences between them in terms of transformation geometry test ($p = .895 > .05$).

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