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Advancements in Fuzzy Research in Pakistan: A Bibliometric Perspective between 1989 and 2023

*Muhammad Saqlain
Department of Mathematics,
King Mongkut's University of Technology Thonburi, 126 Pracha Uthit Rd, Bangmod, Thung Khru, Bangkok 10140, Thailand
School of Computer Science, Faculty of Engineering and Information Technology, University of Technology Sydney, 81 Broadway, Ultimo 2007, NSW, Australia

José M. Merigó
School of Computer Science, Faculty of Engineering and Information Technology, University of Technology Sydney, 81 Broadway, Ultimo 2007, NSW, Australia

Poom Kumam
Department of Mathematics,
King Mongkut's University of Technology Thonburi, 126 Pracha Uthit Rd, Bangmod, Thung Khru, Bangkok 10140, Thailand

Muhammad Riaz
Institute of Mathematics,
University of the Punjab, Lahore 54000, Lahore, Pakistan

*Corresponding Author
muhammad.saql@kmutt.ac.th; Muhammad.Saqlain@uts.edu.au

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Abstract. Pakistan, as a developing nation, has contributed to fuzzy research through its researchers, whose work reflects a growing interest in both the theoretical and applied aspects. However, the extent and impact of these contributions remain unexplored in scholarly evaluations. Thus, this study aims to provide a bibliometric analysis of fuzzy research in Pakistan from 1989–2023, utilizing the Web of Science (WoS) core collection database and *VOS viewer* software. The key bibliometric indicators, including *h*-index, co-citation analysis, bibliographic coupling, and keyword co-occurrence, have been evaluated. The main findings show a significant growth, with total publications increasing from 346 (1989–2014) to 2,609

(2019–2023). Early research focused on foundational concepts like fuzzy sets and fuzzy logic, while recent studies emphasize applied domains such as decision-making, artificial intelligence, and machine learning. Leading contributors include Muhammad Akram (349 publications, 10,224 citations) and Tahir Mahmood (217 publications, 6,508 citations), with the University of the Punjab (PU) and COMSATS University Islamabad (CUI) emerging as the most productive institutions. Collaboration analysis identifies Saudi Arabia as the primary partner, contributing 50% of co-authored works, and Dragan Pamucar, a prominent collaborator from the University of Belgrade, Serbia. The topical and keyword analysis reveals the focus of the research area and its alignment with global trends in fuzzy theories, as well as a significant contribution across diverse disciplines. Subject areas like Business Economics, Optics, Robotics, Food Sciences, Biochemistry, and Social Sciences have fewer contributors and appear less aligned with global research trends. Institutions can encourage strategic publishing to improve global rankings and citation impact.

AMS (MOS) Subject Classification Codes: 62B86, 62A86, 03E72, 01A90
Key Words: Bibliometrics; Web of Science; Scopus; fuzzy; *VOSviewer*.

1. INTRODUCTION

Fuzzy sets have played a crucial role in addressing the challenges of modeling uncertainty and imprecision in real-world problems. Lotfi A. Zadeh contribution to fuzzy sets [64] laid the groundwork for representing vagueness mathematically [65], significantly impacting research and applications in areas like control systems and decision-making [66]. The study of ordered fuzzy numbers [16] and the integration of multivalued logic with probability theory have expanded the theoretical foundation of fuzzy systems [46]. Buckley's research demonstrated the practical application of neural networks within fuzzy systems [10], while [27] highlighted the importance of fuzzy simulations for continuous systems. Recent developments, such as [29] study on monotonic fuzzy systems and [31] work on intelligent factories using fuzzy expert systems, illustrate the growing power of fuzzy logic as a decision-making tool. A comprehensive study by [60] and [42] further emphasizes the versatility and adaptability of fuzzy systems across different fields.

Bibliometric studies have been severely employed in different research fields to identify leading trends, keywords, authors, institutes, and collaborators with rising interest. The concept and definition of bibliometrics were introduced by [9], whereas a broader review of the literature on bibliometrics, scientometrics, and informetrics was proposed by [26]. Topic modelling studies spanning nearly two decades were investigated by [34]. The influence of bibliometric analysis on scholarly production was analysed by [19]. Bibliometric methods were used to assess research in education sciences by [17], and [30] examined bibliometric patterns in tourism research. Over the past decade, the field of soft and neutrosophic set theory has seen remarkable progress, driven by diverse mathematical innovations and real-world applications. Saqlain [51] provides a comprehensive overview of

this evolution, highlighting the scientific growth within Neutrosophic Sets and Systems. Meanwhile, foundational surveys like [1] critically examine soft point definitions, emphasizing their limitations and future challenges. The development of advanced models such as the linguistic fuzzy-valued hypersoft set-based MCDM approach by [52] further illustrates the practical relevance of these theories. Earlier works, including [50], have laid essential groundwork in extended soft set frameworks, supporting the ongoing expansion of this dynamic research area.

Recently, many bibliometric studies have enhanced the fuzzy research and its application in multidisciplinary problems. The bibliometric indicators of fuzzy research were reviewed by [37] and highlighted the significance of the research, while [44] explored its global contribution in the scientific databases. The study of [40] showcases the growth of research outputs and global collaborations with fuzzy logic and AI in financial analysis. Bibliometric analyses on fuzzy decision-making were done by [7] and [32], while [4] performed a citation analysis of journals focused on fuzzy set theory. Trends in fuzzy sets research were examined by [13], while [8] analysed aggregation operators. The bibliometric study of the ordered weighted averaging operator explored by [23], and [33] compared citation analysis across major databases. To understand the growing relevance of hesitant fuzzy sets [11] investigated the area in decision sciences. The applications of fuzzy techniques in big data were examined by [49]. To analyse the fuzzy logic trends and patterns [44] examined and identified the growing role of countries like China and Iran. [12] analysed the fuzzy inference systems and highlighted the development and diverse applications. Rostami [47] visualized the output in fuzzy research, and publications of the Iran University of Medical Sciences from 1980-2020 examined its institutional growth and impact. These studies collectively map the global, interdisciplinary, and evolving nature of fuzzy research.

Recent bibliometric studies offer valuable insights into the academic contributions of Pakistan's researchers across various disciplines. Faiz [21] identified the trends and growth of computer science literature in Pakistan, and [36] in their bibliometric analysis highlighted the contribution of the Institute of Communications and Technology (ICT) in the country's economic growth. Desktop Research (DR) output in Pakistan, as part of the growing field of research globally, by [58]. The contribution of the University of Punjab's scholarly output by doing a bibliometric analysis was analysed by [2], the bibliometric study of library philosophy and practice contributions by Pakistani authors by [48], and the historical development of information science research by [53].

Scientific structure and evaluation of fuzzy research in Spain was highlighted by [35], [63] analysed 30 years of evolution of fuzzy research in China, and an overview of fuzzy systems research in the United States of America and Canada, explored by [38], showcase the evolution of fuzzy research in these countries. The motivation for selecting Pakistan as a case study stems from the noticeable increase in research output from this developing country over the past decades. Yet, despite this growth, there remains a gap in understanding the nature, scope, and global alignment of Pakistan's contributions to fuzzy research.

Addressing this gap is crucial for mapping emerging research landscapes in developing nations and identifying opportunities for strategic academic advancement.

Thus, this study develops a bibliometric analysis of Pakistan's fuzzy research to analyse the productivity, collaboration patterns, thematic focus, and its global visibility and impact. Fuzzy theory offers a powerful framework for modelling uncertainty and imprecision, which is particularly relevant to complex real-world problems encountered in diverse sectors such as decision-making, engineering, and artificial intelligence.

The proposed study is organized as follows: Section 2 presents the methodology employed. Section 3 showcases the results, including the total number of papers, the total number of citations, highly cited authors, key contributors, authors, institutions, and countries. Section 4 presents the visual study of bibliometric indicators through *VOS viewer* software. Finally, section 5 discusses the findings and conclusions of the study.

2. METHODS

Bibliometric analysis [18] is a tool used to evaluate the current trends [55], visibility, and citation impact [59] of a research topic statistically [43] and graphically [24]. It is used to identify influential works, patterns in the research domain, and areas for potential growth. It helps decision-makers, and stakeholders to formulate the policies, guide them for funding acquisitions, and research areas for scientific progress in the future. The quantitative study of the data includes the number of documents, citations, h-index, cites per paper, cites per year (C/Y), and publication frequencies [22]. The h-index [25, 5] measures a scholar's productivity and citation impact to identify the most cited authors, documents, institutes, co-authors, journals, keywords, and collaborative networks to analyse the global standings of the research area. This method was further employed by [5] and extended its use across fields.

Graphically, bibliometric analysis is often used to plot networks of citation, co-authorships, keyword clustering, institutional collaboration, and collaborative countries to show the connections between various indicators and metrics. The visual study enhances understanding by highlighting the interconnected nodes and networks of keywords, journals, authors, and institutes. It also helps to understand the influential authors, research areas, and diversification of the research domain. Databases like Scopus and Web of Science (WoS) core collection provide reliable data for analysing the research patterns, and identifying the trends [14, 20, 15] while *VOSviewer* [57] visualizes the bibliometrics including; co-citation [54] to identify intellectual relationships by shared citations, co-occurrence analysis to identify main research and emerging topics by examining keyword patterns, and bibliographic coupling [28] highlights the link of documents through shared references by plotting the network maps and density nodes.

A bibliometric analysis of fuzzy research in Pakistan was conducted using the Web of Science Core Collection database, chosen for its rigorous indexing standards and relevance for bibliometric research for the period 1989-2023 on the topic "Fuzz*", and 312,128 documents were found. Filtering the results to exclude final year publication for 2024-2025,

early access articles, and by selecting articles and reviews, narrows down the data set to 179,342 documents. Only peer-reviewed journal articles and review papers were included in the analysis to maintain the academic quality and comparability of the data. Document types such as "conference proceedings, book chapters, editorials, and letters" were excluded to avoid inconsistencies in citation metrics and peer-review standards. Finally, from the country filters by selecting "Pakistan," it gives us 3,805 documents. Additional searches for keywords are done in the Scopus database with the same filters, and 3,853 papers were found, placing Pakistan at number 18 globally in fuzzy research in both databases. These publications indicate the growing role of fuzzy researchers from Pakistan, which has significant potential for further impact and collaboration.

The framework and rationale for systematic literature reviews employed in this study, specifically the SPAR-4-SLR methodology [18, 41, 3] are illustrated in Figure 1, providing a comprehensive overview of the methodological approach adopted in this research.

3. Results

In this section, we discuss the bibliometric analysis of fuzzy research in Pakistan. It is divided into three subsections. The first subsection represents the metric analysis of the publication and citation structure of fuzzy research from Pakistan and most publications in research journals. Subsection 2 showcases the most cited documents of Pakistani authors and influential research papers cited by Pakistani researchers. Subsection 3 represents the most productive authors and co-authors, influential institutes from Pakistan, the most productive co-author institutes, and the most productive collaborating countries.

3.1. **Publication and citation structure of fuzzy research in Pakistan.**

The data presented in Table 1 shows the growth and impact of fuzzy research in Pakistan as reflected by TP & TC during the period 1989–2023. The data consists of TP 3,805 with TC 72,223. Over the 34-year period, there is a clear escalation in TP and TC. The first phase (1989–2004) demonstrates an unripe stage of research related to fuzzy with 10 publications and 98 citations within these 15 years. This shows low engagement with fuzzy in the early adoption period. There is a clear increase in fuzzy research from 2005 onwards. From then on, the publications and citations started to grow consistently. We observed that the total citations reached 9,040 by the end of 2014, aided by a gradual increase in moderate publications ($\geq$ 20 citations). This indicates both increased volume of research and higher quality of research. Between 2015 and 2019, a transition period indicated by the numbers TP 850 publications and 24,982 citations.

The highest publications came between 2020-2023 (2,609 publications and 38,201 citations), indicating a clear rise in research output and impact. The years 2019 and 2021 crossed 12,000 citations per year and remain influential in terms of citations. The years 2022 and 2023 stand out especially: in 2023, there were the number of publications (775) and citations (5,493), and the year 2022, 756 publications with 9,223 citations.

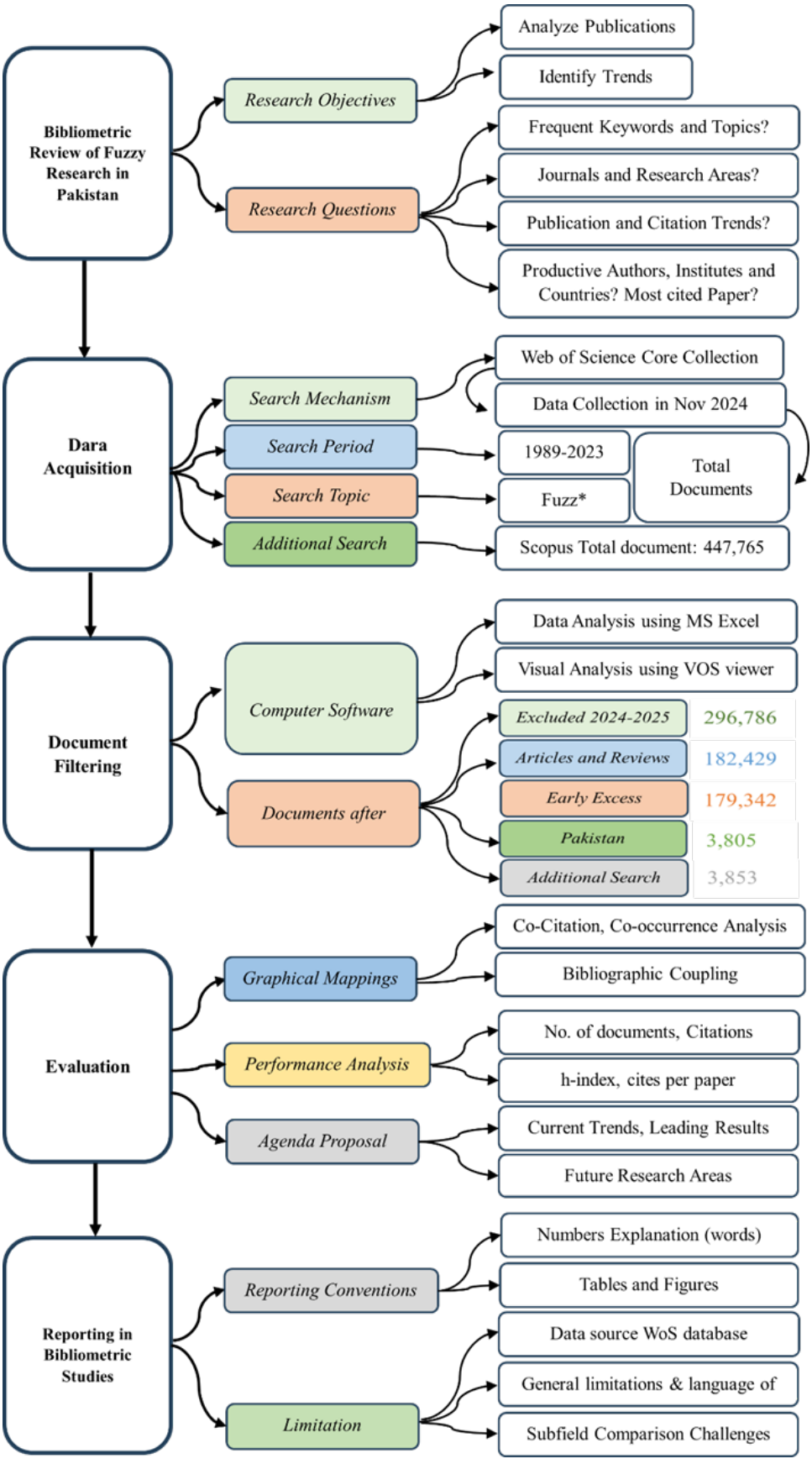

FIGURE 1. Procedure of the study based on the SPAR-4-SLR protocol.

TABLE 1. Annual publications and citation structure of fuzzy research in Pakistan

Year	TP	TC	≥ 200	≥ 100	≥ 50	≥ 20	≥ 10	≥ 5	≥ 1
1989-2004	10	98	0	0	0	1	3	7	9
2005	3	43	0	0	0	1	2	2	3
2006	7	64	0	0	0	1	2	4	7
2007	6	118	0	0	1	1	3	5	6
2008	6	168	0	0	1	4	5	6	6
2009	15	510	0	0	5	11	12	13	14
2010	28	1,058	1	1	3	11	17	23	26
2011	34	1,278	2	4	4	13	22	25	33
2012	61	1,881	0	2	6	16	26	39	52
2013	62	1,873	2	5	9	21	33	40	58
2014	114	1,949	0	1	11	31	49	77	108
2015	62	1,623	1	2	5	25	36	45	57
2016	84	2,311	2	3	11	33	47	62	80
2017	102	3,087	1	9	14	41	72	85	100
2018	195	5,812	0	12	33	90	134	166	190
2019	407	12,149	6	18	66	177	273	339	408
2020	445	10,921	2	16	59	204	321	388	477
2021	633	12,564	0	8	68	241	434	551	673
2022	756	9,223	0	4	29	156	344	548	766
2023	775	5,493	0	1	9	60	165	345	673
1989-2014	346	9,040	5	13	40	111	174	241	322
2015-2019	850	24,982	10	44	129	366	562	697	835
2020-2023	2,609	38,201	2	29	165	661	1,264	1,832	2,589
Total	3,805	72,223	17	86	334	1,138	2,000	2,770	3,746
%	100%	100%	0.45%	2.26%	8.78%	29.91%	52.56%	72.80%	98.45%

Abbreviations: TP and TC = Total papers and citations; ≥ 200, ≥ 100, ≥ 50, ≥ 20, ≥ 10, ≥ 5, ≥ 1 = Number of papers with equal or more than 200, 100, 50, 20, 10, 5, and 1 citation.

The steady increase of citations surpassing the ≥ 10, ≥ 50, and even ≥ 100 for single documents signifies the international acknowledgment of Pakistan's fuzzy research contributions in this time interval. Yet, the marginal drop in total citations recorded in 2023 (5,493) compared to prior years may reflect saturation from research. Data demonstrates that fuzzy research in Pakistan is consistently developing, the rise in total publications and highly cited works over the past 10 years is a sign of higher quality research and international collaborations.

The annual box-and-whisker plot for Table1 is presented in Figure 2. It shows detailed trends in total citations (TC) and total publications (TP) for fuzzy research in Pakistan from 1989 to 2023. This visualization effectively illustrates how the distribution of citations has evolved, reflecting key statistical measures such as the median and the presence of outliers [56]. The graph shows that the productivity of research during the period 1989-2004 was at the initial stage, and the number of published documents was very low as compared to the last decade. The short citation whiskers show that the overall impact of the fuzzy research

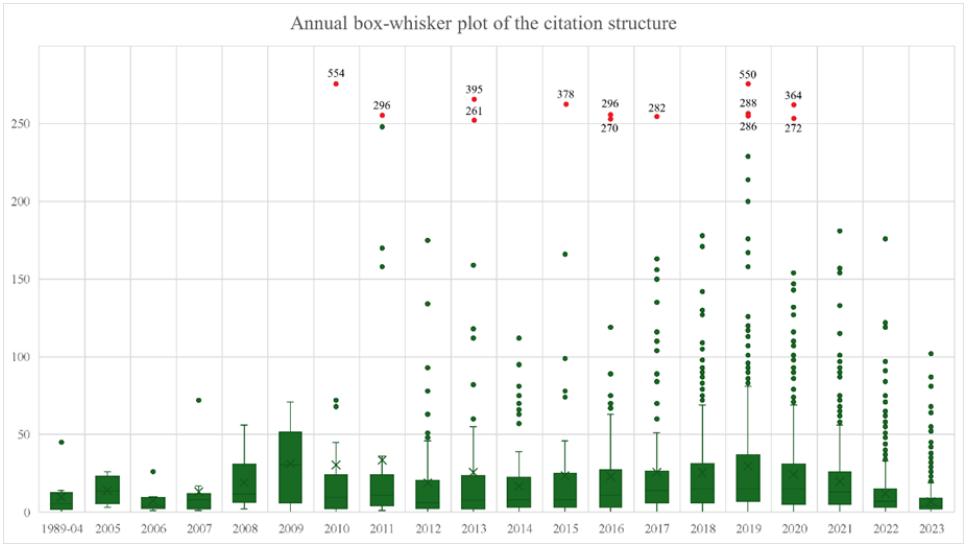

FIGURE 2. Annual box-whisker plot citation structure of the documents published on fuzzy research in Pakistan.

by Pakistani authors was globally low.

The whiskers for the period 2005-2014 show a steady growth in the number of citations of the published papers. The appearance of outliers during 2010, 2011, and 2013 indicates the publications that gained notable citations and global impact.

Longer whiskers during this time indicate increased variability in citation counts, as some studies achieved higher impact than others. Fuzzy research in Pakistan underwent significant growth for 2015-2019 years. The box plots expanded markedly, pointing to a larger interquartile range and greater citation variability. Medians rose significantly, indicating a higher average impact of research publications. The increasing number of outliers suggests the emergence of landmark studies that achieved widespread recognition. This period aligns with sharp growth in both TP and TC, reflecting enhanced maturity and international recognition of the field.

Pakistani researchers have demonstrated significant progress in fuzzy research, with a notable publication in prestigious academic journals presented in Table 2. The *Journal of Intelligent & Fuzzy Systems* leads as the most popular journal, featuring 303 publications, 5,721 citations, and an impressive *h*-index of 39. This journal has become a cornerstone for Pakistani scholars in the field. Other leading journals, such as *IEEE Access* and *Symmetry Basel*, follow with 215 and 160 publications, respectively, showcasing the innovation of fuzzy research. High-impact journals like *Soft Computing* and *Neural Computing Applications* underline the quality of Pakistani contributions, with 30.43 and 44.68 citations per paper, and an *h*-index of 28 for *Neural Computing Applications*, highlighting its significant

impact despite a relatively smaller number of 37 papers.

The period from 2020 to 2023 has witnessed a surge in publications, particularly in journals such as *Symmetry Basel* (160 papers), *AIMS Mathematics* (157 papers), *Mathematics* (125 papers), and *Granular Computing* (76 papers). The selection of journals, ranging from subject-specific to multidisciplinary, has showcased the versatility and adaptability of Pakistani researchers. The impact and the productivity of the research increased with the academic collaboration, funding, and global visibility. However, journals like *CMC Computers, Materials Continua*, and *Axioms* show lower citation rates per paper, indicating areas for improvement in publication quality.

TABLE 2. Publication record of journals in fuzzy research from Pakistan

R	Journal name	JIF	TP	TC	C/P	H	≥ 100	≥ 10	D1	D2	D3
1	J Intelligent Fuzzy Syst	1.7	303	5,721	18.88	39	10	144	29	128	146
2	IEEE Access	3.4	215	3,120	14.51	29	1	114	0	41	174
3	Symmetry Basel	2.2	160	3,214	20.09	33	4	86	0	44	116
4	AIMS Mathematics	1.8	157	1,015	6.46	16	0	32	0	0	157
5	Mathematics	2.3	125	2,132	17.06	23	1	59	0	37	88
6	Soft Computing	3.1	117	3,560	30.43	29	6	70	2	24	91
7	Mathematical Problems Engin	DL	92	995	10.82	17	1	29	4	6	82
8	Granular Computing	0.0	76	1,549	20.38	23	0	52	0	6	70
9	Comput. Applied Mathematics	2.5	73	1,459	19.99	23	2	43	0	15	58
10	J Function Spaces	1.9	62	394	6.35	12	0	14	0	2	60
11	J Mathematics	1.3	58	681	11.74	15	0	25	0	0	58
12	Energies	3.0	52	1,212	23.31	19	1	31	0	6	36
13	Int J Fuzzy Syst	3.6	52	1,817	34.94	24	4	41	4	20	28
14	CMC	2.1	49	347	7.08	10	0	12	0	0	49
15	Sustainability	3.3	45	701	15.58	16	0	25	0	6	39
16	Int J Intelligent Syst	5.0	44	2,240	50.91	26	7	32	2	15	27
17	Neural Computing Applications	4.5	37	1,653	44.68	19	2	28	13	10	15
18	Complexity	1.7	36	330	9.17	11	0	13	0	3	33
19	Applied Soft Computing	7.2	33	1,305	39.55	18	2	23	10	9	14
20	Axioms	1.9	33	233	7.06	9	0	9	0	2	31
21	Complex Intelligent Syst	5.0	31	715	23.06	13	1	21	0	3	28
22	Int J Comput Intelligence Syst	2.5	31	618	19.94	15	0	19	0	5	26
23	Scientific Reports	3.8	31	216	6.97	9	0	8	0	1	30
24	Applied Sciences Basel	2.8	29	355	12.24	12	0	13	0	4	25
25	Punjab University J Mathematics	0.6	29	241	8.31	9	0	9	0	8	21
26	Fractal and Fractional	3.6	28	371	13.25	12	0	17	0	0	28
27	PLOS One	2.9	28	365	13.04	11	0	11	0	5	23
28	Sensors	3.4	28	482	17.21	13	0	17	0	4	24
29	Expert Syst with Applications	7.5	26	765	29.42	16	1	19	4	5	17
30	Electronics	2.6	24	317	13.21	9	0	9	0	5	19

Abbreviations: JIF, TP, TC, C/P and H = Journal impact factor, Total publications, total citations, cites per paper and *h*-index available in WoS; ≥ 100 and ≥ 10 = Number of articles with equal or more than 100 and 10 citations; D1 = 1989-2014; D2 = 2015-2019; D3 = 2020-2023.

High publication volumes in platforms like the *Journal of Intelligent & Fuzzy Systems* and *Symmetry Basel* highlight productivity, yet focusing on fewer, high-impact papers, as seen with *Neural Computing Applications* and *Soft Computing* could elevate academic recognition. Emerging interdisciplinary fields such as AI-integrated fuzzy systems, decision-making under uncertainty, and IoT-based applications are gaining traction, as evidenced by increased publications in journals like *Sustainability* and *Applied Sciences Basel*. Overall, Pakistani researchers have made substantial contributions to fuzzy research, with significant potential to enhance their global standing through impactful publications, strategic international collaborations, and engagement with top-tier journals.

3.2. **Influential papers of fuzzy research in Pakistan.** Table 3 provides an in-depth analysis of the top 50 most influential papers authored by Pakistani researchers, highlighting their total citations (TC), Journals, co-authors, the ratio of citations per year (C/Y), and year of publication. The published work showcases the application of fuzzy set theory to areas like decision-making, renewable energy, environmental sustainability, healthcare, and human resource selection. By providing the solution to real-world problems, Pakistani researchers have expanded fuzzy set theory and its extensions, such as Spherical fuzzy sets, T-Spherical fuzzy sets, Hesitant fuzzy sets, Pythagorean fuzzy sets, q-rung orthopair fuzzy sets, Linear Diophantine sets by (Riaz and Hashmi, 2019), and rough sets. It includes two papers that received ≥ 500 citations, three that received ≥ 300 citations, 12 that received ≥ 200, and 55% received ≥ 100 citations. More than 75% of papers from Table 3 have received C/Y above 20.

The top cited work "Soft Sets Combined with Fuzzy Sets and Rough Sets: A Tentative Approach" by Ali M.I., along with co-authors, published in 2019 in the prestigious journal of *Soft Computing*(SC), with 554 citations, shows the critical impact of the foundation of fuzzy research in Pakistan.

Table 3: The 50 influential and most cited documents of fuzzy research in Pakistan

R	TC	J	Title	Author/s	Year	C/Y
1	554	SC	Soft sets combined with fuzzy sets and rough sets: a tentative approach	Feng, F; Li, CX; Davvaz, B; Ali, MI	2010	36.93
2	550	NCA	An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets	Mahmood, T; Ullah, K; Khan, Q; Jan, N	2019	91.67
3	395	RSER	A review of maximum power point tracking techniques of PV system for uniform insolation and partial shading condition	Ishaque, K; Salam, Z	2013	32.92
4	378	SEJE	Wavelet-based EEG processing for computer-aided seizure detection and epilepsy diagnosis	Faust, O; Acharya, UR; Adeli, H; Adeli, A	2015	37.80

5	364	IF	A smart healthcare monitoring system for heart disease prediction based on ensemble deep learning and feature fusion	Ali, F; El-Sappagh, S; Islam, SMR; Kwak, D; Ali, A; Imran, M; Kwak, KS	2020	72.80
6	296	SC	Numerical solutions of fuzzy differential equations using reproducing kernel Hilbert space method	Abu Arqub, O; AL-Smadi, M; Momani, S; Hayat, T	2016	32.89
7	296	IS	Bipolar fuzzy graphs	Akram, M	2011	21.14
8	288	JIFS	Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems	Riaz, M; Hashmi, MR	2019	48
9	286	JIFS	Spherical fuzzy sets and their applications in multi-attribute decision making problems	Ashraf, S; Abdullah, S; Mahmood, T; Ghani, F; Mahmood, T	2019	47.67
10	282	SC	Application of reproducing kernel algorithm for solving second-order, two-point fuzzy boundary value problems	Abu Arqub, O; Al-Smadi, M; Momani, S; Hayat, T	2017	35.25
11	272	CIS	On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition	Ullah, K; Mahmood, T; Ali, Z; Jan, N	2020	54.4
12	270	RSER	Load forecasting, dynamic pricing and DSM in smart grid: A review	Khan, AR; Mahmood, A; Safdar, A; Khan, ZA; Khan, NA	2016	30
13	261	IJIS	TOPSIS for Hesitant Fuzzy Linguistic Term Sets	Beg, I; Rashid, T	2013	21.75
14	248	ASC	A note on soft sets, rough soft sets and fuzzy soft sets	Ali, MI	2011	17.71
15	229	ENE	Towards a Smarter Battery Management System for Electric Vehicle Applications: A Critical Review of Lithium-Ion Battery State of Charge Estimation	Ali, MU; Zafar, A; Nengroo, SH; Hussain, S; Alvi, MJ; Kim, HJ	2019	38.17
16	214	TFS	Another View on Generalized Intuitionistic Fuzzy Soft Sets and Related Multiattribute Decision Making Methods	Feng, F; Fujita, H; Ali, MI; Yager, RR; Liu, XY	2019	35.67
17	200	JCP	Evaluating the strategies for sustainable energy planning in Pakistan: An integrated SWOT-AHP and Fuzzy-TOPSIS approach	Solangi, YA; Tan, QM; Mirjat, NH; Ali, S	2019	33.33
18	181	KBS	Group decision-making based on complex spherical fuzzy VIKOR approach	Akram, M; Kahraman, C; Zahid, K	2021	45.25

19	178	IJML	Projection models for multiple attribute decision making with picture fuzzy information	Wei, GW; Alsaadi, FE; Hayat, T; Alsaedi, A	2018	25.43
20	176	EF	Recent Advances in Machine Learning Research for Nanofluid-Based Heat Transfer in Renewable Energy System	Sharma, P; Said, Z; Kumar, A; Nizetic, S; Pandey, A; Hoang, AT; Huang, ZH; Afzal, A; Li, CH; Le, AT; Nguyen, XP; Tran, VD	2022	58.67
21	176	IJIS	Spherical aggregation operators and their application in multiattribute group decision-making	Ashraf, S; Abdullah, S	2019	29.33
22	175	FIL	Strong Intuitionistic Fuzzy Graphs	Akram, M; Davvaz, B	2012	13.46
23	171	IJIS	Another view on q-rung orthopair fuzzy sets	Ali, MI	2018	24.43
24	170	NA	On the fractional differential equations with uncertainty	Arshad, S; Lupulescu, V	2011	12.14
25	167	SCM	Barriers to circular food supply chains in China	Farooque, M; Zhang, A; Liu, YP	2019	27.83
26	166	RSER	A comprehensive overview on signal processing and artificial intelligence techniques applications in classification of power quality disturbances	Khokhar, S; Zin, AABM; Mokhtar, ASB; Pesaran, M	2015	16.6
27	163	ASC	On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision-making methods	Zhan, JM; Ali, MI; Mehmood, N	2017	20.38
28	159	BMS	Different Approaches to Multi-Criteria Group Decision Making Problems for Picture Fuzzy Environment	Ashraf, S; Mahmood, T; Abdullah, S; Khan, Q	2019	26.5
29	159	IS	Intuitionistic fuzzy hypergraphs with applications	Akram, M; Dudek, WA	2013	13.25
30	158	IJIS	Group decision-making based on Pythagorean fuzzy TOPSIS method	Akram, M; Dudek, WA; Ilyas, F	2019	26.33
31	158	CMA	Interval-valued fuzzy graphs	Akram, M; Dudek, WA	2011	11.29
32	157	JEM	Predictive modeling of swell-strength of expansive soils using artificial intelligence approaches: ANN, ANFIS and GEP	Jalal, FE; Xu, YF; Iqbal, M; Javed, MF; Jamhiri, B	2021	39.25
33	156	NCA	Complex neutrosophic set	Ali, M; Smarandache, F	2017	19.5

34	154	RSER	Intelligent building control systems for thermal comfort and energy-efficiency: A systematic review of artificial intelligence-assisted techniques	Merabet, GH; Essaaidi, M; Ben Haddou, M; Qolomany, B; Qadir, J; Anan, M; Al-Fuqaha, A; Abid, MR; Benhaddou, D	2021	38.5
35	154	AIHC	Spherical fuzzy Dombi aggregation operators and their application in group decision making problems	Ashraf, S; Abdullah, S; Mahmood, T	2020	30.8
36	147	SC	Multi-criteria group decision-making based on ELECTRE I method in Pythagorean fuzzy information	Akram, M; Ilyas, F; Garg, H	2020	29.4
37	145	CIE	Fuzzy DEMATEL analysis of barriers to Blockchain-based life cycle assessment in China	Farooque, M; Jain, V; Zhang, A; Li, Z	2020	29
38	143	SC	Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making	Ullah, K; Garg, H; Mahmood, T; Jan, N; Ali, Z	2020	28.6
39	143	IJFS	Bipolar Fuzzy Hamacher Aggregation Operators in Multiple Attribute Decision Making	Wei, GW; Alsaadi, FE; Hayat, T; Alsaedi, A	2018	20.43
40	142	IJFS	Decision Making with Uncertainty Using Hesitant Fuzzy Sets	Faizi, S; Rashid, T; Salabun, W; Zafar, S; Watróbski, J	2018	20.29
41	134	PPL	Discriminating Outer Membrane Proteins with Fuzzy K-Nearest Neighbor Algorithms Based on the General Form of Chou's PseAAC	Hayat, M; Khan, A	2012	10.31
42	133	IJHE	Dynamic planning, conversion, and management strategy of different renewable energy sources: A Sustainable Solution for Severe Energy Crises in Emerging Economies	Chien, FS; Kamran, HW; Albashar, G; Iqbal, W	2021	33.25
43	132	MAT	TOPSIS Method Based on Complex Spherical Fuzzy Sets with Bonferroni Mean Operators	Ali, Z; Mahmood, T; Yang, MS	2020	26.4
44	132	MPE	Decision-Making Analysis Based on Fermatean Fuzzy Yager Aggregation Operators with Application in COVID-19 Testing Facility	Garg, H; Shahzadi, G; Akram, M	2020	26.4
45	132	SC	Picture 2-tuple linguistic aggregation operators in multiple attribute decision making	Wei, GW; Alsaadi, FE; Hayat, T; Alsaedi, A	2018	18.86

46	130	SYM	Similarity Measures for T-Spherical Fuzzy Sets with Applications in Pattern Recognition	Ullah, K; Mahmood, T; Jan, N	2018	18.57
47	127	JEPM	Impact of knowledge absorptive capacity on corporate sustainability with mediating role of CSR: analysis from the Asian context	Shahzad, M; Ying, Q; Rehman, SU; Zafar, A; Ding, XA; Abbas, J	2020	21.17
48	127	JIFS	Fuzzy N-soft sets: A novel model with applications	Akram, M; Adeel, A; Alcantud, JCR	2018	18.14
49	126	MRT	Brain tumor detection and classification: A framework of marker-based watershed algorithm and multilevel priority features selection	Khan, MA; Lali, IU; Rehman, A; Ishaq, M; Sharif, M; Saba, T; Zahoor, S; Akram, T	2019	21
50	122	EN	The determinants of renewable energy sources for the fueling of green and sustainable economy	Zhao, J; Patwary, AK; Qayyum, A; Alharthi, M; Bashir, F; Mohsin, M; Hanif, I; Abbas, Q	2022	30.5

Mahmood et al.'s publication "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets" in *Neural Computing & Applications* (NCA), published in 2019, received the highest 91.67 citations per year (C/Y) and stands first among influential papers data list. Ali et al.'s publication "A Smart Healthcare Monitoring System for Heart Disease Prediction Based on Ensemble Deep Learning and Feature Fusion" published in *Information Fusion* (IF) in 2020, received 72.8 C/Y and stands second in terms of C/Y among Pakistani influential papers. There are some more notable papers in terms of C/Y. It includes publications in *Energy and Fuels* (EF) in 2022 by Afzal A., achieving 58.67 C/Y. Another significant paper in Complex & Intelligent Systems (CIS) in 2020 by Ullah et al.'s with 54.4 C/Y, which advanced pattern recognition using complex Pythagorean fuzzy sets. Riaz and Hashmi's 2019 work in the *Journal of Intelligent & Fuzzy Systems* (JIFS), received 48 C/Y, focused on linear Diophantine fuzzy sets in decision-making. Another publication in the same journal by Ashraf et al.'s 2019 study on spherical fuzzy sets, achieved 47.67 C/Y. Akram et al.'s 2021 study in *Knowledge-Based Systems* (KBS) on spherical fuzzy VIKOR received 45.25 C/Y.

Additionally, the research paper in *Energies* (ENE) by Ali et al.'s 2019 review, Salam's 2013 review in *Renewable & Sustainable Energy Reviews* (RSER), in Supply Chain Management (SCM) by Farooque et al.'s in 2019, shows the applicability of fuzzy research in providing the solution to the real-life problems and the acknowledgment of Pakistani researchers globally. Table 3, also shows that the influential papers are published in highly reputed journals including *IEEE Transactions on Fuzzy Systems* (TFS), *Knowledge-Based Systems* (KBS), *Soft Computing* (SC), *Information Fusion* (IF), *Renewable & Sustainable Energy Reviews* (RSER), *Information Sciences* (IS), *Journal of Intelligent & Fuzzy Systems* (JIFS), and *International Journal of Hydrogen Energy*(IJHE) representing the maturity of researchers and the influence of the work. The *Journal of Intelligent & Fuzzy Systems*

(JIFS), *Soft Computing* (SC), and *International Journal of Fuzzy Systems* (IJFS) are the most occurring journals where influential papers have been published by fuzzy researchers of Pakistan.

In Table 4, the top 40 most cited documents in fuzzy research referenced by Pakistani researchers are presented. The key observations are drawn in terms of influential authors, citations, and journals. It shows the diversity of fuzzy research in Pakistan from foundational work to the most recent advancements, extensions, and applications in real-life problems. The data highlight the most influential paper authored by Zadeh (1965), which proposed the concept of Fuzzy Sets (FS), received 2260 citations, and was published in Information & Control. Atanassov's (1986) contribution to intuitionistic fuzzy sets (IFS), published in 1986, received the second highest recognition with 1,032 citations and was published in the *journal of Fuzzy Sets and Systems* and stands second most influential paper. The contribution of Yager (2013) on Pythagorean Fuzzy Subsets (PFSs) published in the *Proceedings of the 2013 Joint IFSA World Congress and NAFIPS Annual Meeting* (IFSA/NAFIPS) received 542 citations. These foundational works emphasize the early theoretical developments that continue to shape fuzzy research globally.

Several other papers, like Soft Set-theory by Molodtsov (1999), cited by 450 articles, and Pythagorean membership grades by Yager (2014) in *IEEE Transactions on Fuzzy Systems* with 444 citations, are considered as foundational papers in computational and fuzzy systems research. These papers highlight the diversification of the field in later years, addressing computational challenges and applied methodologies.

Almost half of the papers have received equal to or more than 200 citations. Notably, Ronald R. Yager and Lotfi A. Zadeh appear multiple times in the top 40 papers, showcasing their reputation as leading researchers in fuzzy systems and acknowledgment by Pakistani researchers. It also shows that the focus of many authors from Pakistan is linked with their foundational papers. Another key point is that many Pakistani authors like Tahir Mahmood, Muhammad Irfan Ali, Kifayat Ullah, Muhammad Riaz, and Muhammad Akram, because they are Pakistani researchers and have received high recognition by the Pakistani fuzzy research community. Data in Table 4 shows that foundational theories in fuzzy and applied research are the focus of Pakistani researchers in this field. The journals that feature prominently in this list are *IEEE Transactions on Fuzzy Systems, International Journal of Intelligent Systems, Computers, and Mathematics with Applications*, and *Fuzzy Sets and Systems*. In term of period, from 1989-2014, it highlights the foundational nature of work. From 2015 onwards, the clustering of most cited papers shows the advancement of fuzzy research and its growing recognition internationally.

The data reveals a shift from the theoretical development of fuzzy research to its applications. The concentration of the earlier research papers of Zadeh and Atanassov was defining fuzzy and intuitionistic fuzzy sets, while later works explore computational and real-world applications of these theories, as seen in Mahmood's and Xu's papers. The repeated presence of the authors in the list shows the contribution of researchers over the decade and continuous scholarly output in fuzzy research. The standing of high-impact

TABLE 4. Top 40 most cited documents of fuzzy research in Pakistani publications

Rank	Year	First Author	Reference	Vol	Page	Type	TC
1	1965	Zadeh LA	Inform Control	v8	p338	A	2,260
2	1986	Atanassov KT	Fuzzy Set Syst	v20	p87	A	1,032
3	2013	Yager RR	Proc IFSA – NAFIPS		p57	C	542
4	1999	Molodtsov D	Comput Math Appl	v37	p19	A	450
5	2014	Yager RR	IEEE T Fuzzy Syst	v22	p958	A	444
6	2017	Yager RR	IEEE T Fuzzy Syst	v25	p1222	A	397
7	2007	Xu ZS	IEEE T Fuzzy Syst	v15	p1179	A	298
8	1975	Zadeh LA	Inform Sciences	v8	p199	A	288
9	2002	Ramot D	IEEE T Fuzzy Syst	v10	p171	A	272
10	2001	Maji PK	J Fuzzy Math	v9	p589	A	267
11	2006	Xu ZS	Int J Gen Syst	v35	p417	A	258
12	1982	Pawlak Z	Int J Comput Inf Sci	v11	p341	A	257
13	2013	Yager RR	Int J Intell Syst	v28	p436	A	252
14	1986	Atanassov KT	Fuzzy Set Syst	v20	p87	A	247
15	2014	Zhang XL	Int J Intell Syst	v29	p1061	A	232
16	2019	Mahmood T	Neural Comput Appl	v31	p7041	A	225
17	2010	Torra V	Int J Intell Syst	v25	p529	A	219
18	2015	Peng XD	Int J Intell Syst	v30	p1133	A	214
19	2003	Maji PK	Comput Math Appl	v45	p555	A	204
20	2012	Alkouri AS	AIP Conf Proc	v1482	p464	C	196
21	2018	Liu PD	Int J Intell Syst	v33	p259	A	194
22	1975	Rosenfeld A	Fuzzy Sets Their App		p77	BC	192
23	1971	Rosenfeld A	J Math Anal Appl	v35	p512	A	184
24	2009	Ali MI	Comput Math Appl	v57	p1547	A	176
25	2020	Ullah K	Complex Intell Syst	v6	p15	A	149
26	2016	Garg H	Int J Intell Syst	v31	p886	A	147
27	1989	Atanassov K	Fuzzy Set Syst	v31	p343	A	143
28	1987	Bhattacharya P	Pattern Recogn Lett	v6	p297	A	143
29	2002	Maji PK	Comput Math Appl	v44	p1077	A	142
30	1971	Zadeh LA	Inform Sciences	v3	p177	A	140
31	2019	Riaz M	J Intell Fuzzy Syst	v37	p5417	A	135
32	2010	Feng F	Soft Comput	v14	p899	A	131
33	2007	Roy AR	J Comput Appl Math	v203	p412	A	126
34	1987	Kaleva O	Fuzzy Set Syst	v24	p301	A	125
35	1994	Wen-Ran Zhang	Proc NAFIPS		p305	C	117
36	2011	Akram M	Inform Sciences	v181	p5548	A	116
37	2013	Cuong BC	3rd WICT World Cong		p1	C	116
38	1990	Dubois D	Int J Gen Syst	v17	p191	A	111
39	2017	Garg H	Arab J Sci Eng	v42	p5275	A	110
40	1975	Kramosil I	Kybernetika	v11	p336	A	110

Abbreviations: TC = Total citations; A = Article; B = Book; BC = Book chapter; C = Conference proceedings.

journals and the citation statistics in the last few years reflect the growth and prominence within the field.

3.3. **Most productive authors, institutions, and countries.** In Table 5, the data showcase the top 30 researchers from Pakistan in Fuzzy research, and it also presents the influence and institutional affiliations. The data shows that only one author has published more than 300 research papers, one more than 200, and two with more than 100 research articles. The rest 70% of the authors have published research papers in the field. The leading researcher from Pakistan is Muhammad Akram from the University of Punjab stands out with an impressive 349 publications, 10,224 citations, and the highest h-index of 51, with 14 papers having ≥ 100 citations and 269 with ≥ 10 citations. The most productive period is D3 with a total of 205 publications. These stats highlight his significant contribution to the field with recognition. Tahir Mahmood from International Islamic University, with 217 publications, 6,508 citations, and an h-index of 41, with 15 papers having citations and 127 with citations. The most productive period is D3 with a total of 175 publications. Saleem Abdullah from Abdul Wali Khan University, with 193 publications, 5,328 citations, and an h-index of 41, 8 papers having 100 citations and 125 with ≥ 10 citations. Muhammad Riaz from the University of Punjab, with 122 publications, 2,622 citations, and an h-index of 29, 1 paper having ≥ 100 citations and 69 with ≥ 10 citations. The most productive period is D3 with a total of 111 publications, which also shows that Riaz's contribution started in the D2 period. Their significant output and impact underscore their pivotal roles in advancing fuzzy research in Pakistan. A notable trend in recent years is the surge in research productivity during the D3 period (2020–2023), and 80% of research is produced during this period.

Emerging researchers like Zeeshan Ali, from the International Islamic University, Muhammad Saeed, Attique-Ur-Rahman, and Umer Ishtiaq, from the University of Management and Technology, and Hafiz Muhammad Athar Farid, from the University of the Punjab have started their fuzzy research career during the period D3 (2020-2023) and are prominent researcher in the area, also received a worldwide recognition. Tasawar Hayat from Quaid I Azam University, with 31 publications, 2,484 citations, and an h-index of 23, has the highest C/P ratio of 80.13, and Muhammad Irfan Ali from Islamabad Model College for Boys authored 55 publications, 2,675 citations, and an h-index of 26 has the second highest C/P ratio of 48.64.

The Institutional affiliations of the authors show that prominent universities like the University of Management and Technology act as fuzzy research hubs in Pakistan, Riphah International University, the University of Punjab, and the International Islamic University have a fostering and thriving environment for fuzzy systems research. Researchers from these universities and their consistent presence in the data highlight the importance of institutional support and infrastructure in nurturing research excellence. Overall, the data reveals a dynamic and evolving research landscape. Senior researchers like Akram have laid a solid foundation for fuzzy systems research in Pakistan, while the rising contributions of younger researchers signal a promising future. The focus on high-impact

TABLE 5. Top 30 most productive authors in fuzzy research in Pakistan

R	Author Name	University	Country	TP	TC	H	C/P	≥ 100	≥ 10	D1	D2	D3
1	Akram M	U Punjab	PAK	349	10,224	51	29.3	14	269	30	114	205
2	Mahmood T	Int Isl U	PAK	217	6,508	41	29.99	15	127	4	38	175
3	Abdullah S	AW Khan U	PAK	193	5,328	41	27.61	8	125	9	69	115
4	Riaz M	U Punjab	PAK	122	2,622	29	21.49	1	69	0	11	111
5	Rashid T	U Mang Tech	PAK	76	1,455	19	19.14	2	35	4	30	42
6	Ashraf S	KFUE Inf Tech	PAK	76	2,818	28	37.08	6	46	0	21	55
7	Ullah K	Riphah Int U	PAK	74	2,614	24	35.32	6	43	0	16	58
8	Ali Z	Int Isl U	PAK	72	1,698	23	23.58	3	48	0	2	70
9	Saeed M	U Mang Tech	PAK	71	689	16	9.7	0	25	0	3	68
10	Shabbir M	Quaid I Azam U	PAK	57	1,021	18	17.91	0	32	38	16	3
11	Ali MI	Isl M C Boys	PAK	55	2,675	26	48.64	7	41	10	28	17
12	Amin F	Hazara U	PAK	49	1,121	17	22.88	0	29	0	25	24
13	Qiyas M	Riphah Int U	PAK	49	914	17	18.65	2	31	0	7	42
14	Naeem M	Isl U Bahawalpur	PAK	47	457	13	9.72	0	18	0	1	46
15	Khan AQ	U Mang Tech	PAK	40	394	11	9.85	0	13	4	12	24
16	Rehman N	Riphah Int U	PAK	38	276	11	7.26	0	12	2	14	22
17	Zulqarnain RM	U Mang Tech	PAK	38	549	15	14.45	0	22	0	0	38
18	Baig I	LS Econ	PAK	36	898	14	24.94	1	20	11	20	5
19	Yaqoob N	Riphah Int U	PAK	36	488	14	13.56	0	11	4	11	21
20	Ali G	U Education	PAK	35	749	19	21.40	0	25	0	5	30
21	Farid HMA	U Punjab	PAK	34	728	15	21.41	0	22	0	0	34
22	Rahman AUR	U Mang Tech	PAK	34	317	11	9.32	0	13	0	0	34
23	Naz S	U Education	PAK	34	894	18	26.29	0	27	0	11	23
24	Ali Y	GIK Inst	PAK	33	732	19	22.18	0	26	0	7	26
25	Rahman K	Shaheed BB U	PAK	32	578	15	18.06	1	19	0	11	21
26	Hayat T	Quaid I Azam U	PAK	31	2,484	23	80.13	11	30	1	26	4
27	Gulistan M	Hazara U	PAK	31	367	12	11.84	0	14	0	15	16
28	Jaffar A	Superior U Lahore	PAK	30	492	15	16.40	0	17	20	8	2
29	Ishtiaq U	U Mang Tech	PAK	28	132	7	4.71	0	4	0	0	28
30	Safdar B	Ghazi U	PAK	27	201	10	7.44	0	10	0	2	25
R	**Co-Author**	**University**	**Country**	**TP**	**TC**	**H**	**C/P**	≥ 100	≥ 10	**D1**	**D2**	**D3**
1	Pamucar D	U Belgrade	SER	79	1,294	22	16.38	0	37	0	0	79
2	Aslam M	King Khalid U	KSA	63	1,010	16	16.03	2	27	8	10	45
3	Kausar N	Yildiz Tech U	TUR	62	419	10	6.76	0	10	2	6	54
4	Garg H	Thapar U	IND	58	1,701	21	29.33	6	37	0	4	54
5	Jan N	Korea N U Trans	ROK	51	1,839	18	36.06	5	26	0	13	38
6	Khan MB	U Bahrain	BHR	42	661	13	15.74	0	23	0	0	42
7	Alcantud JCR	U Salamanca	ESP	38	1,513	24	39.82	2	34	0	12	26
8	Smarandache F	U New Mexico	USA	33	700	16	21.21	1	17	0	21	12
9	Alkenani AN	King Abdulaziz U	KSA	32	550	14	17.19	0	16	0	4	28
10	Gustavo SG	U Salamanca	ESP	29	447	12	15.41	0	11	0	0	29

Abbreviations: TP = Total publications; TC = Total citations; H = *h*-index; C/P = Citations per publication; D1 = 1989-2014; D2 = 2015-2019; D3 = 2020-2023.

publications and increasing international collaborations underscores the growth and maturity of this research community, setting the stage for even greater advancements in the field.

Among the top 10 international collaborators, it's observed that Dragan Pamucar, from the University of Belgrade, Serbia, stands first and has co-authored 79 publications, received 1,294 citations, with an *h*-index of 22, and 37 papers having citations ≥ 10. The collaborative period for Pamucar is from 2020-2023, and he has published all of his research work during this time, witnessing the fuzzy development and collaboration during this period in Pakistan. Muhammad Aslam from King Khalid University, Saudi Arabia, is the second leading co-author and has published 63 research papers, with 1,010 citations, an *h*-index of 16, 2 papers having ≥ 100 citations, and 27 with ≥ 10 citations. Aslam is the only co-author among the listed collaborators who has long-term relations with fuzzy researchers from Pakistan across all the periods D1-D3. Kausar N., from Yildiz Technical University (Turkey), Harish Garg, from Thapar University (India), Naeem Jan, from Korea National University of Transportation (South Korea), Alcantud JCR., from the University of Salamanca (Spain), Florentin Smarandache, from the University of New Mexico (USA) and Alkkeani AN., from King Abdulaziz University (Saudi Arabia) are the key collaborators and have made significant contributions. Together, these researchers have strengthened Pakistan's global research presence, promoting innovation and cross-disciplinary applications in fuzzy systems. Table 6 lists the 30 most productive universities in fuzzy research in Pakistan, showcasing a range of metrics such as total publications (TP), total citations (TC), *h*-index (H), and the distribution of the number of citations per article. It shows that only one university with more than 500 research papers has 12,787 citations. Similarly, the list consists of one university with one with ≥ 300 publications, two with ≥ 200 publications, and 6 with ≥ 100 publications. University of Punjab (PU) stands first with 509 total publications (TP), 12,787 citations, and the highest *h*-index of 55. The quality of research is evident with 14 publications having ≥ 100 citations and 345 with ≥ 10 citations. During the most recent period, 2020-2023 (D3), the institute contributed 349 publications.

COMSATS University Islamabad (CUI), stands second among the top contributors, with 470 TP, 7,878 TC, and an *h*-index of 42. It has 6 research papers with ≥ 100 citations, and 227 with ≥ 10 citations. International Islamic University (IIUI) is the third most contributor to fuzzy research in Pakistan and has 366 publications, 8,170 citations, and an *h*-index of 44. University of Management and Technology (UMT) placed at number four with TP 291, 3,370 TC, and an *h*-index of 29. Quaid-I-Azam University (QAU) stands at number five with TP 240, 6,302 TC, and an impressive *h*-index of 40. Further, Riphah International University, Abdul Wali Khan University, National University of Science and Technology (NUST), and the University of Lahore (UoL) are showing steady productivity. The metrics of the youngest Lahore Garrison University (LGU) are relatively modest compared to the top institutions, with 47 publications, but its focus on recent contributions has demonstrated its dedication to advancing research in fuzzy systems. Pakistan experienced a rise in fuzzy research over the last four years from 2020-2023 (D3), and a significant rise in fuzzy research has been noticed in Table 6. The increased publication during this time shows the growing innovation and establishing the country as an emerging hub in this field.

TABLE 6. The most productive and influential institutions in fuzzy research in Pakistan

R	Institution	TP	TC	H	C/P	≥ 100	≥ 10	D1	D2	D3
1	U Punjab	509	12,787	55	25.12	14	345	34	126	349
2	Comsats U Islamabad	470	7,878	42	16.76	6	227	66	124	280
3	Int Islamic U Pakistan	366	8,170	44	22.32	17	179	27	67	272
4	U Management Technology	291	3,370	29	11.58	1	110	3	45	243
5	Quaid I Azam U	240	6,302	40	26.26	15	116	65	68	107
6	National U Sci Tech	196	3,452	34	17.61	2	93	30	62	104
7	Riphah Int U	168	1,804	21	10.74	0	64	3	28	137
8	Abdul W K U	157	4,262	38	27.15	6	95	7	57	93
9	Hazara U	140	2,343	26	16.74	0	76	0	57	83
10	U Lahore	125	1,167	19	9.34	0	42	13	19	93
11	U Education	100	1,363	21	13.63	0	47	0	4	96
12	Govt College U Faisalabad	97	811	15	8.36	0	28	0	12	85
13	Govt College U Lahore	90	1,018	15	11.31	2	26	23	11	56
14	Natl U Comp Emerging Sci	77	1,557	21	20.22	2	42	21	23	33
15	Air U Islamabad	74	1,105	18	14.93	0	35	8	16	50
16	U Eng Tech Taxila	67	1,433	21	21.39	1	33	10	18	39
17	U Eng Tech Lahore	62	1,043	17	16.82	1	28	2	16	44
18	GIK Inst Eng Sci Tech	57	1,134	23	19.89	0	39	6	13	38
19	U Peshawar	53	727	12	13.72	1	14	1	18	34
20	U Agriculture Faisalabad	51	970	17	19.02	1	27	1	13	37
21	U Gujrat	50	709	14	14.18	1	22	0	9	41
22	Kohat U Sci Tech	49	678	16	13.84	0	22	0	12	37
23	Virtual U Pakistan	48	596	13	12.42	0	18	1	5	42
24	Lahore Garrison U	47	577	14	12.28	0	23	0	7	40
25	Gomal U	47	387	12	8.23	0	17	0	3	44
26	Lahore U Management Sci	45	734	15	16.31	0	19	25	10	10
27	Khwaja Fareed U Eng Info Tech	44	468	11	10.64	0	13	0	2	42
28	U Malakand	44	373	12	8.48	0	14	0	7	37
29	Bahria U	42	680	16	16.19	0	23	0	12	30
30	Bahauddin Zakariya U	40	962	18	24.05	1	24	2	10	28

Abbreviations in previous tables.

Table 7 presents the 20 most productive co-author institutions in fuzzy research in Pakistan. it includes Institution names, country, TP, TC, *h*-index, different citation metrics, QS world ranking, and Academic Ranking of World Universities (ARWU). Saudi Arabian institutions contribute 50% of the total collaborative publications listed in Table 7, King Abdulaziz University (24%), King Saud University (13%), and King Khalid University (13%) respectively. Among other contributors are China Medical University, China Medical University Hospital, and Shandong University of Finance and Economics (15%) collectively. India's Thapar Institute of Engineering and Technology (5%), Cankaya University Turkey (5%), University of Yazd Iran (5%), University of Belgrade Serbia (5%), Prince Songkla

University Thailand (5%), and Spain's University of Salamanca (5%), respectively.

TABLE 7. The most productive co-author institutions in fuzzy research in Pakistan

R	Institution	Country	TP	TC	H	C/P	≥ 100	≥ 10	QS	ARWU
1	King Abdulaziz U	KSA	266	5,701	38	21.43	13	127	149	151-200
2	King Khalid U	KSA	145	1,814	20	12.51	2	54	601	401-500
3	King Saud U	KSA	144	1,946	20	13.51	2	50	200	101-150
4	China Medical U	CHN	118	1,423	22	12.06	0	53		201-300
5	Umm Al Qura U	KSA	90	885	16	9.83	1	35	559	801-900
6	Taif U	KSA	89	1,021	16	11.47	0	36		201-300
7	China Medical U Hospital	CHN	79	963	19	12.19	0	35		
8	Thapar Inst Eng Tech	IND	73	1,946	23	26.66	7	42	851	
9	Qassim U	KSA	68	637	14	9.37	0	25	781	801-900
10	U Salamanca	ESP	66	1,892	26	28.67	2	49	539	501-600
11	Prince Sultan U	KSA	62	1,148	19	18.52	1	33	681	101-150
12	Princess Nou Abdul U	KSA	61	403	11	6.61	0	15	681	301-400
13	Prince Sattam Abdul U	KSA	58	1,004	18	17.31	0	23		501-600
14	U Belgrade	SRB	47	393	11	8.36	0	12	731	401-500
15	U Yazd	IRN	45	1,417	17	31.49	2	29		
16	Cankaya U	TRK	39	366	12	9.38	0	15		
17	King Faisal U	KSA	39	303	10	7.77	0	10	761	801-900
18	Shandong U Fin Eco	CHN	39	1,258	20	32.26	2	33	316	101-150
19	Hanyang U	KOR	38	533	14	14.03	0	19	162	301-400
20	Prince Songkla U	THA	38	790	17	20.79	0	25	951	801-900

Abbreviations: ARWU = Academic Ranking of World Universities; QS = Quacquarelli & Symonds University Ranking.

Among top QS rankings only King Abdulaziz University lies in the list with QS 149 and an ARWU ranking of 151–200. King Abdulaziz University stood first in the list with 266 publications, 5,701 citations, and an *h*-index of 38. It ranks 149 in QS and 151–200 in ARWU rankings. King Saud University, with 144 TP, 1,946 TC, an *h*-index of 20, and a QS ranking of 200, shows the significance of the collaboration. King Khalid University, with 145 total papers, 1,814 total citations, an *h*-index of 20, and a QS ranking of 601–800, demonstrates the influence of Saudi universities in advancing fuzzy research in Pakistan.

The data in Table 8 showcase the top 30 global collaborations of Pakistan in fuzzy research, indicating TP, TC, *h*-index, C/P, and different periods D1-D3. Among dominant collaborators, Saudi Arabia (KSA) leads with the highest number of publications, 1,124, 16,830 citations, *h*-index of 57, and 14.97 C/P value. KSA contributing 480 papers with $\geq$ 10 citations and published 942 research articles during D3 (2020-2023) period represents the strength of collaborations in fuzzy research in Pakistan. China follows with 679 TP, 17,058 TC, *h*-index 67, and 25.12 C/P value. Contributing 400 papers with 10 citations

and 500 papers in the last four years highlights the depth of their research collaborations. South Korea, Turkey, India, Taiwan, and Malaysia also emerge as productive collaborators and underscore their role in high-quality research collaboration. Globally, Saudi Arabia, China, South Korea, India, Malaysia, and the USA stand out as the most influential co-author countries in terms of TP, TC, *h*-index, and C/P values. The collaboration of Pakistan with East Asia, Middle East and in Europe are 25%, 45% and 15% respectively. The remaining 15% are distributed across North America, South Asia, and other regions.

During D3 (2020-2023) period 75% of the total fuzzy research of Pakistan has been co-authored, reflecting a strong emphasis on collaboration and interdisciplinary efforts. This period highlights the increasing importance of collaboration in addressing intricate research problems and diverse expertise.

4. Mapping fuzzy research in Pakistan with *VOS viewer*

This section presents the visual analysis of various bibliometric indicators. To understand research patterns and trends of any topic, visual bibliometric analysis is very important and can be done using *VOSviewer* software [57]. With the help of *VOSviewer*, one can identify the most cited journals, authors, collaborators, keywords, research areas, and research papers by uncovering the intellectual structure of the research area. The visual analysis of fuzzy research in Pakistan has been done using the Web-of-Science (WoS) core collection database and *VOSviewer* software, and we analysed the key insights of the fuzzy research field. It is divided into three subsections. Subsection 1 presents the visual co-citation analysis of research journals, authors, and documents. Subsection 2 showcases the bibliographic coupling of documents published on fuzzy research in Pakistan, journals publishing fuzzy research in Pakistan, authors publishing fuzzy research in Pakistan, institutions publishing fuzzy research in Pakistan, and countries publishing fuzzy research in Pakistan. Subsection 3 represents the keywords and topical analysis of fuzzy research in Pakistan.

4.1. **Co-citation analysis.**

To visualize and understand the trends of fuzzy research in Pakistan, the co-citation analysis [54] of the authors, documents, and journals is presented in this subsection. Analysing the co-citation of authors, journals, and documents showcases the key contributors, influential works, and research networks in fuzzy set theory and its applications in Pakistan. The co-citation analysis of authors indicates the influential researchers, their collaborative patterns, and their interconnection. It identifies the emerging researchers and their research direction. The network and co-citation analysis of the documents indicate the influential research papers, theories, and methodologies that have been widely accepted and cited by new researchers in writing their work. The co-citation analysis of journals provides an overview of the reputed journals and focus of the Pakistani researchers in advancing fuzzy research to showcase the global academic audience.

Figure 3 presents the co-citation clusters of research journals in fuzzy research in Pakistan, visualized using a *VOS viewer* with a minimum citation threshold of 100 and 100 links. The large green node of the *Fuzzy Sets and Systems* journal identifies the influence

TABLE 8. The most productive country collaborators of Pakistan in fuzzy research

R	Country	TP	TC	H	C/P	≥ 100	≥ 10	D1	D2	D3
-	Pakistan	3,805	72,223	94	18.46	82	1,864	346	850	2,609
1	Saudi Arabia	1,124	16,830	57	14.97	20	480	44	138	942
2	Peoples R China	679	17,058	67	25.12	28	400	24	155	500
3	South Korea	245	4,581	35	18.70	4	128	27	58	160
4	Turkey +Turkiye64	230	2,937	26	12.77	1	100	3	20	207
5	India	194	4,084	32	21.05	8	114	6	26	162
6	Taiwan	187	2,774	28	14.83	3	88	0	12	175
7	Malaysia	179	4,228	35	23.62	5	98	16	43	120
8	Egypt	138	1,988	25	14.41	1	47	2	12	124
9	USA	132	3,797	31	28.77	6	80	17	45	70
10	Iran	125	2,804	28	22.43	2	71	27	35	63
11	Spain	123	2,900	30	23.58	4	78	3	25	95
12	UK+14+9+2	117	2,160	26	18.46	2	61	9	30	78
13	Serbia	104	1,476	23	14.19	0	44	2	4	98
14	Thailand	101	1,777	25	17.59	0	61	3	6	92
15	U Arab Emirates	90	1,353	18	15.03	2	40	0	15	75
16	Yemen	82	658	15	8.02	0	26	0	1	81
17	Italy	71	1,291	21	18.18	1	39	3	14	54
18	Romania	65	954	15	14.68	1	29	10	8	47
19	Australia	64	1,623	21	25.36	2	40	2	19	43
20	Canada	64	1,048	18	16.38	1	31	2	17	45
21	Poland	61	1,937	24	31.75	5	40	9	8	44
22	South Africa	54	434	13	8.04	0	17	3	7	44
23	Iraq	51	549	14	10.76	0	22	0	1	50
24	Ireland	44	560	13	12.73	0	18	3	2	39
25	Vietnam	41	917	17	22.37	1	25	0	7	34
26	Jordan	39	975	12	25.00	2	17	0	3	36
27	Tunisia	35	269	10	7.69	0	11	0	0	35
28	Lebanon	31	338	8	10.90	0	8	0	4	27
29	Nigeria	30	200	7	6.67	0	5	1	4	25
30	Belgium	26	478	9	18.38	1	9	0	6	20

Abbreviations: TP = Total publications; TC = Total citations; H = *h*-index; C/P = Citations per publication; D1 = 1989-2014; D2 = 2015-2019; D3 = 2020-2023.

and significant role in advancing fuzzy research in Pakistan. Another central node in purple represents the co-citations of fuzzy research in *Information Sciences*, and the J*ournal of Intelligent and Fuzzy Systems* reflects the foundational role. The dense co-citation in blue shows that the *International Journal of Intelligent Systems*, the *International Journal of Fuzzy Systems*, *IEEE Transactions on Fuzzy Systems*, and *Symmetry Basel* played an

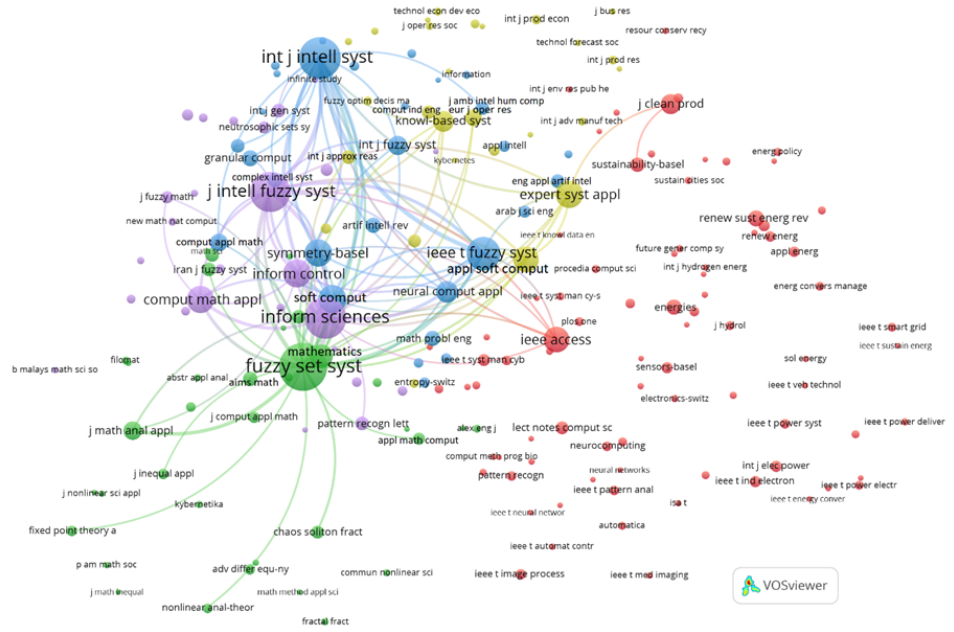

FIGURE 3. Co-citation of journals of fuzzy research in Pakistan: minimum citation threshold of 100 and 100 links.

important role in building fuzzy research.

The interconnection of large links shows that these journals are interrelated and indicate a synergistic growth of the topics, and have a strong hold. The minimum 100 citations threshold represents the expansion of the topic in recent years and the influential journals. The dense and interconnected links underscore the participation of Pakistan in fuzzy research and showcase the interdisciplinary research within the global fuzzy research community. The journals with small nodes in red, which are less co-cited and not interlinked to each other, show the diversification of fuzzy research, its application in recent years, and the focus of the research topics. These smaller interrelated journals have made their dominance in the most recent period 2020-2023.

In Figure 4, the clusters showcase the co-citation threshold of 50 citations and a minimum of 100 documents. The prominent colours like blue, red, and green represent fuzzy set theory, intuitionistic set theory, and soft set theory, along with their extensions, respectively. The dense cluster in blue represents the core research paper of L.A Zadeh was published in *Information & Control*, by Pakistani authors in their work. The interlinks of the journal and documents show that the focus of Pakistani researchers i.e. a foundational work. Another node in red represents the foundational work of Atanassov on intuitionistic fuzzy sets that dominate the network. Similarly, Pythagorean fuzzy sets by Yager, showcase the direction of fuzzy research in Pakistan, and the interlinking of lines represents the influence of the work. The small clusters linked with the dense represent the multidisciplinary approach

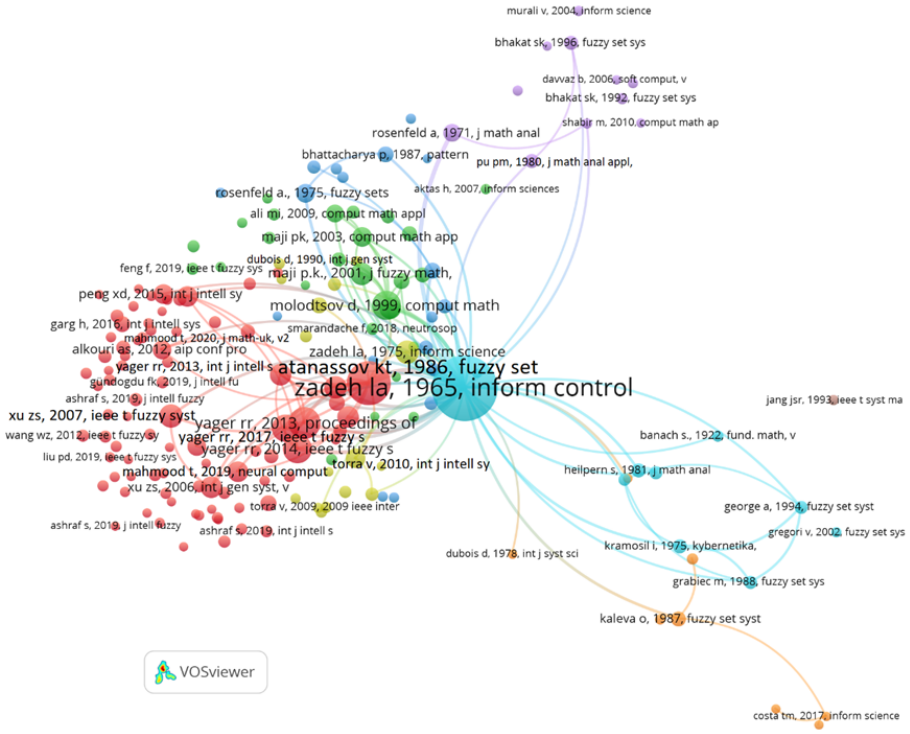

FIGURE 4. Co-citation of documents of fuzzy research in Pakistan: min citation threshold of 50 and 100 links.

of Pakistani researchers. The varying densities of the clusters represent the collaboration and focus of their work in the research domain and foster advancements in both theoretical and applied fuzzy logic domains. Several distinct clusters in green represent the soft set theory and its different extensions, highlighting the key contribution in the area. The interconnection of the work suggests that these documents are cited frequently and indicate the collective influence in shaping the research domain. The co-citation analysis in figure 5 indicates the co-citations of authors in Pakistan's fuzzy research, with a minimum citation threshold of 100 and 100 links. The cluster highlights the key contributors and collaborators in fuzzy research in Pakistan. Lotfi A. Zadeh remains the most influential author with dense connections and strong interlinks with other set theories, indicating the direction of fuzzy research in Pakistan. The connections with the other set theories published by Krassimir T. Atanassov, Harish Garg, Muhammad Akram, Ronald R. Yager, Muhammad Riaz, Tahir Mahmood, and Peide Liu, indicate the expansion of fuzzy research and highlight the extensions and applications providing a stable intellectual foundation in Pakistani research. The dispersed clusters in blue and red showcase the multidisciplinary approach of the researchers in the domain. The clusters of foundational and older documents with recent research underpin the intellectual contribution. Notable Pakistani authors Akram, Mahmood,

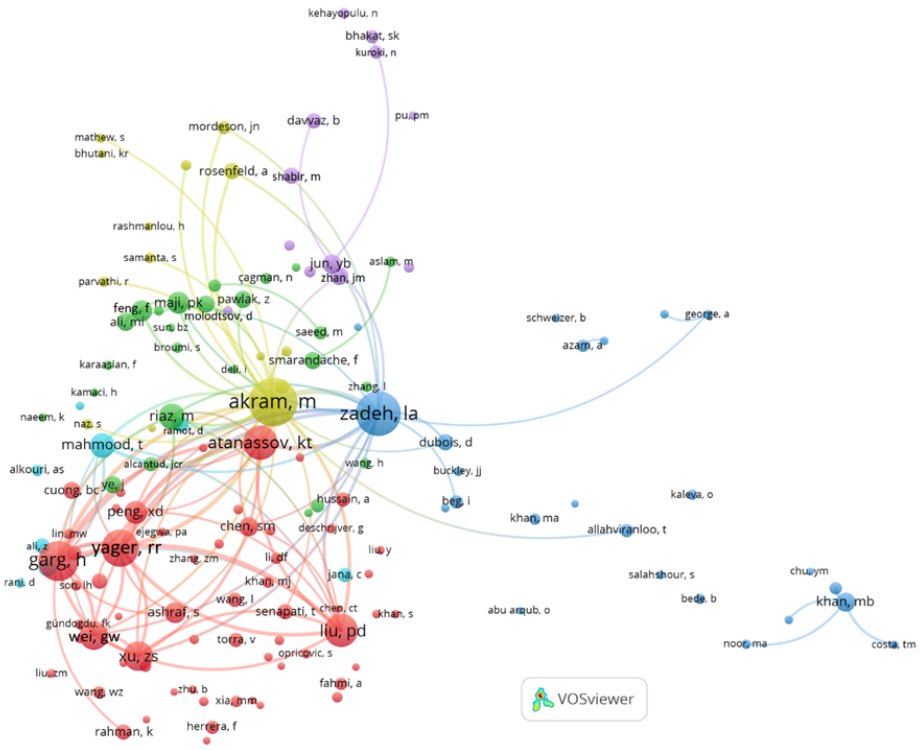

FIGURE 5. Co-citation of authors of fuzzy research in Pakistan: minimum citation threshold of 100 and 100 links.

and Riaz's network clusters and interlinks highlight the contribution in paving the fuzzy research in Pakistan. Other key highlighted clusters and interlinks indicate the collaborative and cumulative approach to advancing fuzzy research in Pakistan, blending theoretical developments with practical implementations. The clusters and networks highlight the fuzzy research development, and the published work is interconnected with Chinese researchers.

The less-dense clusters in light blue represent the application of the work proposed by Mahmood, clusters in green represent the contribution by Riaz, yellow clusters showcase the interconnection of the work published by Akram, and purple networks represent the focus of the research area by Muhammad Shabbir, and collaborators. Overall, dense clusters and strong interlinks indicate the interdisciplinarity of fuzzy research in Pakistan. *VOSviewer* analysis reveals a vibrant academic output by analysing Pakistan's fuzzy research. It shows that 70% of research publications have been published in D3 (2020-2023), underscoring the collaborative and research interest of the topic in Pakistan. The clusters in its visualization highlight the interdisciplinary focus of researchers.

4.2. **Bibliographic coupling.** Bibliographic coupling in bibliometric analysis identifies the research connections and intellectual similarities among the published works [28]. In

this analysis, the similarities between the documents, research journals, authors, contributing countries, and institutions can be explored. The cited references indicate how different works are interconnected conceptually. The analysis helps to find the emerging topics, researchers, and collaborators and map of the growth of knowledge within the field. The bibliographic coupling helps in understanding the interdisciplinary connections and structure, and dynamics of research. The bibliographic coupling of fuzzy research in Pakistan is essential for understanding the research direction, advancements, and collaborative dynamics within this field. This analysis indicates the influential publications, research journals, active researchers or authors, contributing countries, and institutions that are shaping fuzzy research in Pakistan. Moreover, with the help of bibliographic coupling, the interconnectedness of Pakistan's fuzzy research with international collaborators is identified visually.

Figure 6 illustrates the bibliographic coupling of research papers published on fuzzy research in Pakistan. The clusters are formed with a threshold of a minimum of 50 citations and 100 documents. The clusters show the influential documents in nine different colours, indicating the common ten research directions in Pakistan. The influential and prominent paper by Mahmood (2011) on dense green clusters, Faust (2015) on yellow dense nodes, and Feng (2010) on red dense clusters represent the foundational role of these documents in advancing fuzzy research in Pakistan. Other emerging nodes with high densities like purple, orange, pink, blue, and brown clusters and interlinks represent the focus of the studies in decision-making, set theories with their extensions like (Intuitionistic, Pythagorean, q-Rung Orthopair sets), aggregate operators, and application of fuzzy set theories in real life decision problems. The smaller nodes orbiting these dense clusters symbolize the emergence of new contributors, novel research directions, and interdisciplinary applications. They highlight the dynamic and evolving nature of fuzzy research, with scholars branching into unexplored domains and enriching the field. These nodes represent the emerging contributors, interdisciplinary applications, and research directions in fuzzy research.

Overall, the bibliographic coupling analysis underscores the influence and contribution of authors in fuzzy research in different directions and advances the area of research in Pakistan. The map also shows that the documents citing each other are not only expanding the foundational research direction but also extending the works published by Chinese and international authors.

Figure 7 illustrates the bibliographic coupling analysis of journals publishing fuzzy research in Pakistan, with a minimum threshold of 5 documents and 100 links. The emerging and prominent green cluster represents the J*ournal of Intelligent & Fuzzy Systems*, showcasing the influence and prominent role in advancing fuzzy research in Pakistan. Another notable node, represented in orange, is the *IEEE Access* journal, which showcases the importance of fuzzy research. Another dense node around the core clusters represents journals like *AIMS Mathematics, Symmetry Basel*, and *Mathematics*, and their interconnected lines with each other signify the key contribution of the fuzzy research published in them. Some more closely interconnected nodes around the core clusters of the journals *Soft Computing, Granular Computing*, and *Applied Mathematics and Computation* indicate the focus on computational methods in fuzzy research. The link between the journals showcases the

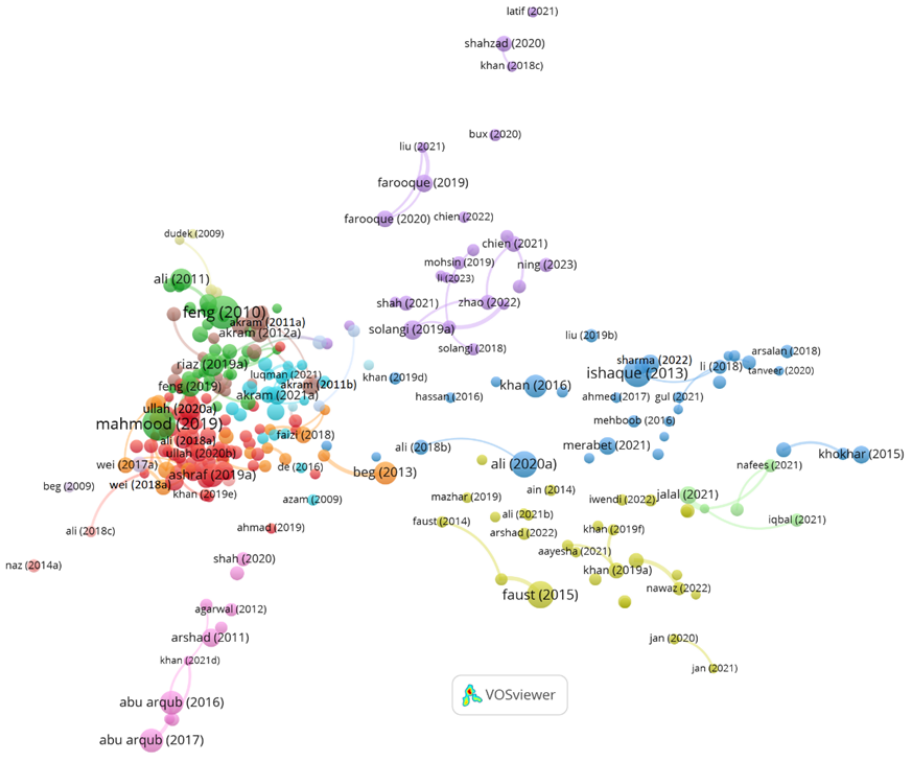

FIGURE 6. Bibliographic coupling of documents published of fuzzy research in Pakistan: minimum threshold of 50 citations and 100 links.

multidisciplinary collaboration of Pakistani researchers in fuzzy research.

Similarly, journals like *Mathematical Problems in Engineering* are in the field of foundational fuzzy mathematics and decision-making. Some non-connected nodes around the prominent clusters indicate the diversification of fuzzy research in recent years. It also shows the collaborative nature and focuses on highly reputed journals of young researchers in the field. The color gradient in the visualization indicates the timeline of publications, red represents the most recent publications, and strong linkage with well-reputed journals. The older contributions in blue and their separate cluster show that the new researchers are focusing the application-based research work in the fuzzy domain.

Figure 8 illustrates the Bibliographic coupling of authors contributing to fuzzy research in Pakistan with a minimum threshold of 15 documents and 100 links. The most prominent and central cluster of the network shows that Muhammad Akram is the most influential and contributing author in fuzzy research from Pakistan. The interlinks with other authors' nodes indicate the groundbreaking development of Muhammad Akram in the fuzzy

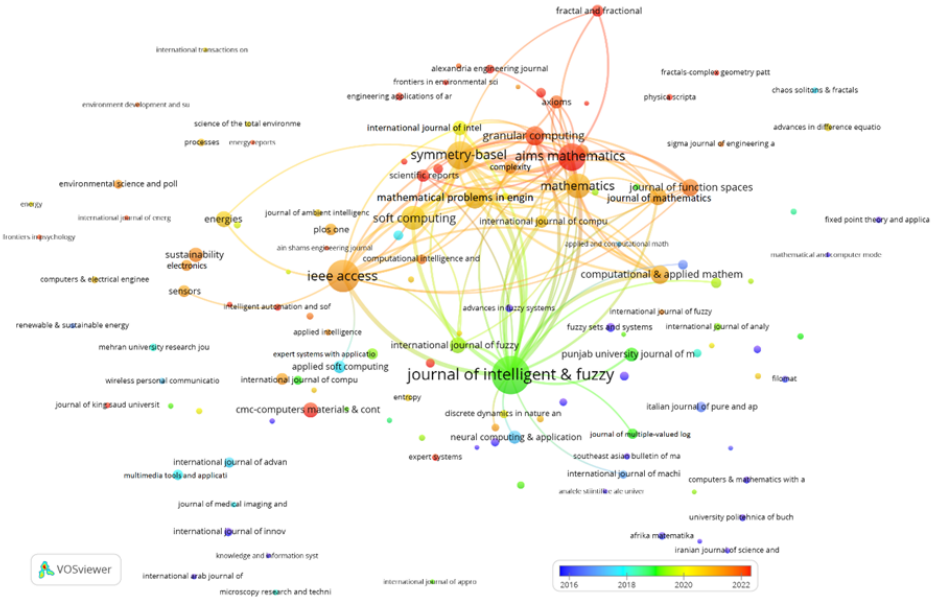

FIGURE 7. Bibliographic coupling of journals publishing fuzzy research in Pakistan: minimum threshold of 5 documents and 100

domain. Among other prominent clusters, the dense node in green represents Saleem Abduallah, the dense node in yellow represents Tahir Mahmood, and the orange cluster representing Muhammad Riaz is also significant in research groups and publications. Among low-density clusters, the nodes of Zeeshan Ali represent the youngest researcher with good achievements in the research field. The cluster of Tabasam Rasid, Muhammad Saeed, Muhammad Shabbir, Kifayat Ullah, and Asghar Khan has shown development in recent years. Notables and some less interconnected nodes representing the contribution in specific fields within the fuzzy research domain. The separate clusters reflect their contributions to specific subfields of fuzzy research, such as decision-making and computational mathematics. The strength of interconnected links between authors showcases the number of co-authorships and intellectual or mutual influences. The colour gradient timeline representing shades ranging from red shows the period of publication from 2020-2023 (D3). The colour gradient with green and blue shows the early development periods. The colour and temporal analysis show the emerging authors and those who are active in fuzzy research. The map also represents the key co-authors from other countries like Harish Garg, Florentin Smarandache, Dragan Pamucar, Muhammad Aslam, Hamiden Abd El-Wahed Khalifa, and Gustavo show the collaborations and country contribution in the development of fuzzy research in Pakistan.

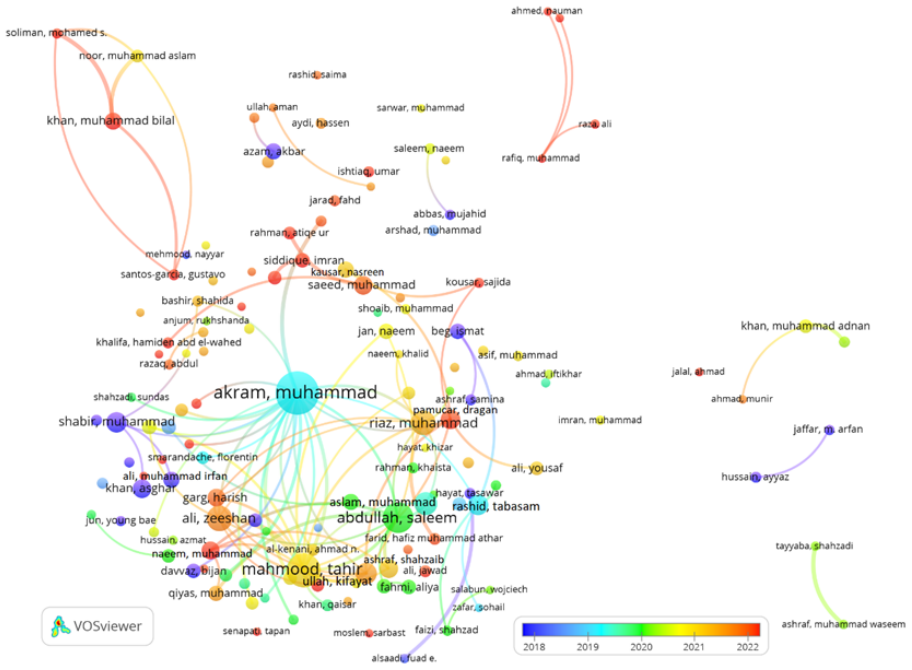

FIGURE 8. Bibliographic coupling of authors publishing of fuzzy research in Pakistan: minimum publication threshold of 15 documents and 100 links.

Figure 9 represents the bibliographic coupling network of institutions publishing fuzzy research in Pakistan with a minimum threshold of 10 documents and 100 links. The emerging cluster in green represents the University of Punjab, indicating its significant contribution to fuzzy research, and the interconnected links between other national and international institutions showcase the extensive collaboration and provide a platform for fuzzy research. Other prominent nodes representing COMSATS University Islamabad, University of Management and Technology, Riphah International University, Quaid-i-Azam University, International Islamic University Islamabad, and Abdul Wali Khan University highlight the fuzzy research participation and collaboration with institutes and authors. Some less dense but interconnected links show the collaboration and influence of the research domain. The nodes that are separate and not connected to many institutes are also contributing to fuzzy research in Pakistan, but in some specific research areas and applications. These visual networks, nodes, and interlinks highlight the leading role of the Universities and collaboration with national and international fuzzy research communities. The map also indicates the key contributing international institutes like King Abdulaziz University, King Khalid University, King Saud University, China Medical University, and Taif University. It also shows the interconnected links between Pakistani Universities and indicates the collaboration and extension support with each other. The color gradient of the network map showcases the

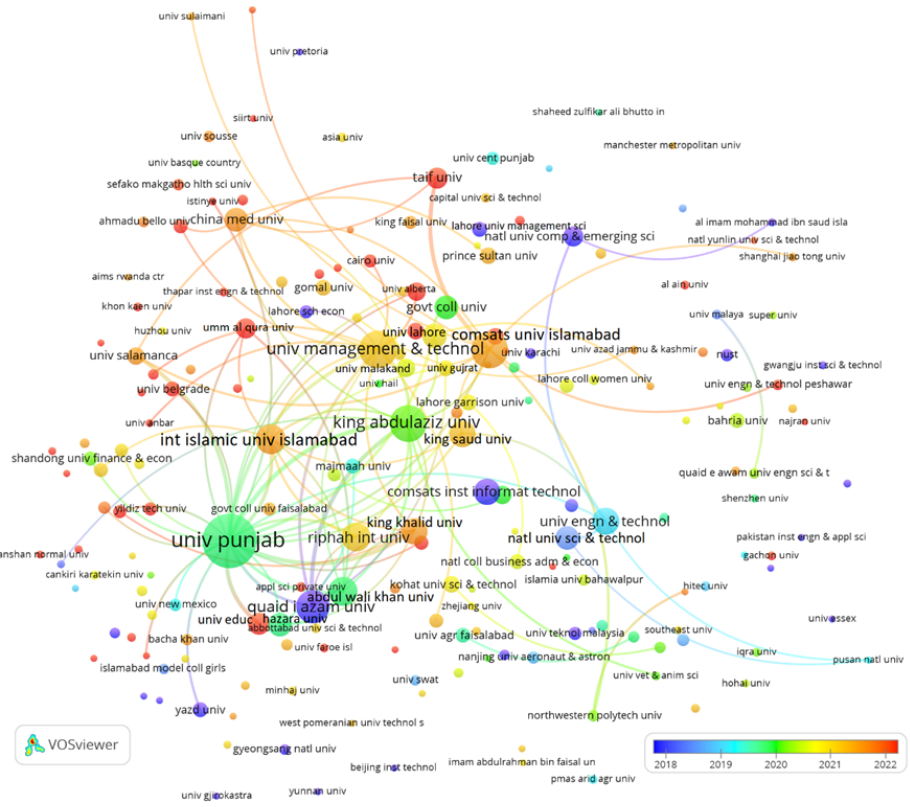

FIGURE 9. Bibliographic coupling of institutions publishing of fuzzy research in Pakistan: minimum publication threshold of 10 documents and 100 links.

timeline of fuzzy research publications in Pakistan, and it shows that the recent period 2020-2023 (D3) is the most productive and emerging year in the contribution of fuzzy research globally. The strong connection between the institutes is represented by lines and shows the intensity of mutual documents and citations.

Figure 10 shows the network of bibliographic coupling of countries publishing fuzzy research with Pakistan, plotted with a minimum publication threshold of 5 documents and 100 links. The bibliographic coupling shows how research papers and references are linked with each other.

The most prominent and emerging node in yellow, representing Saudi Arabia and its interconnected links with Pakistan, shows the influence and contribution of fuzzy research in Pakistan. Another dense node in green representing the People's Republic of China indicates the well-established contribution and collaboration in fuzzy research. Some more notable clusters on the map reveal strong relations with South Korea, Malaysia, the USA,

Egypt, USA, India, Iran, Turkey, and Serbia. The interlinks of these countries with thick lines show that the documents from these countries cited fuzzy documents of Pakistan most frequently, and thin lines show fewer citations. Interestingly, the contribution of European countries like Italy, Spain, Germany, Serbia, Belgium, Hungary, and Ireland reflects broader international engagement in fuzzy research. The visual analysis also reflects the global participation and the impact of the research published by Pakistani authors. The less connected or separate nodes show the diversity and impact of research in their documents.

Analysing the colour gradient of the map representing grades ranging from blue to red indicates the most recent years (2020-2023). The density maps, along with networks, show the overall impact and contribution of the Pakistani authors in fuzzy research globally. The emerging and new connections with Canada and Vietnam. Australia and Bangladesh show growing research interest and mutual contribution.

The map indicates the global relations of Pakistan spanning from Asia, Europe, North America, Africa, and Australia, making it an emerging country in the development of fuzzy research. The interconnected links between countries on the map with less dense nodes as collaborators show the growing interest in fuzzy research, making it a pivotal hub for advancing research in this field.

4.3. **Keyword and topical analysis.** This subsection presents an insightful analysis of fuzzy-related keywords indexed in Scopus filtered through specific criteria. Using the keyword "fuzz*" in the search category of keywords, documents, and abstracts, a total of 449,320 documents were found. To further refine the data, we excluded the documents from "2024-2025", resulting in 418,115 documents. By filtering the data, only "articles" and "review articles" were selected, resulting the 233, 139 documents. Further selecting the publication stage to "finalized" results 232,398 documents, finally, from the countries and territories filtering option, "Pakistan" was selected, resulting in 3,853 documents. Further discussion and explorations are based on these numbers, including graphical and bibliometric analysis of the Table. 9, Table. 10, and Figure 11-14. The study presents the co-occurrence analysis of the topical and keyword studies of fuzzy research in Pakistan, which indicate the thematic structure and insights along with research-focused areas. The analysis includes the study of the co-occurrence of author keywords of fuzzy research in Pakistan in different time frames ranging from 1989 to 2023, and research areas have been compared with global standing subject-wise. The analysis reveals that the fuzzy research revolves around prominent keywords like fuzzy sets, aggregate operators, decision-making, multi-criteria decision-making (MCDM), fuzzy modelling, and fuzzy neural networks. These keywords underscore the importance of fuzzy theories in various disciplines of real-world problems. The most emerging and prominent keywords in fuzzy research in Pakistan are fuzzy logic, fuzzy sets, and fuzzy systems, and used to solve uncertainties in decision-making problems like engineering, management sciences, and public health problems that align with global trends. Machine Learning, Neural Networks, fuzzy neural networks, Artificial Intelligence (AI), and fuzzy clustering are the prominent keywords during the 2020-2023 period, indicating the growing focus of the studies with the integration of fuzzy

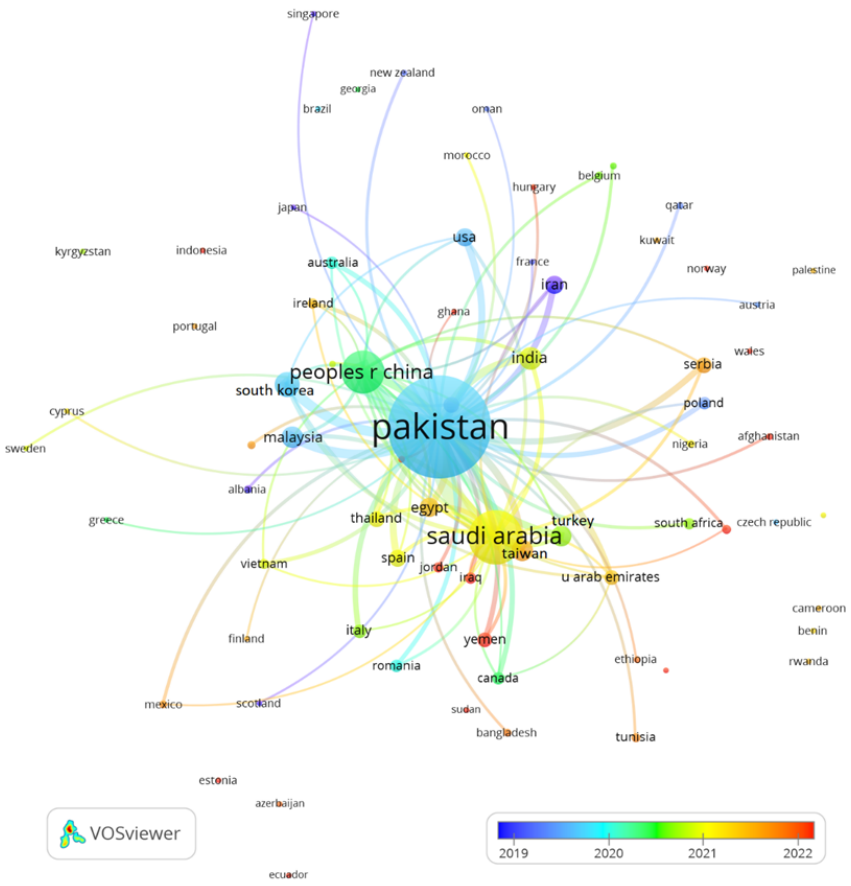

FIGURE 10. Bibliographic coupling of countries publishing of fuzzy research in Pakistan: minimum publication threshold of 5 documents and 100 links.

systems. It also indicates that the researchers from Pakistan are exploring the application of fuzzy research and its application in solving real-world problems, and using AI models to enhance the handling of problems. Table. 9 presents the frequently used keywords by Pakistani authors in fuzzy research, which indicate the trends and focus of the research domain. The metrics in the table show that the focus of Pakistani researchers on fuzzy is inclined toward foundational theories. The keywords "Fuzzy Set + Fuzzy Sets + Fuzzy Systems + Fuzzy Information" with 1,258 publications, 33,438 citations, and an *h*-index of 81, reflect the foundational role in the theory. Followed by the keywords "Decision Making + Decision Makings + Decision-Making + Decision Problem + Decision Support System + Decision Theory" with 1,059 publications, 27,351 citations, and an *h*-index of 75, showcase the significant contribution in decision-making methods. Similarly, "Fuzzy Control + Fuzzy Inference + Fuzzy Logic Controller + Fuzzy Logic System + Fuzzy Rules"

indicates the foundation role in control systems. It indicates the emerging keywords like "Mathematical Operators", "Aggregation Operator + Aggregation Operators", "Algorithm + Algorithms", "Soft Set + Soft Sets", and "Intuitionistic Fuzzy Set" with high citations-per-publication (C/P) ratio. The keywords "Artificial Intelligence", "Computer Circuits", "Diagnosis", "Numerical Methods", "Graph Theory", and "Geometry" represent the interdisciplinary research within the domain. The practical implication of the theory can be reflected with the keywords "Uncertainty", "Multi-Criteria Decision-Making + MCDM", and "Multi-Criteria Group Decision Making + MCGDM". Some keywords with smaller but promising values include "Membership Functions", "Algorithms", "Optimization", Fuzzy Neural Networks", and Statistical Methods" showcase the focus on refining computational methods and merging fuzzy theories with modern technologies and methodologies. Overall, the table. 9 indicates the balance in advancing the foundational fuzzy theories with application to emerging computational, uncertainty, and decision-making studies.

Figure 11 presents the co-occurrence map of the author keywords in fuzzy research from Pakistan, and it's plotted with a minimum occurrence threshold of 10 and 200 links. The network map indicates the thematic structure and trends of fuzzy research by Pakistani authors. The dense cluster in blue reveals that the most prominent topic of discussion in fuzzy research in Pakistan is "fuzzy logic." The density of the cluster and the commonly cited documents that are interconnected showcase the influence of the word and its role in the development of the literature. Among other prominent clusters on the map representing decision-making, fuzzy sets, aggregation operators, MCDM, and soft sets, showcase the backbone of fuzzy research in Pakistan. Notably, the cluster of machine learning (ML) and artificial Intelligence is also emerging, representing the growing interest of researchers in this domain. The map reveals that some less dense but connected links indicate the authors' focus on multidisciplinary research in solving complex problems. The colour gradient of the map ranges from green to red on the map representing the most recent years (2020-2023), which shows the diverse study and application of fuzzy research in Pakistan. The analysis reveals that Pakistani authors are paving the future studies by solving unexplored areas of research, and strategically focusing on AI and ML techniques.

Figure 12 represents the co-occurrence network of author keywords of fuzzy research in Pakistan from 1989 to 2014 with a minimum occurrence threshold of 2 and 100 links. The visualization shows the distinct clusters of keywords, like fuzzy logic, fuzzy mapping, fuzzy sets, soft sets, fuzzy fixed points, intuitionistic fuzzy sets, and fixed points, highlighting the theoretical advancements and the prominence of mathematical foundations. The small clusters show the extensions of fuzzy algebra and its applications, indicating the unique and strong networks of the fuzzy algebra interest and its development in Pakistan. During the period 1989-2014, fuzzy research in Pakistan was at the foundational stage and it was growing towards the application-oriented research areas, such as image restoration, fuzzy logic controllers, segmentation, and computer-aided diagnosis.

The clusters and interlinks of the map indicate the historical development of theoretical and applied computation and the direction of fuzzy research. The distinct cluster showcases the explorations of interdisciplinary applications during the early developmental stages of

TABLE 9. The most productive and influential keywords

R	Keyword	TP	TC	H	C/P	≥ 100	≥ 10	D1	D2	D3
1	Fuzzy Sets	1,258	33,438	81	26.58	52	759	120	305	833
2	Decision Making	1,059	27,351	75	25.82	40	656	7	195	776
3	FLC	885	21,077	66	23.82	22	516	104	231	550
4	Mathematical Operators	400	10,415	53	26.04	17	258	3	53	344
5	Aggregation Operator + Operators	355	9,808	54	27.63	17	232	0	48	307
6	Algorithm + Algorithms	175	4,432	37	25.33	3	115	20	65	90
7	MCDM	160	4,096	37	25.60	6	107	0	21	139
8	Computer Circuits	158	3,084	32	19.52	2	87	7	60	91
9	Uncertainty	155	2,640	27	17.03	3	82	3	9	143
10	Soft Set + Soft Sets	148	3,880	31	26.22	6	72	13	30	105
11	Intuitionistic Fuzzy Sets	147	5,162	38	35.12	12	92	11	35	101
12	Artificial Intelligence	136	3,638	35	26.75	6	77	9	37	90
13	Statistical Methods	129	3,928	37	30.45	7	81	0	22	107
14	Fuzzy Neural Networks	127	4,265	37	33.58	6	87	4	33	90
15	Group Decision Making	123	3,436	32	27.93	3	94	0	38	85
16	Fuzzy Set Theory	122	3,561	27	29.19	9	69	4	25	93
17	Numerical Methods	120	3,175	31	26.46	4	83	1	21	98
18	Graph Theory	117	3,108	29	26.56	5	80	9	44	64
19	Linguistics	116	3,099	29	26.72	3	73	3	23	90
20	Controllers	112	2,398	28	21.41	1	67	5	34	73
21	Multi Attribute Decision Making	106	3,253	33	30.69	4	82	0	17	89
22	Intuitionistic Fuzzy	104	2,233	27	21.47	1	61	6	31	67
23	Uncertainty Analysis	104	2,736	29	26.31	4	56	4	15	85
24	Algebra	103	1,495	20	14.51	1	44	30	32	41
25	Membership Functions	101	2,204	26	21.82	3	59	9	28	64
26	Diagnosis	97	3,182	29	32.80	7	60	3	17	77
27	Fuzzy Information	96	2,863	32	29.82	4	60	0	14	82
28	Geometry	96	3,100	32	32.29	5	62	0	20	76
29	Optimizations + Optimisation	95	1,590	24	16.74	0	44	4	27	64
30	MCGDM	64	1,255	21	19.61	0	34	0	0	64

Abbreviations: TP = Total publications; TC = Total citations; H = *h*-index; C/P = Citations per publication; D1 = 1989-2014; D2 = 2015-2019; D3 = 2020-2023.

fuzzy research in Pakistan. It shows the bidirectional influence of fundamental fuzzy set theories, their extensions, and the concepts on real-world problem-solving and vice versa. Fig. 13 illustrates the co-occurrence map of authors' keywords in fuzzy research from Pakistan plotted for 2015–2019, with a minimum occurrence threshold of 3 and 100 links. This phase of fuzzy development in Pakistan shows more interlinks and connected nodes with emerging keywords like fuzzy logic, fuzzy sets, decision-making, algorithms, aggregation operators, MCDM, soft sets, fuzzy fixed points, and the TOPSIS technique.

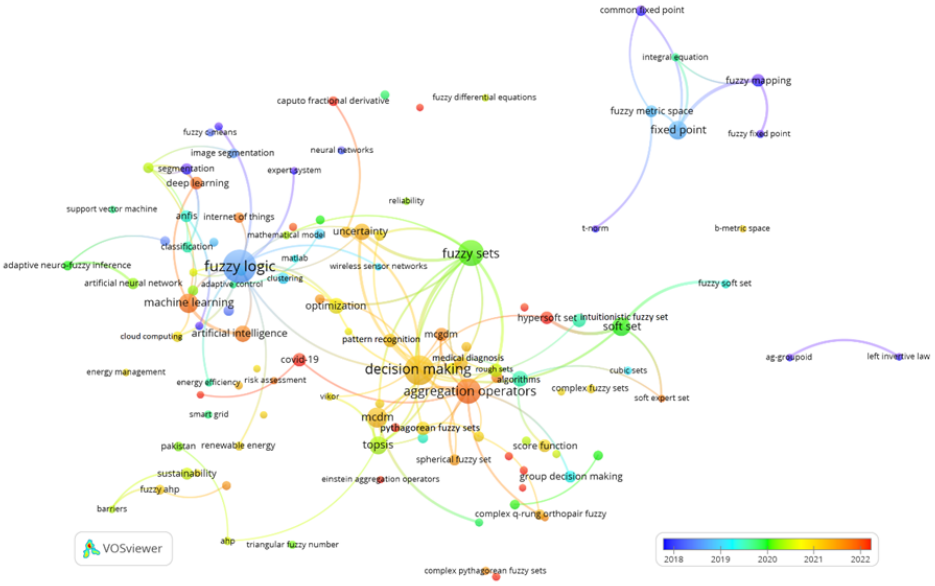

FIGURE 11. . Co-occurrence of author keywords of fuzzy research in Pakistan: minimum occurrence threshold of 10 and 200 links.

This illustrates the application of early theoretical works and multidisciplinary research in the domain. It's proof of the growing fuzzy research methodologies with optimization techniques during the 2015-2019 period by exploring more areas of relevance. The map highlights the extensions of fuzzy set theory, including cubic sets, bi-polar fuzzy sets, intuitionistic fuzzy sets, rough sets, spherical fuzzy sets, hesitant fuzzy sets, m-polar fuzzy sets, and application of these sets by proposing MCDM methodologies like TOPSIS, aggregation operators, distance and similarity measures. This period showcases the researcher from Pakistan exploring the interdisciplinary applications by proposing a mathematical model with a fuzzy research domain. The map highlights the transition towards problem-solving and fuzzy methodologies as proof of the diversification of fuzzy research in Pakistan.

Figure 14 presents the co-occurrence map of author keywords in fuzzy research in Pakistan, ranging from 2020–2023, showing significant insights into the research focus and interconnected links of documents, authors, and institutes within the field. The emerging cluster on the map indicates that the decision-making means that most documents are published or cited with this keyword. Other prominent keywords are fuzzy sets, aggregation operators, fuzzy logic, and soft sets, indicating the central role in fuzzy research. The interconnected nodes emphasize the development of decision methods to support the multi-criteria decision-making (MCDM) process in complex and real-life problems. The small cluster around the fuzzy set and soft set represents the active development of the extension and its application in the decision process.

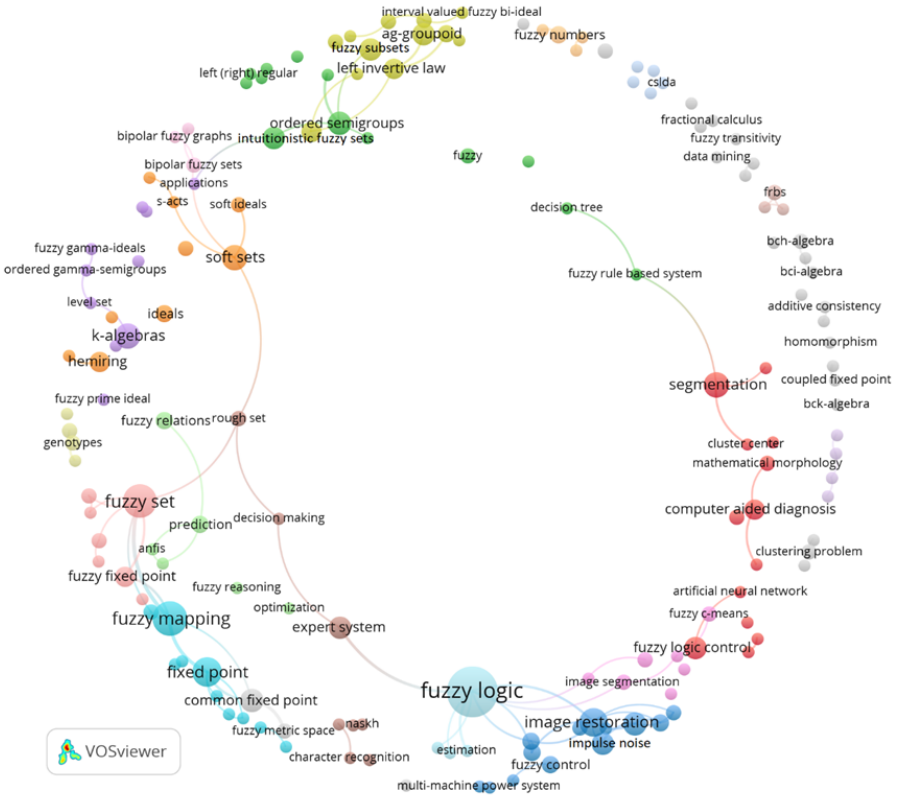

FIGURE 12. Co-occurrence of author keywords of fuzzy research in Pakistan (1989-2014): minimum occurrence threshold of 2 and 100 links.

Notably, the nodes of Artificial Intelligence (AI) and Machine Learning (ML) are also emerging, showcasing the authors' interest in developing methods to solve data-driven problems, such as prediction, classification, and ranking. The indication of neural networks and deep learning shows the trends of fuzzy research toward advanced and hybrid ML techniques.

The less interconnected clusters showcase the diverse applications of fuzzy theories in tackling the uncertainty and decision-making problems in real. Overall, the map illustrates vibrant and interconnected research in Pakistan's fuzzy research community, with a balance between theoretical advancements and practical applications.

Table 10 presents the top 30 research areas in fuzzy research from Pakistan, revealing significant contributions across diverse disciplines. The data shows that Computer Science is a leading and emerging field with 1,544 documents and 34,845 citations. The emphasis on computational methods in fuzzy research and their application is the cornerstone of

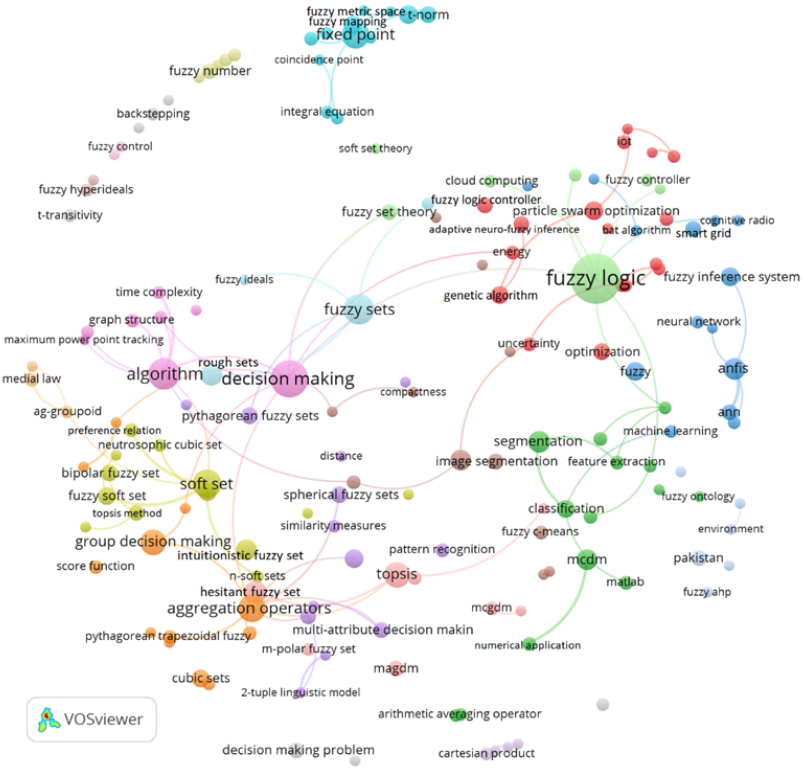

FIGURE 13. Co-occurrence of author keywords of fuzzy research in Pakistan (2015-2019): minimum occurrence threshold of 3 and 100 links.

the study. Globally, Computer Science is also a leading research area that witnesses the alignment of Pakistani fuzzy research. The second most productive area is Mathematics, showcasing the importance of the theoretical and methodological development of the study. Globally, it ranks three while Pakistan at number two illustrates the backbone of the foundational development and progress of the research during different periods. Engineering ranks third, with 847 documents and 14,733 citations showcasing the focus on integrating fuzzy systems into industrial and practical scenarios. It stands at number two globally, showing the growing influence of research in the field. The research area Science Technology (Other Topics) highlights the multidisciplinary applications of fuzzy to indicate the influence of emerging technologies. The analysis highlights the notable contribution of the Pakistani fuzzy research community to the Science & Technology, Mathematics, Physics, Chemistry, Computer Science, Environmental, and Telecommunication sectors, which are well-aligned with global research trends and rankings. These areas highlight the focus on advancing theoretical works, applications to real-life issues, and innovation in these areas.

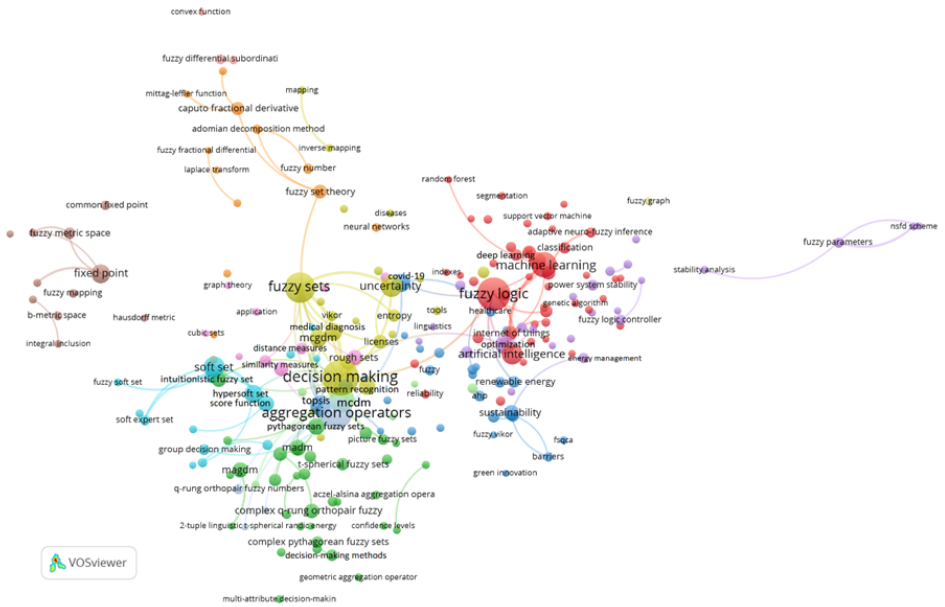

FIGURE 14. Co-occurrence of author keywords of fuzzy research in Pakistan (2020-2023): minimum occurrence threshold of 5 and 100 links.

On the other hand, subject areas like Business Economics, Optics, Robotics, Food Sciences, Biochemistry, and Social Sciences are fewer contributors and appear to be less aligned with global research trends. The limited contribution and representation of these fields in global research communities need enhanced focus and strategic investment. The exploration of global economic trends and business models, and emphasis on sustainable agriculture practices, nutritional advancements, and biotechnology should be addressed to provide Economic, Food, and Health security and stability. The study of Social Sciences indicates a gap in research addressing societal, cultural, and economic issues from a global perspective. Overall, the topical and keyword analysis reveals the focus of the area of research and its alignment with global trends in fuzzy theories. Pakistani researchers are advancing fuzzy research by exploring interdisciplinary applications and their practical implementations to robust the solution to solve real-life issues and generative AI models.

Figures 10-14 and Table 9 reveal a shift in research priorities over time. Early studies (1989–2004) were primarily centred around foundational concepts such as fuzzy sets, fuzzy logic, and membership functions, indicating a strong theoretical orientation. In contrast, recent years (2015–2023) show a marked rise in applied themes, with keywords like decision-making, artificial intelligence, machine learning, and multi-criteria decision analysis emerging as dominant. Meanwhile, research topics such as control systems, automata, and approximate reasoning appear to have declined in frequency, suggesting a transition

TABLE 10. Top 30 leading research areas in fuzzy research from Pakistan

R	Research Area	TP	TC	H	C/P	≥ 100	≥ 10	D1	D2	D3
1	Computer Science	1,544	34,845	77	22.57	49	868	149	421	974
2	Mathematics	1,088	13,693	47	12.59	9	406	112	170	806
3	Engineering	847	14,733	52	17.39	11	440	58	175	614
4	Science Technology Other Topics	493	8,748	44	17.74	11	228	36	114	343
5	Telecommunications	289	4,485	31	15.52	3	155	6	68	215
6	Environmental Sciences Ecology	152	3,886	36	25.57	4	95	0	27	125
7	Physics	130	2,089	26	16.07	0	60	16	21	93
8	Materials Science	118	1,498	21	12.69	0	49	3	8	107
9	Automation Control Systems	113	2,962	31	26.21	4	74	9	36	68
10	Energy Fuels	112	4,197	35	37.47	10	72	3	28	81
11	Chemistry	73	1,408	21	19.29	1	41	1	12	60
12	Mathematical Computational Biology	68	760	16	11.18	0	28	2	11	55
13	Instruments Instrumentation	53	981	19	18.51	0	31	0	13	40
14	Operations Research Management Science	46	1,166	20	25.35	1	31	8	9	29
15	Business Economics	44	1,299	21	29.52	1	34	0	6	38
16	Water Resources	35	544	13	15.54	0	15	0	8	27
17	Neurosciences Neurology	28	800	13	28.57	1	13	1	6	21
18	Geology	21	317	8	15.10	0	8	0	2	19
19	Agriculture	18	165	8	9.17	0	4	4	7	7
20	Imaging Science Photographic Technology	18	256	10	14.22	0	10	6	6	6
21	Life Sciences Biomedicine Other Topics	17	469	8	27.59	1	7	8	4	5
22	Radiology Nuclear Medicine Medical Imaging	15	173	8	11.53	0	8	2	6	7
23	Construction Building Technology	13	344	11	26.46	0	12	0	2	11
24	Mechanics	13	263	6	20.23	1	5	4	5	4
25	Meteorology Atmospheric Sciences	13	143	6	11.00	0	4	0	1	12
26	Thermodynamics	13	481	8	37.00	2	8	2	2	9
27	Plant Sciences	11	83	6	7.55	0	1	10	0	1
28	Remote Sensing	11	220	7	20.00	0	7	3	2	6
29	Biotechnology Applied Microbiology	10	81	4	8.10	0	3	3	2	5
30	Medical Informatics	10	346	8	34.60	1	8	3	4	3

Abbreviations:TP = Total publications; TC = Total citations; H = *h*-index; C/P = Citations per publication; D1 = 1989-2014; D2 = 2015-2019; D3 = 2020-2023.

from classical fuzzy theory to more application-driven areas aligned with global technological trends. Table 10 highlights the emerging trends in fuzzy research in Pakistan. At the top level, subject areas such as "Computer Science," "Mathematics," and "Engineering" have shown a significant increase in the total number of publications during 2020–2023, as indicated in column D3. At a secondary level, areas like "Materials Science," "Environmental Sciences & Ecology," and "Telecommunications" have also emerged as growing fields of interest.

5. Conclusion

The section is divided into three subsections. The first subsection describes the main findings, and the contribution of this article. Second, the subsection investigates the practical implications of the study. Lastly, the results are summarized with future directions.

5.1. **General Discussion.** The article provides a bibliometric analysis of fuzzy research in Pakistan over 34 years, ranging from 1989-2023, highlighting the significant development and growing research interest. The early research from 1989 to 2014 evolved around foundational concepts like fuzzy sets, fuzzy logic, fixed points, and fuzzy algebra, and published only 346 documents. However, most recent years, 2019-2023 the number of published documents reached 2,609. The study demonstrates the diversification of fuzzy research into applied domains such as decision-making, multi-criteria decision-making (MCDM), artificial intelligence (AI), machine learning (ML), image processing, and segmentation, indicating the configuration of academic networks and the growing importance of collaboration.

The analysis reveals that Akram, with 349 documents and 10,224 citations is the most productive researcher from Pakistan, followed by Mahmood, with 217 documents and 6,508 citations. Saleem Abdullah, Muhammad Riaz, and Tabasam Rashid are prominent researchers in the field. Emerging researchers like Saeed, and Zeeshan, who published 95% of their work during the 2019-2023 period, are contributing to the fuzzy world. In the list of most productive and influential institutions from Pakistan in fuzzy research, the University of the Punjab (PU) ranked first with 509 publications and 12,787 citations, followed by Comsat University Islamabad (CUI) with 470 documents and 7,878 citations. International Islamic University Islamabad (IIUI), University of Management and Technology (UMT), Quaid-I-Azam University (QAU), Riphah International University, Abdul Wali Khan University (AWKU), National University of Science and Technology (NUST), and the University of Lahore (UoL) are also prominent universities in fuzzy research productivity from Pakistan.

The most prominent co-authors are Pamucar, from the University of Belgrade (Serbia), with 79 publications, and Aslam, from King Khalid University (KSA) with 63 documents respectively. Kausar from Yildiz Technical University (Turkey), Garg from Thapar University (India), Jan from Korea National University of Transportation (South Korea), Alcantud from the University of Salamanca (Spain), Smarandache from the University of New Mexico (USA) and Alkkeani from King Abdulaziz University (Saudi Arabia) are the key collaborators and have made significant contributions. Saudi Arabia appears as a most collaborative country, and its institutes contribute to 50% of the total collaborative work, King Abdulaziz University 24%, King Saud University 13% , and King Khalid University 13%, with 266, 145, 144 co-authored documents respectively. China Medical University and China Medical University Hospital reflect the prominent collaborators from the People's Republic of China. Among other collaborators, South Korea, India, Malaysia, Thailand, Iran, Turkey, Taiwan, Egypt, Spain, Serbia, Australia, Jordan, Nigeria, and the USA as the

most influential co-author countries. In the top QS rankings, only King Abdulaziz University lies with QS 149 and an ARWU 151–200.

The *Journal of Intelligent & Fuzzy Systems* leads as the most popular journal, featuring 303 publications, 5,721 citations, and an impressive *h*-index of 39. The period 2020-2023 has observed a surge in publications, particularly in journals like *Symmetry Basel* (160 papers), as well as *AIMS Mathematics* (157 papers), *Mathematics* (125 papers), and *Granular Computing* (76 papers). The top cited work, "Soft Sets Combined with Fuzzy Sets and Rough Sets: A Tentative Approach" by Ali MI along with co-authors, published in 2019 in the prestigious journal of *Soft Computing* (SC), with 554 citations, shows the critical impact of the foundation of fuzzy research in Pakistan. Mahmood et al.'s publication "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets" in *Neural Computing & Applications*, published in 2019, received the highest 91.67 citations per year (C/Y). Table 3, also shows that the influential papers are published in highly reputed journals including *IEEE Transactions on Fuzzy Systems, Knowledge-Based Systems, Soft Computing, Information Fusion, Renewable & Sustainable Energy Reviews, Information Sciences, Journal of Intelligent & Fuzzy Systems*, and *International Journal of Hydrogen Energy*, representing the maturity of researchers and the influence of the work. The *Journal of Intelligent & Fuzzy Systems, Soft Computing*, and *International Journal of Fuzzy Systems* are the most frequent journals where influential papers have been published by fuzzy researchers in Pakistan. In particular, Yager and Zadeh appear multiple times in the top 40 papers, showcasing their reputation as leading researchers in fuzzy systems and their recognition by Pakistani researchers. It also shows that the focus of many authors in Pakistan is linked to their foundational papers. Another key point is that many Pakistani authors, such as Mahmood, Ali, Ullah, Riaz, and Akram, are Pakistani researchers and have received high recognition from the Pakistani fuzzy research community.

The visual analysis of fuzzy research in Pakistan has been done using the Web of Science (WoS) core collection database and *VOSviewer* software, and we analyzed the key insights of the fuzzy research field. The co-citation analysis of the documents, authors, and journals indicates the influential research papers, theories, and methodologies opted by the authors in reputed journals by Pakistani researchers in advancing fuzzy research and to showcase to a global academic audience. The bibliographic coupling analysis of documents, authors, institutes, and journals underscores the influence and contribution of authors in fuzzy research in different directions and advancing the area of research in Pakistan. The co-occurrence analysis of the topical and keyword studies of fuzzy research in Pakistan, indicate the thematic structure and insights along with research-focused areas. The analysis reveals that the fuzzy research revolves around prominent keywords like fuzzy sets, aggregate operators, decision-making, multi-criteria decision-making (MCDM), fuzzy modelling, and fuzzy neural networks.

5.2. **Practical Implications.** This study offers practical implications for decision-makers and institutions to enhance fuzzy research in Pakistan. Institutions like the University of

Punjab, COMSATS University Islamabad, and the International Islamic University Islamabad have the potential to secure funding from foreign research funding agencies to support collaborative research initiatives.
Educational institutions should introduce courses at the graduate level on MCDM and fuzzy modeling, and can use the findings to update the curriculum by integrating fuzzy logic and fuzzy sets into ML and AI programs. This can increase the number of publications and elevate their global standing.

Specialized courses and certifications can equip students to solve multi-disciplinary research problems and collaborate in industry. The study highlights the opportunities to attract foreign collaboration by strengthening ties with countries like China, Saudi Arabia, South Korea, Malaysia, and Iran to access advanced research programs and methodologies to expand the global network. Universities and industry partnerships can drive innovation and create social impact. The use of fuzzy logic in real-world problems provides opportunities to industries like energy, health care, agriculture, economics, and robotics and gains benefit from the growing impact of the application of fuzzy research. Publishing trends in high-impact journals can offer a roadmap for improving research visibility. Institutions can encourage strategic publishing to improve global rankings and citation impact. Tools like VOSviewer can help monitor progress and identify gaps, fostering a culture of evidence-based decision-making in research planning.

Overall, the findings offer actionable insights for funding agencies, policymakers, and universities in Pakistan. Funding bodies can support emerging areas and underrepresented disciplines, such as fuzzy applications in social sciences and economics. Universities should encourage international collaborations and strategic publishing to enhance global visibility and citation impact.

5.3. **Limitations and Future Research.** This study provides a general overview of fuzzy research in Pakistan up to 2023; future emerging topics, authors, and institutions may vary due to publication trends. The study is done using the Web of Science (WoS) core collection database, which may exclude significant contributions from platforms like Scopus or Google Scholar. The focus on citations, number of documents, and collaboration can overlook the qualitative impact of research, as well as the practical applications and societal benefits of fuzzy research in Pakistan. The current ranking is based on the total number of research papers (TRP), and incorporating total citations could lead to a different ranking in the future.

Future research could address these limitations by incorporating data from multiple bibliographic databases to provide a more comprehensive view. Qualitative assessments and case studies on industrial or societal applications would enrich the findings, while comparisons with leading countries could identify gaps and growth opportunities.

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