Punjab University Journal of Mathematics (2024), 56(11), 684- 699
https://doi.org/10.52280/pujm.2024.56(11)01

Domain Wall's Impact on a Type III Bianchi Universe with f(R, T) Gravity

M. Junaid
Department of Mathematics, Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur India
Department of Mathematics, Anjuman College of Engineering and Technology, Nagpur India
Email: junaidmaths1987@rediffmail.com
N. P. Gaikwad
Department of Mathematics, Dharampeth M.P. Deo Memorial Science College, Nagpur India
Email: np_gaikwad@rediffmail.com

Received: 03 September, 2024 / Accepted: 26 May, 2025 / Published online: 20 June, 2025

Abstract. In this research we employ first model of f(R,T) gravitational theory $f(R,T) = R + 2f(T)$ to explore the type III Bianchi Universe featuring domain wall. It was assumed that the expansion scalar is directly related to the shear scalar to find solution of the field equation. This results the metric potentials connection $Z = kY^n$ and the consideration of two scenarios for the integrating constant D. We observed that the domain wall existed early in the universe's history and eventually vanished. A few of the model's physical parameters are also covered.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Type III Bianchi Universe, Domain wall , f(R,T) gravitational theory.

1. Introduction

Riess et al. [1] analyzed photometric and spectral data from 10 new Type Ia supernovae (SNe Ia) at high-redshift along with previously collected data. By comparing the luminosity distances of these supernovae, they presented observational data indicating that the universe is expanding more quickly, posed a fundamental challenge to the standard model of cosmology based on General Relativity(GR). While GR predictions are accurate under weak gravitational fields (inside the Solar System gravity), but in strong gravitational fields like the early Universe or near singularities such as black holes, they are not accurate. Thus GR fails to probe gravity at all scale [17].Because of above reason there was need to modify GR which help us to understand gravity beyond GR. As an continuation of GR, Harko et al. [7] introduced gravitation theory by replacing scalar curvature R with a function f(R,T) which is dependent on R stand for Ricci scalar and T for trace of matter tensor. The additional term T account for effect of matter on the curvature of space-time, going beyond the minimal coupling assumption in GR. This distinguishes f(R,T) gravity from both GR and f(R) gravity, where matter and geometry are minimally coupled. The significance of f(R,T) gravity lies in its ability to model the accelerated expansion of the universe without requiring a cosmological constant and provide a unified structure for studying both early-time inflation and late time cosmic acceleration.Mule et al. [15] formulated the type-III Bianchi model with pressureless fluid, holographic dark energy in the gravitational f(R,T) theory.To explore the LRS type-I Bianchi spacetime in gravitation f(R, T) theory, Halife [4] incorporated matter of type strange quark and normal into matter of domain wall distributions. In the line of gravitation f(R, T) thoery, Sahoo et al. [16] formulated the type I Bianchi Universe with bulk viscosity and found precise solutions to the Einstein's field equations depending on deceleration parameter.Matter and geometry Non-minimal coupling utilized by Moraes et al. [14] to formulate a cosmological model in the gravitational f(R, T) theory. They also address application of energy conditions and the model's cosmological viability, showing that the deceleration parameter aligns with observational constraints and predicts a shift in the universe's expansion from deceleration to acceleration. Zubair et al.[23] found the solution of type I and V Bianchi in f(R,T) gravity with time dependent deceleration parameter. Zubair et al. [22] discussed the dynamics of type I and III Bianchi in f(R,T) gravity.

Prior to the Big Bang the Universe had extremely high temperatures, after that it started to cool speedily due to that the phase transition occurred and symmetry of Universe instinctively broken and as a result the topological defects formed called as domain wall. Domain wall carry energy momentum that can alter the space-time structure, potentially seeding inhomogeneties or imitating dark energy due to their negative pressure. Since domain walls naturally break isotropy, studying them in anisotropic backgrounds such as Bianchi type models provides a more realistic representation of early Universe dynamics. Until now lots of work has been done on domain walls. According to Hatkar et al. [8] analysis of the type-VI Bianchi Universe along domain walls in context of modified f(R,T) gravitation, domain wall's negative pressure may cause them to function as dark energy. A study of magnetized domain walls in general relativity is conducted by Adhav et al. [2]. The gravitational impact inside and outside of a domain wall under the f(R) gravitation theory examined by Natsuki [20] and found that domain wall may cause topological inflation. Hatker et al.[8] studied Bianchi type VI Universe in f(R,T) gravity and discussed the effect of domain wall. Given that domain walls inherently induce anisotropy, their effects are best studied in the context of anisotropic cosmological models. Bianchi type Universe which generalize the isotropic Fridmann Lemaitre Robertson Walker (FLRW) models,provide an appropriate geometric framwork. Specifically, Bianchi type III model allows for spatial homogeneity along with directional anisotrpy and has been applied in numerous studies to investigate early Universe dynamics, structure formation and deviations from cosmic isotropy. Akarsu and Battal [3] examined the type-III Bianchi model with dark energy and It was observed that the universe can evolve toward isotropy monotonically, even when an anisotropic fluid is present. Type III Bianchi model along with bulk viscosity, magnetic field was examined by Wang [19].

By incorporating a domain wall configuration into the Bianchi Type III cosmological model under the f(R,T) framework, we aim to explore how such domain influence the anisotropic expansion dynamics of the early universe, and whether they contribute to mechanisms resembling dark energy behavior. This is how this paper is put together: The mathematical derivation of gravitational f(R,T) theory is given in section 2, Type III Bianchi Universe's field equation in the presence of a domain wall is solved in section 3, the field equation for the cases $D = 0, n \neq 0$ and $D \neq 0, n = 0$ is solved in sections 4 and 5, and a conclusion is presented in section 6.

2. Mathematical Formulation of Gravitational f(R,T) Theory

The source action principle of mathematical formulation of gravitational f(R, T) theory is expressed as follows

$$S = \int \sqrt{-g} \left[\frac{1}{\kappa} f(R,T) + L_m \right] \, d^4y \tag{2. 1}$$

where $\kappa = 16\pi G$ and L_m reprsents density of matter.
To make different type of cosmological model, Harko et al. present three different model of gravitational f(R,T) theory as given below:

$$f(R,T) = \begin{cases} R + 2f(T) \\ f(R) + g(T) \\ f(R) + h(R)g(T) \end{cases}$$

Among above three model we utilized first model:

$$f(R,T) = R + 2f(T) \tag{2. 2}$$

This minimal coupling is among the most studied in the literature due to its simplicity and tractability. It introduces a non-minimal interaction between matter and curvature, which has been proposed to account for late-time cosmic acceleration, where f(T) represents the trace of stress-energy matter's tensor and $f(T) = \alpha T$ is our choice, with α serving as a constant. Numerous author used this linear relation such as Zubair et al.[23], Hatker et al.[8], Zubair et al. [22] and Sahoo et al. [16] to study the dynamics of early Universe.
We obtain the following field equation by differentiating action (S) with respective to g_{ij} and from equation (2.2).

$$G_{ij} = 8\pi T_{ij} + 2f^{'}(T) + [2f^{'}(T)p + f(T)]g_{ij} \tag{2. 3}$$

The energy-momentum tensor representing domain walls is supported by early universe phase transition theories. Domain walls could act as topological defects and influence the evolution of anisotropic backgrounds. Since domain walls naturally break isotropy, studying them in anisotropic backgrounds such as Bianchi type models provides a more realistic representation of early Universe dynamics. T_{ij} represents the tensor for energy momentum of domain wall which expressed as follows:

$$T_{ij} = \rho(g_{ij} + u_i u^i) + p u_i u^i \tag{2. 4}$$

Where ρ is energy density and p stands for pressure of domain wall. Additionally $\rho = \rho_b + \sigma_d$, $p = p_b - \sigma_d$ where p_b and ρ_b denote the pressure and energy density of barotropic fluid, while σ_d represents the tension of domain wall. A unit space vector u_i in the same direction has the property $u_i\ u^i$ = -1.

3. Mathematical Derivation of Field Equations

In the standpoint of GR, the type-III Bianchi cosmological model reveals a homogeneous but anisotropic cosmos as one of the solutions to Einstein's field equations. Anisotropies may have been a significant factor in the early universe's dynamics, but Bianchi type-III cosmological models are especially interesting for understanding the dynamics of the early cosmos. These models also provide a useful framework for exploring potential deviations from isotropy in the cosmic microwave background radiation, thereby offering insights into the universe's initial conditions and large-scale structure. The type-III Bianchi spatially homogenous and anisotropic Universe's line elements are provided by:

$$ds^2 = dt^2 - X^2 dx^2 - Y^2 e^{-2ax} dy^2 - Z^2 dz^2 \tag{3. 5}$$

The functions X, Y, Z are the functions of t and the constant a is a non-zero.
The following field equations are obtained by using the equation (2.3), (2.4) and (3.5).

$$\frac{Y_{44}}{Y} + \frac{Z_{44}}{Z} + \frac{Y_4 Z_4}{YZ} = -(8\pi + 5\alpha)\rho - \alpha p \tag{3. 6}$$

$$\frac{X_{44}}{X} + \frac{Z_{44}}{Z} + \frac{X_4 Z_4}{XZ} = -(8\pi + 5\alpha)\rho - \alpha p \tag{3. 7}$$

$$\frac{X_{44}}{X} + \frac{Y_{44}}{Y} + \frac{X_4 Y_4}{XY} - \frac{a^2}{X^2} = -(8\pi + 5\alpha)\rho - \alpha p \tag{3. 8}$$

$$\frac{X_4 Y_4}{XY} + \frac{Y_4 Z_4}{YZ} + \frac{X_4 Z_4}{XZ} - \frac{a^2}{X^2} = -(8\pi + \alpha)p - 3\alpha\rho \tag{3. 9}$$

$$\frac{X_4}{X} - \frac{Y_4}{Y} = 0 \tag{3. 10}$$

Here suffix 4 refers to the derivative at cosmic time t.
Solving equation (3.10) yields

$$X = mY \tag{3. 11}$$

where m is the integrating constant.
The mathematical formula of expansion scalar θ is expressed as

$$\theta = 3H = \ 2\frac{Y_4}{Y} + \frac{Z_4}{Z} \tag{3. 12}$$

and for Shear scalar σ is defind as

$$\sigma^2 = \frac{1}{2}\left[\sum_{k=1}^{3} H_k^2 - \frac{1}{3}\theta^2\right] = \frac{1}{\sqrt{3}}\left(\frac{Y_4}{Y} - \frac{Z_4}{Z}\right) \tag{3. 13}$$

We have four field equation [3.6 - 3.9] with five unknown X, Y, Z, ρ and p . So to solve the system of field equation we need one more condition.The physical property "expansion scalar θ is directly related to shear scalar σ", is frequently employed in anisotropic cosmological models to simplify field equations and retain consistency with large-scale homogeneity.e.g. Collins [6], sharif et al.[18], and Koussar et al.[11] etc. applied this property to get the solution of field euations.

Using equations (3.11), (3.12), (3.13) and with the help of physical property that expansion scalar θ is directly related to shear scalar σ , we get following expression.

$$\frac{Z_4}{Z} = n\frac{Y_4}{Y} \tag{3. 14}$$

Where $n = \dfrac{1-2l\sqrt{3}}{1+\sqrt{3}l}$
On solving equation (3.14) we get

$$Z = kY^n \tag{3. 15}$$

where k in integrating constant.
Using the equation (3.6), (3.7), (3.9), (3.10) and (3.15) we get

$$Y_{44} + (n+1)\frac{Y_4^2}{Y} = \frac{-a^2}{m^2 Y} \tag{3. 16}$$

Let $Y_4 = L(Y)$, then we get the reduced form of equation (3.16) as

$$Y_4^2 Y^{2(n+1)} = \frac{-a^2 Y^{2(n+1)}}{(n^2-1)} + D \tag{3. 17}$$

4. CASE-I

For $D = 0$, and $n \neq 0$ on solving equation (3.17) we obtained the values of Y, X and Z

$$Y = \frac{at}{\sqrt{1-n^2}} \tag{4. 18}$$

$$X = \frac{amt}{\sqrt{1-n^2}} \tag{4. 19}$$

$$Z = \frac{ka^n t^n}{(1-n^2)^{\frac{n}{2}}} \tag{4. 20}$$

With the help of equation from (3.6) to (3.9) and from (4.18) to (4.20), we obtained the value of density and pressure.

$$\rho = \frac{-8\pi m^2 n(2n-1) - \alpha(2m^2n^2 + m^2 n + m^2 + n^2 - 1)}{m^2 t^2(64\pi^2 + 48\pi\alpha + 8\alpha^2)} \tag{4. 21}$$

$$p = \frac{(8\pi + 5\alpha)(m^2 + n^2 - 1) - 3\alpha m^2(2n^2 - n)}{m^2 t^2(64\pi^2 + 48\pi\alpha + 8\alpha^2)} \tag{4. 22}$$

Based on Figure 1, the density remained positive with complete evolution of Universe and continue decreasing with cosmic time t. Thus we have feasible cosmological model.Furthermore, Sahoo et al. [16] and Chirde and Shekh [5] also showed similar energy density behavior.
The graph for pressure is given in Figure 2, It begins with a large negative and moves towards zero as t approaches infinity. According to observations, unkown energy (dark energy) which is responsible to accelerated expansion of universe is because of negative pressure. Moraes and Sahoo [14], Saho et al. [16], Koussour et al. [13] and Koussour et al. [12] showed the similar behavior of the pressure.

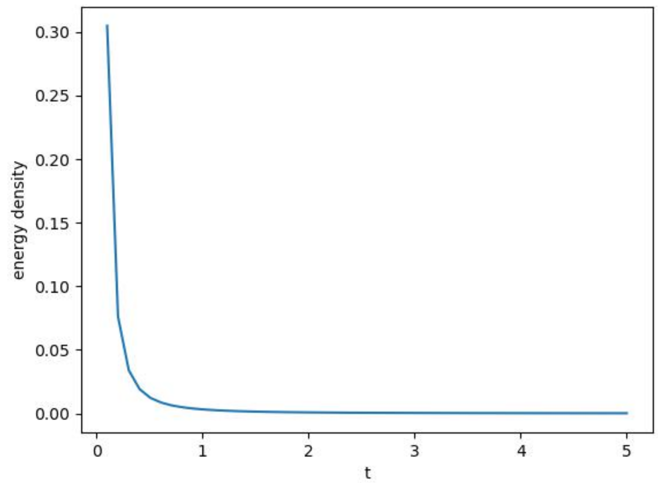

FIGURE 1. Graph of ρ (density) versus cosmic time t for $m = 0.8, \alpha = n = 0.1$

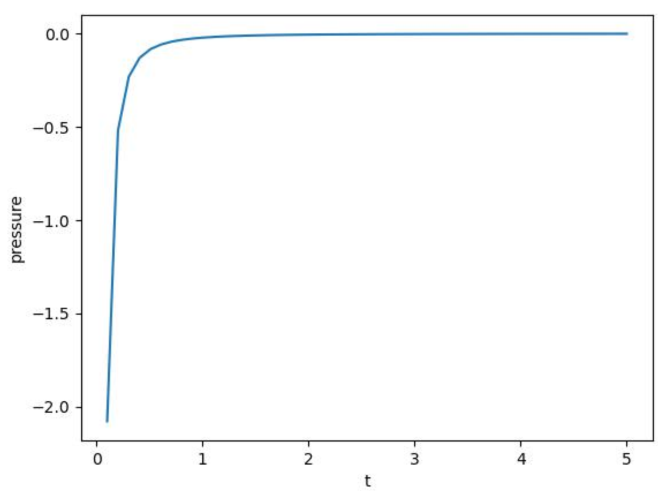

FIGURE 2. Graph of p (Pressure) versus cosmic time t for $m = 0.8, \alpha = n = 0.1$

The relation between pressure and energy density of barotropic fluid is defined as $p_b =$

$(\gamma-1)\rho_b$, where $1 \leq \gamma \leq 2$. The expression for tension of domain wall is obtained as:

$$\sigma_d = \frac{8\pi m^2 n(1-\gamma)(2n-1) - (8\pi+5\alpha)(m^2+n^2-1) - \gamma\alpha s + \alpha(8m^2n^2 - 2m^2n + m^2 + n^2 - 1)}{\gamma m^2 t^2(64\pi^2 + 48\pi\alpha + 8\alpha^2)} \tag{4. 23}$$

where $s = 2m^2n^2 + m^2n + m^2 + n^2 - 1$

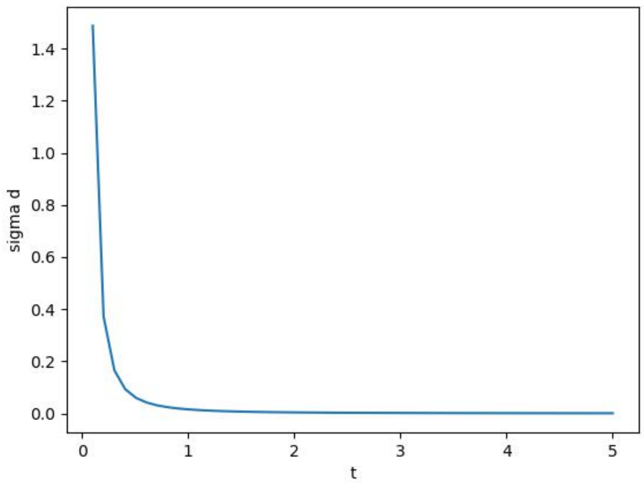

FIGURE 3. Tension of domain wall σ_d plot against time t for $m = 0.8, \alpha = n = 0.1$

From the Figure 3 it seems that the tension of domain wall is very high at the initial stage of the Universe and it continues decreasing to zero along the time.This shows that the domain wall would have been appeared in early stage of the Universe, later it disappeared and energy density of domain wall transformed into another form of energy. The same effect of domain wall is confirmed by Zeldovich [21]. Furthermore, Hatkar et al. [8] and Katore et al. [10] investigated the Bianch type VIII and observed the same effect of tension of domain wall.

By solving equation (3.12) and (3.13) with equation (4.18) and (4.20) we obtained final mathematical form of expansion scalar θ:

$$\theta = \frac{2+n}{t} \tag{4. 24}$$

and for shear scalar σ is

$$\sigma = \frac{1-n}{\sqrt{3}t} \tag{4. 25}$$

The deceleration parameter q which is dependent on time has the form

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{4. 26}$$

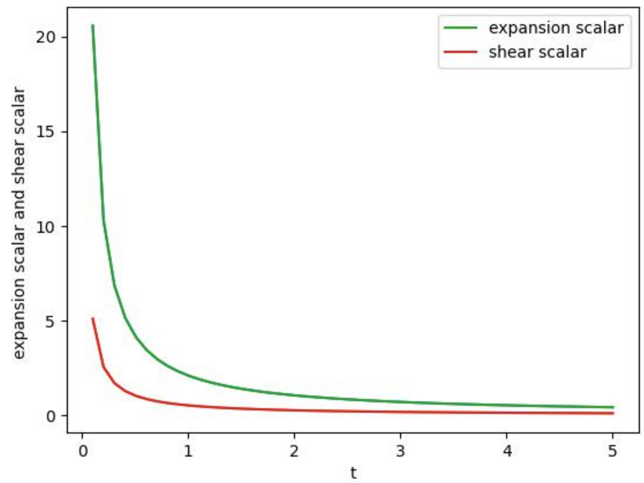

FIGURE 4. Graph of θ (expansion scalar) and σ (shear scalar) versus cosmic time t for $n = 0.1$

On solving equation (4.26) with equation (4.18), (4.19), and (4.20), we get the mathematical expression of deceleration parameter is

$$q = \frac{3}{n+2} - 1 \tag{4. 27}$$

Figure 4 shows the nature of θ (expansion scalar) and σ (shear scalar), they are non-negative function of cosmic time t that decreases over time which means universe is expanding at a relatively rapid rate at this early point in its evolution. Equation (4.24) and (4.25) shows that $\frac{\sigma}{\theta} \neq 0$ and not affected by time t, therefore the Universe is anisotropic. Hatkar et al. [10] showed the same nature of θ (expansion scalar) and σ (shear scalar).
The nature of deceleration parameter shows the accelerating or decelerating behavior of the Universe, if q less than zero, Universe is accelerating and if q greater than zero, Universe is decelerating. Figure 5 shows that $q < 0$ for $n > 1$ and hence the Unverse is accelerating whereas $q > 0$ for $n < 1$, therefore eapansion of Universe is deccelerating. Volume of Universe if obtained as follows:

$$V = \frac{kma^{n+2}t^{n+2}}{(1-n^2)^{\frac{n}{2}+1}} \tag{4. 28}$$

From Figure 6 we can say that volume increases along the time, it mean's that the Universe initialy start from zero volume and expanded with time. The same effect of volume is observed by Hatkar et al. [8] and Chirde and Shekh [5].

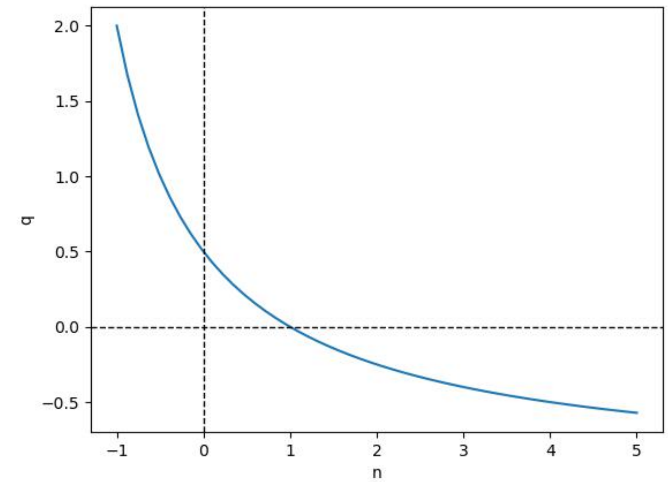

FIGURE 5. Graph of q (deceleration parameter) versus n

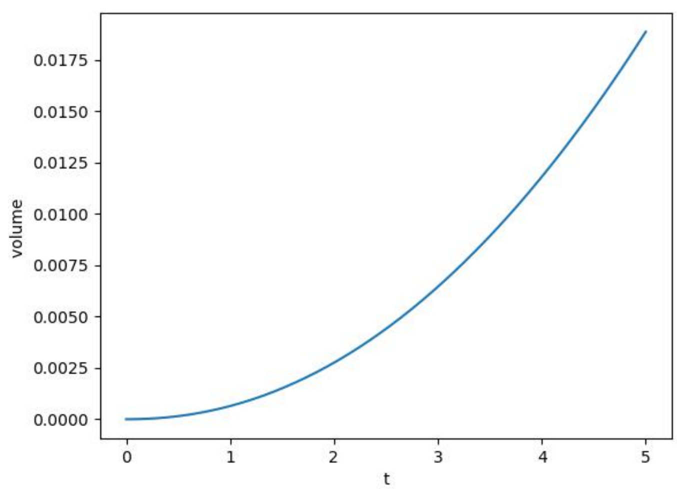

FIGURE 6. Volume V plot against cosmic time t

5. CASE 2

For $D \neq 0$, and $n = 0$ on solving equation (3.17) we obtained the values of Y, X and Z

$$Y = \frac{\sqrt{a^4 t^2 - m^4 D}}{ma} \tag{5. 29}$$

$$X = \frac{\sqrt{a^4t^2 - m^4D}}{a} \tag{5. 30}$$

$$Z = k \tag{5. 31}$$

Using equation from (3.6) to (3.9) and from (5.29) to (5.31) we obtained energy density and pressure as given below

$$\rho = \frac{-a^4m^4D[(8\pi+\alpha)(-2a^4t^2+2Dm^4+1)+\alpha]}{(a^4t^2-Dm^4)^2(64\pi^2+48\pi\alpha+8\alpha^2)} \tag{5. 32}$$

$$p = \frac{-a^4m^4D[3\alpha(-2a^4t^2+2Dm^4+1)-8\pi-5\alpha]}{(a^4t^2-Dm^4)^2(64\pi^2+48\pi\alpha+8\alpha^2)} \tag{5. 33}$$

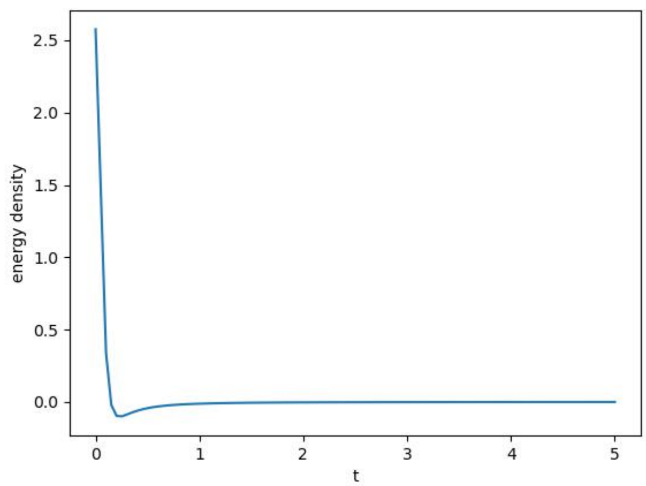

FIGURE 7. Graph of ρ (Energy density) against time t for $\alpha = 0.1$ $m = 0.8$

It is evident from Figure 7, energy density ρ consistently displays a positive, decreasing trend over cosmic time t. Figure 8 indicates that the pressure is begins with large negative and then tends to zero with inceasing cosmic time t.

$$\rho_b = \frac{-a^4m^4D(-2a^4t^2+2Dm^4)(8\pi+4\alpha)}{\gamma(a^4t^2-Dm^4)^2(64\pi^2+48\pi\alpha+8\alpha^2)} \tag{5. 34}$$

$$\sigma_d = \frac{-a^4m^4D[\gamma(8\pi+\alpha)(-2a^4t^2+2Dm^4+1)+\alpha\gamma-(-2a^4t^2+2Dm^4)(8\pi+4\alpha)]}{\gamma(a^4t^2-Dm^4)^2(64\pi^2+48\pi\alpha+8\alpha^2)} \tag{5. 35}$$

Figure 9 demonstrates that the domain wall's tension displays a positive, decreasing trend over cosmic time t, indicating that domain wall existed initially and thereafter vanished. With the help of equation (3.12), (3.13) and (4.26) we obtained final mathematical form of

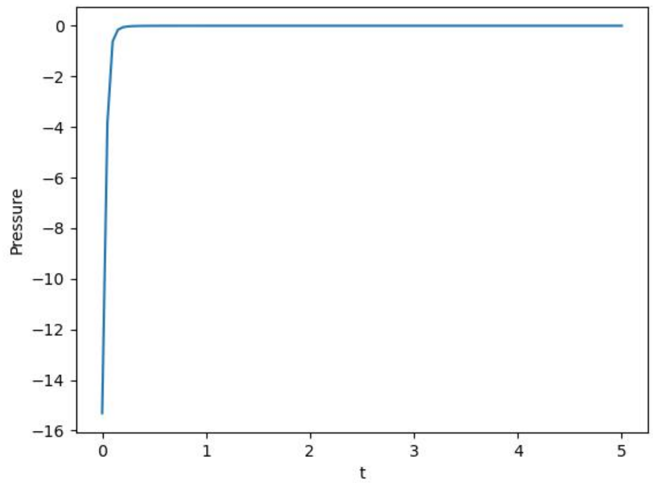

FIGURE 8. Graph of p (Pressure) versus time t for $\alpha = 0.1$ $m = 0.8$

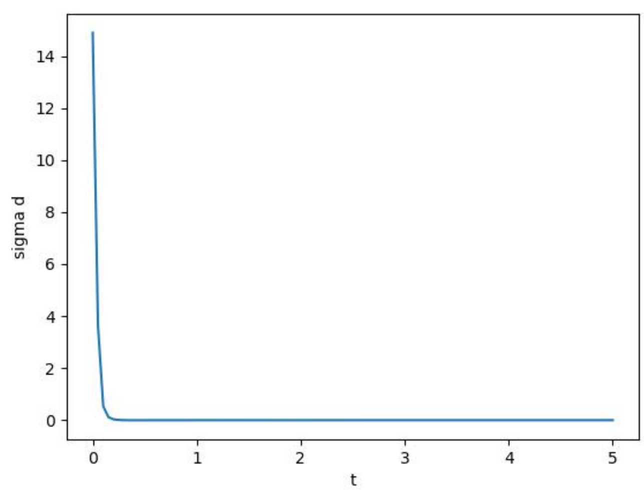

FIGURE 9. Domain wall tension σ_d Plot against time t for $\alpha = 0.1$ $m = 0.8$ $\gamma = 1.5$

θ (expansion scalar)

$$\theta = \frac{2a^4t}{a^4t^2 - Dm^4} \tag{5. 36}$$

q (deceleration parameter)

$$q = \frac{3Dm^4(a^4t^2 - Dm^4)}{2a^4t^2} + \frac{1}{2} \tag{5. 37}$$

and σ (shear scalar)

$$\sigma = \frac{a^4t}{\sqrt{3}(a^4t^2 - Dm^4)} \tag{5. 38}$$

Figure 10 demonstrates that the expansion of universe is decelerating since the deceleration

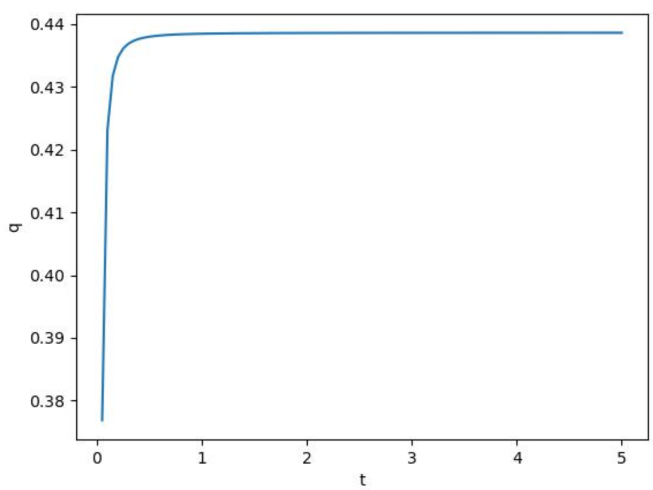

FIGURE 10. Plot of deceleration parameter q against time t for $m = 0.8$

parameter displyas a non-negative value during whole development of universe.
it is discovered from Figure 10, the θ (expansion scalar) and σ (shear scalar) displays non-increasing dependent function of cosmic time t after some time of big bang.It is also observed that $\frac{\sigma}{\theta} \neq 0$ and independent of cosmic time t, that shows the anisotropic nature of Universe. The Universe's volume is obtained as follows:

$$V = \frac{(a^4t^2 - Dm^4)k}{ma^2e^{ax}} \tag{5. 39}$$

Figure 11 illustrates how volume increases with function of cosmic time t, implying that with zero volume universe started and expanded over time.

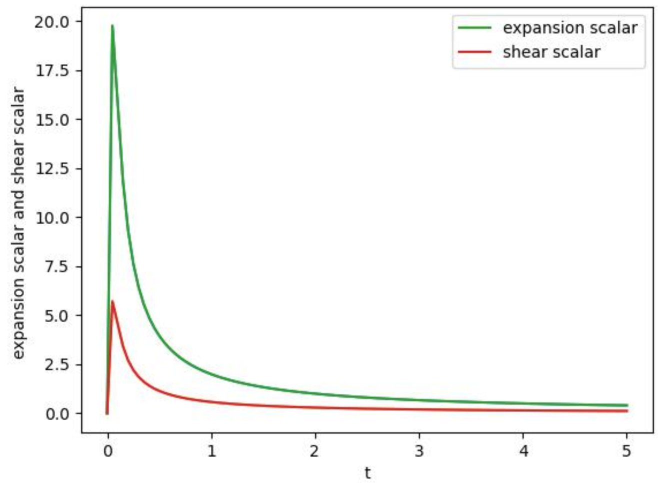

FIGURE 11. Plot of θ (expansion scalar) and σ (shear scalar) against time t for $m = 0.8$

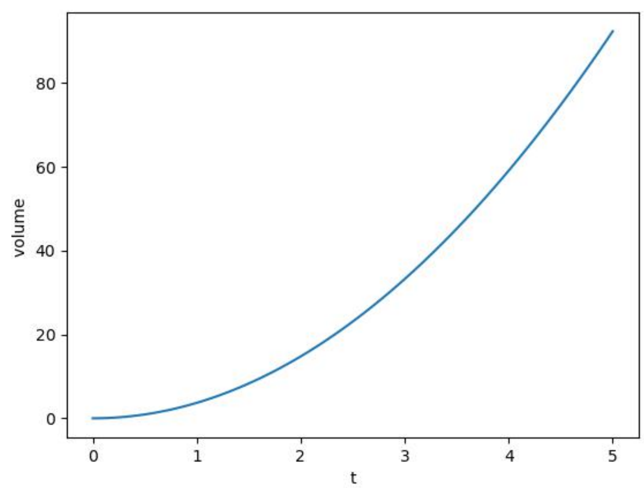

FIGURE 12. volume of Universe plot against cosmic time t for $\alpha = 0.1$ $m = 0.8$

6. CONCLUSION

In this research, we took first model $f(R,T) = R + 2f(T)$ of gravitational f(R,T) theory with a linear matter geometry coupling f(T)=αT to study domain wall in type III

Bianchi Universe, yields physically feasible model, show evidence of anisotropic expansion, decaying domain wall tension and behavior consistent with early Universe cosmological evolution. To get solution to field equation, we take into consideration two cases such as $D = 0, n \neq 0$ and $D \neq 0, n = 0$.

Case 1 : It is observed that with time t, the domain wall's energy density decreases and is positive. As t goes to infinity, the domain wall's pressure p, tends to zero from a big negative starting point. At the beginning of the universe, the domain wall tension is extremely high, then it gradually decreases to zero.This indicates that the domain wall was present during the early stages of the universe., later it disappeared. Deceleration parameter $q < 0$ for $n > 1$, indicating an acceleration of Unverse, whereas $q > 0$ for $n < 1$, indicating a slowing of the Universe. It can be seen from the nature of the expansion and shear scalars that the universe expanded at a very rapid pace in its early stages of evolution before slowing down over time. It is also observed that $\frac{\sigma}{\theta} \neq 0$ and not function of time t, indicating anisotropic nature of the Universe. Volume increases as a function of time t.

Case 2 : Throughout the universe's evolution, the deceleration parameter has been positive, indicating that expansion of universe is slowing down. From value of θ (expansion scalar) and σ (shear scalar) it is observed that $\frac{\sigma}{\theta} \neq 0$ and not dependent on cosmic time t, shows anisotropic Universe. Universe's volume is nonstop increasing with the time.

Comparison with Previous Results To place our results in context, we compared them with the works of Hatkar et al.[8] and Katore et al. [9], who explored domain wall cosmologies in other anisotropic Bianchi types and within the same gravity framework.

Hatkar et al.[8] examined domain walls in a Bianchi Type VI universe using the same linear f(R,T) model. Both their study and ours assume a shear–expansion scalar proportionality, enabling tractable solutions. Like in our model, they found that the domain wall tension starts large and decays over time, disappearing asymptotically—consistent with Zeldovich's [21] predictions. Both studies confirm that negative pressure arises naturally due to domain walls, and such pressure mimics dark energy, driving accelerated expansion. However, in their VI model, the deceleration parameter remains positive in physically viable regimes, whereas our model shows acceleration (q ¡ 0) for appropriate choices of model parameters.Katore [8] also explored Bianchi Type III, but under slightly different assumptions. They adopted a similar f(R,T) model and applied it to both Bianchi Type III and Kantowski–Sachs spacetimes. They found that domain walls appeared in the Bianchi III case while disappeared in Kantowski–Sachs spacetimes. Both their models and ours show that anisotropy persists even at late times, reinforcing the idea that f(R,T) gravity naturally supports anisotropic features in early cosmology.

Our results closely align with both studies in the following aspects:

1.Anisotropy ($\frac{\sigma}{\theta} \neq 0$) persists throughout evolution due to shear scalar dominance. 2.Energy density remains positive and decreases over time. 3.Domain wall tension decays with time (or disappears), aligning with standard cosmological predictions. 4.Negative pressure suggests compatibility with accelerated expansion.

In conclusion, our findings harmonize with the broader landscape of domain wall cosmologies in f(R,T) gravity, contributing novel insights by resolving singularities, capturing decaying wall tension, and offering parameter regimes that allow late-time acceleration—without invoking dark energy explicitly. The consistency across models with different Bianchi types confirms the robustness of f(R,T) frameworks in describing early anisotropic stages of the universe.

Conflict of Interest and Funding: The authors declare that there is no conflict of interest and this research did not receive any specific grant.

REFERENCES

[1] R. Adam G. and Filippenko, Alexei V and Challis, Peter and Clocchiatti, Alejandro and Diercks, Alan and Garnavich, Peter M and Gilliland, Ron L and Hogan, Craig J and Jha, Saurabh and Kirshner, Robert P et al. *Observational Evidence From Supernovae For an Accelerating Universe and a Cosmological Constant.* The astronomical journal **116**, No.3 (1998): 1009-1038.

[2] K.S. Adhav, V.B. Raut, R.S. Thakare and C.B. Kale. *Magnetized Domain Walls Cosmological Model in General Relativity.* International Journal of Theoretical Physics **48**, (2009): 2290-2296.

[3] O. Akarsu and Can Battal Kilinç. *Bianchi Type III Models with Anisotropic Dark Energy.* General Relativity and Gravitation **42**, No.4 (2010): 763-775.

[4] H. Caglar. *LRS Bianchi-I Universe Model with Domain Wall in f (R, T) Gravity.* Physica Scripta, (2024).

[5] V.R. Chirde and S.H. Shekh. *Plane Symmetric Dark Energy Models in the Form of Wet Dark Fluid in f (r, t) Gravity.* Journal of Astrophysics and Astronomy **37**, (2016): 1-16.

[6] C.B. Collins. *Special Exact Solutions of Einstein's Equations a Theorem and Some Observations.* Physics Letters A **60**, No.5 (1977): 397-398.

[7] T. Harko, Francisco SN Lobo, Shin'ichi Nojiri, and Sergei D Odintsov. *f (r, t) Gravity.* Physical Review D. **84**, No.2 (2011): 024020.

[8] S.P. Hatkar, D.P. Tadas, and S.D. Katore. *Domain Wall Bianchi Type VI0 Universe in f(r, t) Gravity.* Astrophysics **67**, (2024): 537-555.

[9] S.D. Katore and S.P. Hatkar. *Bianchi Type III and Kantowski-Sachs Domain Wall Cosmological Models in the f (r, t) Theory of Gravitation.* Progress of Theoretical and Experimental Physics **2016**, No.3 (2016): 033E01.

[10] S.D. Katore, V.R. Chirde, and S.P. Hatkar. *Domain Walls in f (r, t) Theory of Gravitation.* International Journal of Theoretical Physics **54**, (2015): 3654-3664.

[11] M. Koussour, K. El Bourakadi, S.H. Shekh, SKJ Pacif, and M. Bennai. *Late-Time Acceleration in f (q) Gravity: Analysis and Constraints in an Anisotropic Background.* Annals of Physics **445**, (2022): 169092.

[12] M. Koussour, S.H. Shekh, and M. Bennai. *Anisotropic Nature of Space-Time in f(Q) Gravity.* Physics of the Dark Universe **36**, (2022): 101051.

[13] M. Koussour, S.H. Shekh, and M. Bennai. *Cosmic Acceleration and Energy Conditions in Symmetric Teleparallel f (Q) Gravity.* Journal of High Energy Astrophysics **35**, (2022): 43-51.

[14] P.H.R.S. Moraes and P.K. Sahoo. *The Simplest Non-Minimal Matter Geometry Coupling in the f (r, t) Cosmology.* The European Physical Journal C **77**, (2017): 1-8.

[15] K.R. Mule, S..P Gaikwad, and V.G. Mete. *Bianchi Type-iii Holographic Dark Energy Cosmological Model in f (r, t) Theory of Gravitation.* New Astronomy **105**, (2024): 102087.

[16] P.K.Sahoo, Parbati Sahoo, and Binaya Kumar Bishi. *Anisotropic Cosmological Models in f (r, t) Gravity with Variable Deceleration Parameter.* International Journal of Geometric Methods in Modern Physics **14**, No.6 (2017): 1750097.

[17] S. Shankaranarayanan, and Joseph P. Johnson. *Modified Theories of Gravity: Why, How and What?*, General Relativity and Gravitation **54**, No.5 (2022): 44.

[18] M. Sharif and M. Zubair. *Dynamics of Bianchi I Universe with Magnetized Anisotropic Dark Energy.* Astrophysics and Space Science **330**, (2010):399-405.

[19] Wang and Xing-Xiang. *Bianchi Type-III String Cosmological Model with Bulk Viscosity and Magnetic Field.* Chinese Physics Letters **23**, No.7 (2006): 1702.

[20] N. Watanabe. *Domain Wall in f(r) Gravity.* arXiv preprint arXiv:1304.6542. (2013).

[21] Y. B. Zel'dovich, I Yu Kobzarev, and Lev Borisovich Okun. *Cosmological Consequences of Spontaneous Violation of Discrete Symmetry.* Zh. Eksp. Teor. Fiz. **40**, (1974): 3-11.

[22] M. Zubair and Syed M. Ali Hassan. *Dynamics of Bianchi Type I, III and Kantowski-Sachs Solutions in f (r, t) Gravity.* Astrophysics and Space Science **361**, No.4 (2016): 149.

[23] M. Zubair, Syed M. Ali Hassan, and G. Abbas. *Bianchi Type I and V Solutions in f (r, t) Gravity with Time-Dependent Deceleration Parameter.*. Canadian Journal of Physics. **94**, No.12 (2016): 1289-1296.